NetComplex: A Complexity Metric for Networked System Designs

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Abstract

The systems and networking community treasures "simple" system designs, but our evaluation of system simplicity often relies more on intuition and qualitative discussion than rigorous quantitative metrics. In this paper, we develop a prototype metric that seeks to quantify the notion of algorithmic complexity in networked system design. We evaluate several networked system designs through the lens of our proposed complexity metric and demonstrate that our metric quantitatively assesses solutions in a manner compatible with informally articulated design intuition and anecdotal evidence such as real-world adoption.

1 Introduction

The design of a networked system frequently includes a strong algorithmic design component. For example, solutions to a variety of problems – routing, distributed storage, multicast, name resolution, resource discovery, overlays, data processing in sensor networks – require distributed techniques and procedures by which a collection of nodes accomplish a network-wide task.

Design simplicity is a much-valued property in such systems. For example, the literature on networked systems often refers to the importance of simplicity (all emphasis added):

> The advantage of Chord is that it is *substantially less complicated*... (Chord [35])
>
> This paper describes a design for multicast that is *simple to understand*... (Simple Multicast [32])
>
> This paper proposed a *simple* and effective approach... (SOSR [17])

Likewise, engineering maxims stress simplicity:

> All things being equal, the simplest solution tends to be the right one. (Occam's razor)
>
> KISS: Keep It Simple, Stupid! (Apollo program)

However, as the literature reveals, our evaluation of the simplicity (or lack thereof) of design options is often through qualitative discussion or, at best, proof-of-concept implementation. What rigorous metrics we do employ tend to be borrowed from the theory of algorithms. These metrics, however, were intended to capture the overhead or efficiency of an algorithm and are at times incongruent with our notion of simplicity. For example, flooding performs poorly on two of the most common metrics used to calibrate system designs (the amount of state maintained at nodes and the number of messages exchanged across nodes), but most of us would consider flooding a simple, albeit inefficient, solution. Similarly, a piece of state obtained as the result of a distributed consensus protocol feels intuitively more complex than state that holds the IP address of a neighbor in a wireless network.

We conjecture that this mismatch in design aesthetic contributes to the frequent disconnect between the more theoretical and applied research on networked system problems. A good example of this is the work on routing. Routing solutions with small forwarding tables are widely viewed as desirable and the search for improved algorithms has been explored in multiple communities; for instance, a fair fraction of the proceedings at STOC, PODC, and SPAA are devoted to routing problems. The basic distance-vector and link-state protocols incur high routing state ($O(n)$ entries) but are simple and widely employed. By contrast, a rich body of theoretical work has led to a suite of *compact* routing algorithms (*e.g.*, [2,3,10,36]). These algorithms construct optimally small routing tables ($O(\sqrt{n})$ entries) but appear more complex and have seen little adoption.

This is not to suggest existing overhead or efficiency metrics are not relevant or useful. On the contrary, all else being equal, solutions with less state or traffic overhead are strictly more desirable. Our point is merely that design simplicity plays a role in selecting solutions for real-world systems, but existing efficiency or performance-focused metrics can be misaligned with our notion of what constitutes simple system designs.

This paper explores the question of whether we can identify complexity metrics that more directly capture the intuition behind our judgment of system designs. Because the system designs we work with are fairly well-specified, we believe there is no fundamental reason why our appreciation of a design cannot be reinforced by quantifiable measures. Such metrics would not only allow us to more rigorously discriminate between design options, but also to better align the design goals of the theory and systems communities.

We start by reporting on a survey we conducted to understand how system designers evaluate and articulate complexity in system design (Section 2). Building on this, we define a complexity metric in Sections 3 and 4 and evaluate several networked system designs through the lens of our complexity metric in Section 5. Using this analysis, we demonstrate that our metric quantitatively differentiates across flavors of solutions and ranks systems in a manner that is congruent with our survey. We discuss the limitations of our metric in Section 6, review related work in Section 7, and conclude in Section 8.

Finally, it is important to clarify the scope of our work. We intend for our complexity metric to complement – not replace – existing efficiency or performance metrics. For example, in the case of a routing algorithm, our metric might capture the complexity of route construction but reveal little about the quality of computed paths. In addition, while we focus on system design at the algorithmic or procedural level, there are many aspects to a software system that contribute to its ultimate complexity. For example, as the CAP theorem [13] tells us, the careful selection of a system's service model profoundly impacts complexity. The same is true for the sound design of its software implementation. Although at least as important as distributed complexity, these are not aspects we consider in this paper. Lastly, we stress that we view our metric as a prototype: one specific metric that works well with several classes of important systems. We expect that the best-suited metric will emerge in time after much broader discussion and evaluation (similar to the development of standard benchmarks in many communities such as databases and computer architecture). As such, we view our contribution primarily in getting the ball rolling by providing a candidate metric and set of results for further scrutiny.

2 Perceived Complexity

We conducted a survey to explore how system designers perceive complexity of networked system algorithms such as routing, distributed systems, and resource discovery. Nineteen students in a graduate distributed systems class at UC Berkeley participated in the survey. Participants were asked to rank which of two comparable networked system algorithms they viewed as more complex on a scale where 1 means system A is far more complex and 9 means B is far more complex. Participants were also asked to rationalize their choice in 2–3 sentences.

We discuss the algorithms we surveyed in detail in the later sections of this paper; Table 1 briefly summarizes the findings from the survey's quantitative ranking. The one-sample t-test reveals that participants consider distance vector (DV) routing as more complex than link state (LS) routing but less complex than landmark or compact routing. In evaluating classical distributed systems, participants viewed solutions such as quorums, Paxos, multicast, and atomic multicast as more complex than read-one/write-all, two phase commit, gossip, and repeated multicast, respectively. Napster was perceived as simpler than Gnutella and systems such as Gnutella and the Domain Name System (DNS) as simpler than distributed hash tables (DHTs).

Algorithm A	**Algorithm B**	**More complex algorithm**
DV	LS	A ($p < .060$)
DV	Landmark	B ($p < .050$)
DV	Compact	B ($p < .030$)
DV	RCP	not significant
Read one/write all	Quorum	B ($p < .007$)
Two phase commit	Paxos	B ($p < .001$)
Gossip	Multicast	B ($p < .013$)
Atomic multicast	Repeated multicast	A ($p < .001$)
Locking	Lease	not significant
Napster	Gnutella	B ($p < .001$)
DHT	Gnutella	A ($p < .020$)
DNS lookup	DHT lookup	B ($p < .007$)

Table 1: Survey results on comparing networked systems complexity. For each question, we present which algorithm was statistically rated as more complex based on the t test's p-value, which indicates the probability that the result is coincidental. The smaller the p-value, the more significant the result.

The rationales for these rankings shed more insight. Participants found a system was complex if it was hard to "get right," understand, or debug, or if it could not easily cope with failures. For the most part, issues of scalability or performance did not figure in their responses. Some sample answers include: "components have complex interactions," "centralized or hierarchical is simpler than decentralized," "structure is complex," and "requires complex failure and partition handling." Tellingly, participants at times could not clearly articulate why one algorithm was more complex than the other and resorted to *circular* definitions – *e.g.*, "chose system A because it is more complicated" or "B's protocol is more complex."

3 Components of Complexity

A complexity metric could make these arguments objective. A good metric would be based on quantifiable, concrete measurements of the system properties that induce implementation difficulties, complex interactions and failures, and so forth. Many metrics are possible. A perfect metric would be intuitive and easy to calculate, and would correlate with other, more subjective metrics, such as lines of code or system designers' experience.

We build on the observation that much of system design centers on issues of state – the required state must be defined and operations for constructing and using it must be developed – but in distributed systems, one state can derive from states stored on other nodes. To calculate its state, a node must hear from the remote nodes that store the dependencies. This adds additional dependencies on the network and intermediate node states required to relay input states to the node in question. Thus, not only are a given piece of state's dependencies distributed, there are also more of them.

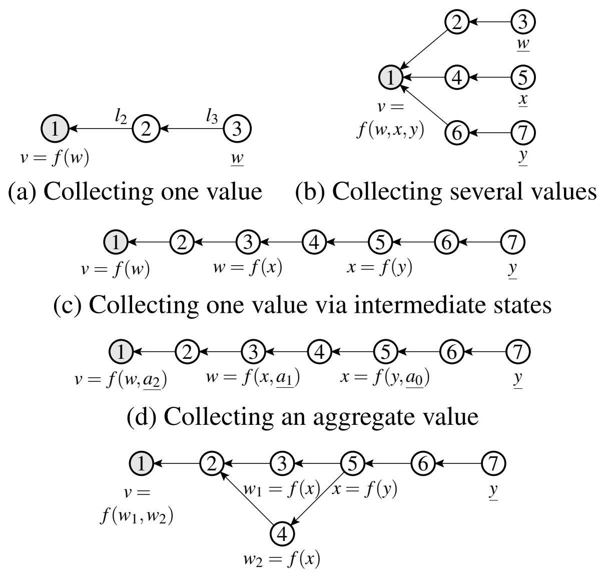
We conjecture that the complexity particular to networked systems arises from the need to ensure state is kept in sync with its distributed dependencies. The metric we develop in this paper reflects this viewpoint and we illustrate several systems for which this dependency-centric approach appears to appropriately reflect system complexity. Alternate approaches are certainly possible however – *e.g.*, based on protocol state machine descriptions, a protocol's state space, and so forth – and we leave a comprehensive exploration of the design space for metrics and their applicability to future work.

Our goal then is to derive a per-state measure c_s that captures the complexity due to the distributed state on which a state s depends. While a natural option would be simply to count s's dependencies, this is not sufficiently discriminating: dependencies, like state, can vary greatly in the burden they impose. Consider Figure 1, which shows dependency relationships between states for several simple networks. In Fig. 1b, a simple distribution tree, v is computed from three dependencies w, x, and y, while in Fig. 1c, which transforms a value over several hops, v has just one *direct* dependency w, which is in turn computed from x, itself computed from y. However, a change in y in Fig. 1b will affect *only* v, while the same change in Fig. 1c must propagate through x and w first. As a system, we argue that Fig. 1c is more complex than Fig. 1b. We therefore *weight* each state, and instead of naively counting dependencies, calculate a state's complexity by *summing* the complexities of its dependencies. The sum includes not only direct dependencies on values, but also dependencies on the *transport states* required to relay those values, accounting for networks whose transport relationships are expensive to maintain.

Some flexibility is required to account for the different types of dependencies in real networked systems, including redundancy, soft state, and so forth, and to differently penalize transport and value dependencies. Nevertheless, our metric is defined exclusively by *counting*; we avoid incorporating intricate probabilistic models of node or link behavior or state machine descriptions and the like. This keeps our metric usable, lending it to evaluation through simple examination and analysis or even empirical simulation, and represents one particular trade-off between a metric's discriminative power and the simplicity of the metric itself. Some of the limitations of our counting-based approach are discussed in Section 6.

4 A Complexity Metric

Given a system that consists of a set of states S, our goal is to assign a complexity metric c_s to each state $s \in S$. We write states as lowercase letters, such as s, v, and w. Where the context is clear, we abuse notation and merge



(a) Collecting one value (b) Collecting several values

(c) Collecting one value via intermediate states

(d) Collecting an aggregate value

(e) Collecting via branches forked and then merged

Figure 1: State relationships in four toy scenarios. For clarity routing table state, written l, is only shown in scenario (a).

the identities of states and nodes; *e.g.*, instead of "delivered to node 1, which stores state x," we simply say "delivered to x." **Local** or **primitive** state can be maintained without network traffic, as in a sensor node's temperature reading. We sometimes indicate primitive state with an underline, as in $\underline{w}$. All other state is **derived** at least partially from states held at other nodes. We call these remote states **direct value dependencies**. In Fig. 1a, w is a primitive state, and v is a derived state with one value dependency, namely w, as indicated by the definition $v = f(w)$. Primitive state is assigned zero complexity, while any derived state has positive complexity. The set of state s's direct value dependencies is written D_s.

A derived state also depends on the transport state required to relay value dependencies through the network. For instance, in Fig. 1a, propagating w to v uses the l_2 and l_3 routing table entries at nodes 2 and 3, respectively. We call these states **transport dependencies** and account for their complexity. The set $T_{s \leftarrow x}$ is defined as the set of transport dependencies involved in relaying x's value to s; it is empty when $x \notin D_s$. In terms of maintaining state consistency, transport dependencies are less of a burden than value dependencies since changes in a state's transport dependencies do not induce costs to keep that state in sync and, as we shall see, our metric reflects this. For instance, in Fig. 1a, any change in w must be communicated to node 1, but a change in l_2 need not, since v depends on the l states only for the delivery of w.

While some value dependencies require state changes be relayed, others need only be established once. For example, if v were defined as a function of w *at some specific time*, rather than of w's *current* value, then once established v is unaffected by changes elsewhere in the network. We say that state x and one of its value dependencies y are **linked** if a change in y must be propagated to x, and **unlinked** otherwise. Linked value dependencies are the major source of network complexity due to the state maintenance they incur and are treated accordingly by our metric.

Evaluating the metric requires determining dependencies among states and defining which dependencies are linked or unlinked. Unused or redundant dependencies, which frequently occur, can be measured in several ways. For example, consider Fig. 1b, where $v = f(w,x,y)$ and let us assume that the value v takes at any point is based on just one of its inputs (for instance, perhaps the active input is chosen based on minimum path length). Then v's value dependencies have distinctly different importance: ensuring consistency requires that v and its active input stay synchronized, while updates from the other dependencies are less critical. When we consider dependencies of state v, we focus on these active states that derive v and ignore unused value dependencies.

We now turn to the metric itself, first defining a submetric u_s which we call the **value dependency impact**. u_s measures the number of remote states on which s is value dependent directly or indirectly. Stated otherwise, these are primitive states that, if they were to change, could result in an update at s and hence one can intuitively view u_s as indicative of the number of updates seen at s for maintaining consistency with its value dependencies. u_s is defined mutually recursively with $u_{s\leftarrow x}$, which measures the number of states on which s is value dependent via some direct value dependency $x \in D_s$. For local state s, we have $D_s = \emptyset$, $u_s = u_{s\leftarrow x} = 0$.

$$u_s = \sum_{x\in D_s} u_{s\leftarrow x} \;;$$

$$u_{s\leftarrow x} = \begin{cases} u_x & \text{if } x \text{ is linked to } s \text{ and} \\ & x \text{ is not dependent on local state} \\ u_x + 1 & \text{if } x \text{ is linked to } s \text{ and} \\ & x \text{ is dependent on local state,} \\ \varepsilon & \text{if } x \text{ is unlinked to } s. \end{cases}$$

If the dependency $s \leftarrow x$ is linked, s must be notified of any change in $x \in D_s$. Applied recursively, changes in any of x's direct or indirect value dependencies must also be passed on to s. Thus, the number of dependencies inherited via x is x's own value dependency impact, u_x, plus one in the event that x was derived (in part) from local state (since a change caused by state local to x would not be accounted for in u_x). For example, states w and x in Fig. 1c, do not include any local inputs while the same states in Fig. 1d do. If s is unlinked to x, then any changes in x are not propagated to s, so we cut off x's value dependency impact. However, to ensure that s is charged for its initial reliance on x, we introduce ε, $0 < \varepsilon \ll 1$, and charge this amount for every non-local, unlinked dependency.

Note that our definition of u_s assumes the dependencies s inherits are independent – a simplifying assumption due to which u_s overcounts in some dependency structures. For example, in Fig. 1e, if $v \leftarrow \{w_1, w_2\} \leftarrow x \leftarrow y$, then y is counted twice in u_s, once via w_1 and once via w_2. This situation arose rarely. Many such branching dependency structures represent unused or redundant dependencies that we model by picking one active input, which leaves the dependency graph in the form of a tree.

The **complexity** of s is then defined as follows:

$$c_s = \sum_{x\in D_s} c_{s\leftarrow x} \;;$$

$$c_{s\leftarrow x} = \begin{cases} u_{s\leftarrow x} + \sum_{y\in T_{s\leftarrow x}} \max(c_y, \varepsilon) + c_x & \text{if } x \text{ linked,} \\ \varepsilon & \text{if } x \text{ unlinked.} \end{cases}$$

This definition accounts for the entire scaffolding of distributed dependencies that maintain changes from s's dependencies to s itself. Suppose s's direct value dependencies are all linked. Then, the first term $\sum_{x\in D_s} u_{s\leftarrow x}$ $(= u_s)$ is s's value dependency impact. The second term $\sum_{x\in D_s} \sum_{y\in T_{s\leftarrow x}} \max(c_y, \varepsilon)$ accounts for the complexity of the transport states from s's dependencies to s itself; ε again ensures that all links are counted, here including transport links that nominally require no state (such as one-hop wireless broadcast). Finally, the last term $\sum_{x\in D_s} c_x$ covers the complexity from inherited (transport and value) dependencies downstream from x. Local state s has $c_s = 0$. Thus intuitively, where u_s was indicative of the updates seen at s, c_s is indicative of the updates seen across all states – value and transport – that maintain changes from s's dependencies to s.

For a chain of linked dependencies from x_0 to x_i (Fig. 1d), which depends on its local state $\underline{a_i}$ (perhaps x_i measures node i's hop count to node 0), and writing c_i for c_{x_i} and so forth, we have $u_i = i$ and

$$c_i = i + \sum_{y\in T_{i\leftarrow(i-1)}} c_y + c_{i-1} \;.$$

Ignoring transport dependencies, the result is $c_i = (i^2 + i)/2$: chained linked dependencies induce complexity proportional to the square of the length of the chain. In Figure 1, if we assume all l states have complexity t, the metric yields $c_v = 1 + 2t$ in Fig. 1a, $c_v = 3 + 6t$ in Fig. 1b, and $c_v = 6 + 6t$ in Fig. 1c.

We sometimes convey intuition about the sources of complexity by writing $c_s = c_s^{\mathrm{V}} + c_s^{\mathrm{T}}$, where c_s^{V} is the complexity contributed by value dependencies and c_s^{T} is the complexity contributed by transport dependencies:

$$c^{\mathrm{V}}_{s\leftarrow x} = u_{s\leftarrow x} + c^{\mathrm{V}}_x \ ,$$
$$c^{\mathrm{T}}_{s\leftarrow x} = \sum_{y\in T_{s\leftarrow x}} \max(c_y, \varepsilon) + c^{\mathrm{T}}_x \ .$$

This split is purely for illustration and does not affect the definition of complexity in any way.

To measure the complexity of an *operation*, such as name resolution, routing, agreement, replication, and so forth, we simply measure the complexity of a state created or updated by that operation. For example, to measure the complexity of multihop routing, we imagine a piece of state, s, derived from one primitive value dependency, x, whose value must be routed across a multihop network. The complexity of routing is defined as c_s, which accounts for the multihop transport dependencies used to route x across the network. Assuming a network with diameter d where every routing table entry has complexity c_r, the resulting complexity is $O(dc_r)$.

This paper evaluates different networked system designs by comparing their complexities for specific operations of interest (*e.g.*, `route`, `write_object`, `find_object`). We found it sufficient to consider one operation at a time for our evaluation. If desired, one might (for example) select the average complexity of key operations as the overall complexity of the networked system. We proceed to evaluating the above metric and defer a discussion of its scope and limitations to Section 6.

4.1 Some Canonical Scenarios

We first examine how the above complexity metric fares in evaluating a few simplified network scenarios and in the following section explore a suite of more complete networked system solutions. Before this, we first introduce two conditions that appear repeatedly in our analysis of system designs and are hence worth calling out.

Redundant inputs and paths Many systems build in redundancy to achieve higher robustness. In our analysis, this manifests itself as some state s that has multiple inputs or paths but only a subset of them are needed to derive s (akin to our discussion of active value dependencies in the previous section). For example, a multiple input scenario could be a node trying to discover the address of a wireless access point (AP) – the node listens for AP beacons but need only hear from a single AP to establish connectivity state. An example involving redundant paths might include two data centers that provision multiple disjoint network paths between them. When a message is encoded with (m,k) erasure code and each code is sent to a distinct path, the destination can construct the message if any k out of m paths work correctly. We call this the k-of-m scenario where m is the total number of inputs (paths) available and k is the number of inputs (paths) required.

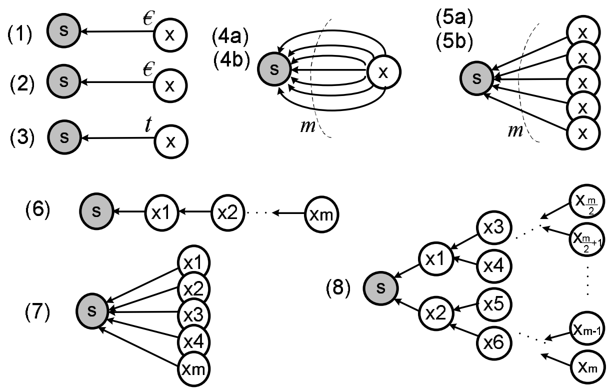


Figure 2: *Canonical scenarios. For clarity we do not show the local and transport state at each node. In all scenarios other than (1) and (2), this transport state is assumed to have complexity t.*

We define the complexity of s derived from k-of-m inputs as being k times the average complexity due to a single input. As shown in [9], this average can be computed simply as $1/m$ times the complexity of s assuming all m inputs were required inputs.

Likewise, for the multipath scenario in which a single input x can be relayed to s using any k of m available paths, we calculate the complexities due to the transport states between x and s as k times the average complexity due to the transport states along any one path.

Recursion In some systems, a piece of state s is derived by an operation that uses states that were themselves set up by the same operation. For example, in DHTs, a node discovers its routing table entries using a `lookup` operation that makes use of state (at other nodes) that was itself set up using `lookup` operations.

We use *two complexity computation passes* for state that involves this kind of recursion. In the first pass, we compute the complexities from value and transport dependencies with the assumption that states used by the operation do not depend on the operation. We compute the final complexities in the second pass in terms of an operation on states whose complexities are computed in the first pass. In Section 5, DHTs and Paxos are canonical examples that involve recursion.

Canonical scenarios We recap the following canonical scenarios, also depicted in Figure 2. In many cases, we can construct dependency structures of networked system algorithms by composing several canonical scenarios. In all scenarios other than (1) and (2), we assume that the transport state at each node has complexity t.

(1) single input, 1-hop broadcast: here s is derived by listening to the broadcast of x. We assume x is local state and hence $c_x = 0$. Moreover, the complexity of transport state at x equals zero since broadcasting does not require any non-local transport state to be es-

Scenario	c_s
(1) 1 input, 1-hop broadcast	$1+\varepsilon$
(2) 1 non-value-dependent input, 1-hop broadcast	ε
(3) 1 input, 1-hop unicast	$1+t$
(4a) 1 input, 1-of-m paths	$1+t$
(4b) 1 input, k-of-m paths	$1+kt$
(5a) 1-of-m inputs, 1 path	$1+t$
(5b) k-of-m inputs, 1 path	$k(1+t)$
(6) m inputs, in series	$\frac{1}{2}m(m+1)+mt$
(7) m inputs, in parallel	$m+mt$
(8) tree	$O(m\log m+mt)$

Table 2: Complexity of canonical scenarios

tablished at x. Correspondingly, state s has complexity $c_s = 1 + c_x + \max(0,\varepsilon) = 1+\varepsilon$.

(2) single unlinked input, 1-hop broadcast: this case is identical to the previous case but here s is unlinked to x (*e.g.*, s stores the value of x as soft-state) and hence $u_{s\leftarrow x} = \varepsilon$ and $c_s = \varepsilon$.

(3) single input, 1-hop unicast: this is identical to the first case, except that instead of broadcasting, x is routed to s using transport state at x which has complexity t and hence $c_s = 1+t$.

(4a) single input, 1-of-m paths: this is identical to the previous case but we now have m identical paths from x to s. As before, the complexity of the transport state for each path is t and hence $c_s = 1+t$.

(4b) single input, k-of-m paths: this is identical to the previous case but here x must be delivered to s along k paths and hence $c_s = 1+kt$.

(5a) 1-of-m inputs, single path per input: 1 input must be delivered to s and hence $c_s = 1+t$.

(5b) k-of-m inputs, single path per input: similar to the previous case but here k inputs must be delivered to s and hence $c_s = k(1+t)$.

(6) m value dependencies; 1 direct, m−1 indirect: similar to scenario-1c in Figure 1, here the value of each x_i is computed from that of x_{i+1} and local state and hence $u_s = m$ and $c_s = \frac{m(m+1)}{2} + mt$.

(7) m direct value dependencies: similar to scenario-1b in Figure 1, s is computed from m inputs each of which is directly connected to s and hence $c_s = m(1+t)$.

(8) tree: each intermediate node has two children and the tree height is $O(\log m)$. Hence $c_s = O(m\log m + mt)$.

The complexities for the above scenarios are summarized in Table 2. Comparing the complexity of s in case (6) to that in case (7), we see that dependencies that accumulate indirectly result in a higher complexity than dependencies that accumulate directly (in keeping with our discussion comparing scenarios (1b) and (1c) in Figure 1). A second observation, based on comparing cases (3) *vs.* (4a) or (3) *vs.* (5a), is that our metric neither penalizes nor rewards the use of redundant state. This decision might seem to warrant discussion. One might argue that redundancy should add to complexity because of the additional effort that goes into creating redundant state. For example, consider a server that must create m replicas of an immutable file instead of just one. While this is true, we note that (in this example) the replicas are not dependent on each other and likewise state derived from one of the replicas is ultimately only dependent on one rather than m replicas and hence neither should have a complexity higher than if there were only a single replica. That said, the additional effort due to creating redundancy would emerge in the complexity of the operation that creates the m copies since this requires maintaining additional state to identify the m nodes at which to store replicas.

In terms of not rewarding redundancy, one might argue (as was done in [34]) that a scenario in which s is derived from k-of-m inputs should have lower complexity than if s were derived from exactly k inputs because having alternate options reduces the extent to which s depends on any single input (and similarly for paths). However, to do so would be conflating robustness and complexity[1] in the sense that having alternate inputs does not ultimately change the number of dependencies for s even though it changes the *extent* to which s might depend on any individual input; *i.e.*, the value of s derived from k-of-m inputs does ultimately depend on some k input states.

5 Analysis

In this section we evaluate a number of networked system designs through the lens of the complexity metric defined in Section 4. Our goal in this is to: (1) illustrate the application of our metric to a broad range of systems and (2) provide concrete examples of the assessments our metric arrives at both in comparing across systems, and relative to traditional metrics.

To the extent possible, our hope is also to validate that our metric matches common design intuition. That said, conclusively validating the goodness of a metric is almost by definition difficult and, in this sense, our results are perhaps better viewed as providing the initial dataset for the future scrutiny of metric performance.

We analyzed the complexity of solutions to four problems that figure prominently in the literature on networked systems: (1) Internet routing, (2) classical distributed systems, (3) resource discovery, and (4) routing in wireless networks. Due to space constraints we only discuss the first two items in this paper; our complete set of results are presented in [9].

5.1 Routing

Routing is one of the fundamental tasks of a networked system and the literature abounds in discussions of routing architectures and algorithms. In this section we analyze a set of routing solutions that represent a range of design options in terms of architecture (*e.g.*, centralized *vs.* distributed), scalability (*e.g.*, small *vs.* large tables), adoption and so forth.

For each solution, we present the complexity of an individual routing entry and a source-to-destination routing operation. For clarity we summarize only the final complexity results here and present the details of their derivation in [9]. For comparison across metrics, we also evaluate each solution using the following traditional measures: (1) per-node state, (2) number of messages and (3) convergence time.[2] In what follows, we consider each routing solution in turn, briefly revise its operation and summarize its complexity. The results of our analysis are summarized in Tables 3 and 4 and we end this section with a discussion examining these results.

Distance-Vector (DV) Used by protocols such as RIP and IGP, distance-vector represents one of the two major classes of IP routing solutions. DV protocols use the Bellman-Ford algorithm to calculate the shortest path between pairs of nodes. Every node maintains an estimate of its shortest distance (and corresponding next-hop) to every destination. Initially, a node is configured with the distance to its immediate neighbors and assumes a distance of infinity for all non-neighbor destinations. Each node then periodically informs its neighbors of its currently estimated distance to all destinations. For each destination, a node picks the neighbor advertising the shortest path to the destination and updates its estimated shortest distance and next-hop accordingly.

For an n node network with diameter d, DV thus requires $O(n)$ per-node state, a total message cost of $O(n^2)$ and convergence time of $O(d)$ in the absence of topology changes. In terms of our complexity measure, a single DV routing entry s has complexity $c_s = O(d^2 + d\varepsilon)$ while a routing operation has a complexity of $c_{route} = O(d^3 + d^2\varepsilon)$.[3]

Link-State (LS) Link-State routing, used in protocols such as OSPF and IS-IS, represents the second major class of widely-deployed IP routing solutions. In LS, each node floods a "link state announcement (LSA)" describing its immediate neighbor connections to the entire network. This allows each node to reconstruct the complete network topology. To compute routes, a node then simply runs Dijkstra's algorithm over this topology map.

LS thus requires $O(nf)$ state per node (where f denotes the average node degree), incurs a total message cost of $O(n^2)$ and convergence time $O(d)$. A routing entry s has complexity $c_s = O(d + d^2\varepsilon)$ while a routing operation has complexity $c_{route} = O(d^2 + d^3\varepsilon)$.[4]

Centralized Architectures The authors of the 4D project [15] argue for architectures that centralize the routing control plane to simplify network management. Several subsequent proposals – RCP [5], SANE/Ethane [7, 8], FCP [25] – present different instantiations of this centralized approach. We analyze two variants of centralized routing solutions inspired by these proposals. Our variants are not identical to any particular proposal but instead adapt their key (routing) insights for a generic network context. We do this because many of the above proposals were targeted at specific contexts which complicates drawing comparisons across solutions if we were to adopt them unchanged. For example, RCP assumes existing intra-domain routing and leverages this to deliver forwarding state from the center to the domain's IGP routers.

In our first "RCP-inspired" variant, a designated center node collects the LSAs flooded by all nodes, reconstructs the complete network map from these LSAs, computes forwarding tables for all nodes and then uses source routing to send each node its forwarding table.[5] When the network topology changes, the center receives the new LSA, recomputes routes and updates the forwarding state at relevant nodes. RCP-inspired has a per-state complexity of $c_s = O(d + d^2\varepsilon)$ and correspondingly, a routing operation complexity of $c_{route} = O(d^2 + d^3\varepsilon)$. This can be intuitively inferred by noting that a routing entry r computed at the center is similar to that at a node in LS; r is then delivered to a node in the network using a source route with the same complexity as r. RCP's performance with traditional metrics is summarized in Table 4.

RCP-inspired centralizes the computation of routes but packet forwarding (*i.e.*, the data plane) still relies on state distributed across nodes along the path. Borrowing from several recent routing proposals [8, 25], our second variant "RCP-inspired + SR" uses source routing to forward packets between pairs of nodes. Routing construction proceeds as before but now the forwarding table sent from the center to a node A contains the entire route (as opposed to just the next hop) from A to each destination and this information is used to source route packets originating at A. Thus, rather than requiring $O(d)$ routing entries (one at each node along the path) for packet forwarding, our second variant requires only the single source-route entry at the source thus retaining the per-state complexity $c_s = O(d + d^2\varepsilon)$ but lowering the complexity of c_{route} to that of a single routing entry and hence $c_{route} = O(d + d^2\varepsilon)$.

Compact routing Compact routing [2, 3, 10, 36] has significantly improved scalability (*i.e.*, small routing tables) relative to deployed solutions but has seen little real-world adoption. Here, we analyze the complexity of a state-of-the-art name-independent[6] routing algorithm by Abraham *et al.* (`AG+_compact`) [2]. `AG+_compact` guarantees optimally small routing tables of $O(\sqrt{n})$ entries, worst-case stretch less than 3.0 for arbitrary topologies and $\approx$1.0 for Internet topologies [23] and hence – as

Algorithm	u_s	c_s^{V}	c_s^{T}	c_s	c_{route}
DV	$O(d)$	$O(d^2)$	$O(d\varepsilon)$	$O(d^2+d\varepsilon)$	$O(d^3+d^2\varepsilon)$
LS	$O(d)$	$O(d)$	$O(d^2\varepsilon)$	$O(d+d^2\varepsilon)$	$O(d^2+d^3\varepsilon)$
RCP-inspired	$O(d)$	$O(d)$	$O(d+d^2\varepsilon)$	$O(d+d^2\varepsilon)$	$O(d^2+d^3\varepsilon)$
RCP-inspired+SR	$O(d)$	$O(d)$	$O(d+d^2\varepsilon)$	$O(d+d^2\varepsilon)$	$O(d+d^2\varepsilon)$
Compact	$O(d\sqrt{n})$	$O(nd^2)$	$O(nd^2)$	$O(nd^2)$	$O(nd^2)$
Hierarchical LS	$O(\log\frac{n}{k})$	$O(\log\frac{n}{k})$	$O(\varepsilon\log^2\frac{n}{k})$	$O(\log\frac{n}{k}+\varepsilon\log^2\frac{n}{k})$	$O(\log^2\frac{n}{k}+\varepsilon\log^3\frac{n}{k})$
Intradomain ROFL	$O(d^2)$	$O(d^2\log n)$	$O(d^3\varepsilon\log n)$	$O((d^2+d^3\varepsilon)\log n)$	$O((d^2+d^3\varepsilon)\log^2 n)$

Table 3: Complexity analysis for routing solutions with the breakdown of the final per-state complexity c_s into its constituent components: u_s, the complexity contributed by value dependencies (c_s^{V}) and the complexity contributed by transport dependencies (c_s^{T}).

Algorithm	State	Message	Convergence time	Complexity
DV	$O(n)$	$O(n^2)$	$O(d)$	$O(d^3+d^2\varepsilon)$
LS	$O(n)$	$O(n^2)$	$O(d)$	$O(d^2+d^3\varepsilon)$
RCP-inspired	$O(n)$, center $O(nf)$	$O(n^2)$	$O(d)$	$O(d^2+d^3\varepsilon)$
RCP-inspired+SR	$O(n)$, center $O(nf)$	$O(n^2)$	$O(d)$	$O(d+d^2\varepsilon)$
Compact	$O(\sqrt{n})$	$O(n\sqrt{n})$	$O(d)$	$O(nd^2)$
Hierarchical LS	$O(\frac{n}{k}+k)$	$O((\frac{n}{k})^2+k^2)$	$O(\log\frac{n}{k})$	$O(\log^2\frac{n}{k}+\varepsilon\log^3\frac{n}{k})$
Intradomain ROFL	$O(\log n)$	$O(n\log^2 n)$	$O(d\log^2 n)$	$O((d^2+d^3\varepsilon)\log^2 n)$

Table 4: Evaluation of routing solutions using different metrics

per standard measures – `AG+_compact` would appear to be an attractive option for IP routing.

Briefly, `AG+_compact` operates as follows: a node A's vicinity ball (denoted VB(A)) is defined as the k nodes closest to A. Node A maintains routing state for every node in its own vicinity ball as well as for every node B such that A $\in$ VB(B). A distributed coloring scheme assigns every node one of c colors. One color, say red, serves as the global backbone and every node in the network maintains routing state for all red nodes. Finally, a node must know how to route to every other node of the same color as itself. For n nodes, vicinity balls of size $k = O(\sqrt{n}\log n)$ and $c = O(\sqrt{n})$ colors, one can show that a node's vicinity ball contains every color. With this construction, a node can always forward to a destination that is either in its own vicinity, is red, or is of the same color as the node itself. If none of these is true, the node forwards the packet to a node in its vicinity that is the same color as the destination. The challenge in `AG+_compact` lies in setting up routes between nodes of the same color without requiring state at intermediate nodes of a different color and yet maintaining bounded stretch for all paths. Loosely, `AG+_compact` achieves this as follows: say nodes A and D share the same color and A is looking to construct a routing entry to D. A explores every vicinity ball to which it belongs (VB(I), A $\in$ VB(I)) and that touches or overlaps the vicinity ball of the destination D (*i.e.*, $\exists$ node X $\in$ VB(I) with neighbor Y and Y $\in$ VB(D)). For such C, A could route to D via C, X and Y. `AG+_compact` considers possible paths for each neighboring vicinity balls VB(C) as well as the path through the red node closest to D and uses the shortest of these for its routing entry to D.

`AG+_compact` incurs $O(\sqrt{n})$ per-node state, total message overhead of $O(n\sqrt{n})$ and converges in $O(d)$ rounds. Derived in [9], `AG+_compact` has per-state complexity $c_s = O(nd^2)$ and $c_{route} = O(nd^2)$.

Hierarchical routing Compact routing represents one effort to reduce routing table size. The approach adopted by IP routing however has been to address scalability through the use of hierarchy. For example, OSPF may partition nodes into OSPF areas and border routers of areas are connected into a backbone network. Identifiers of nodes within a region are assigned to be aggregatable (*i.e.*, sharing a common prefix) so that border routers need only advertise a single prefix to represent all nodes within the region.

For a network partitioned into k areas, hierarchical routing reduces the per-node state to $O(\frac{n}{k}+k)$ and total message overhead to $O((\frac{n}{k})^2+k^2)$. The resultant complexity depends on the network topology. If the diameter of an area scales as $\log\frac{n}{k}$, then, from the LS complexity analysis, we know that routing complexity in an area is $c_a = \log^2\frac{n}{k}+\varepsilon\log^3\frac{n}{k}$. The final routing complexity is $2c_a$, which is asymptotically equivalent to the complexity of non-hierarchical routing $O(\log^2 n+\varepsilon\log^3 n)$. Thus, in this case, hierarchy offers improved scalability at no additional complexity. (If the network is planar, hierarchy as above actually reduces complexity by $O(\sqrt{n})$ [9].)

Intradomain ROFL Hierarchical routing offers improved scalability at the cost of constraining address assignment (giving rise to several well-documented issues). Intradomain ROFL [6] is a scalable routing protocol that retains the ability to route on flat (as opposed to aggregatable) identifiers. Each virtual node maintains its predecessor and successor and a pointer cache that stores source routes of virtual nodes extracted from forwarded packets. In routing a packet, if a node knows a virtual node whose identifier matches the label, it sends the packet directly to the node; otherwise, it forwards the packet to a node whose identifier is closest to the label using a source route. Each node computes source routes of its neighbors from a network topology map obtained from LSAs. To simplify our analysis and comparison, we assume that the pointer cache of a node contains fingers as in Chord [35] to guarantee $O(\log n)$ hops in the flat label space and each node hosts a single virtual node representing itself.

In intradomain ROFL, a node maintains routing entries, each of which is (id, s, r) where id is a particular identifier, s is the successor of id and r is a source route to the node hosting s. Like in LS, $c_r = O(d^2 + d^3\varepsilon)$. Finding s using a lookup operation takes $O(\log n)$ hops thus yielding a complexity of $c_s = O(\log n(d^2 + d^3\varepsilon))$. A routing operation involves $\log n$ such entries, hence results in a complexity of $c_{route} = O(\log^2 n(d^2 + d^3\varepsilon))$. In other metrics, intradomain ROFL requires $O(\log n)$ state per node, incurs a total message cost of $O(n \log^2 n)$, and has convergence time $O(d \log^2 n)$.

5.1.1 Discussion

Tables 3 and 4 summarize our results which we now briefly examine. In drawing comparisons, we generally assume that the network diameter d is O($\log n$) and $\varepsilon \sim 0$.

Complexity *vs.* traditional metrics Our first observation is that none of the traditional metrics yield the same relative ranking of solutions as our complexity metric, confirming that complexity (as defined here) is not the same as scalability or efficiency. Moreover, the ranking due to our complexity metric is in fair agreement with that suggested by real-world adoption and our survey results. For example, DV, LS and hierarchical routing are simpler than either AG+_compact's compact routing algorithm or intradomain ROFL; centralized routing is simpler than DV, compact routing or intradomain ROFL.

Our complexity measure is also more discriminating than the other metrics. For example, DV, LS and both variants of centralized routing fare equally in terms of total state, messages or convergence time while our metric ranks them as DV $>$ LS $=$ RCP-inspired $>$ RCP-inspired + SR. Convergence time in particular appears too coarse-grained – for routing protocols it mostly reflects the scope to which state propagates and hence most solutions have the same value. In some sense, however, this greater discriminative power is to be expected as our metric is somewhat more complicated in the sense of taking more detail into account.

Deconstructing complexity A routing entry at a node A for destination B depends fundamentally on the link connectivity information from the d nodes along the path to B. In DV, the computation mapping these d link states into a single routing entry is *distributed* – occurring in stages at the multiple nodes en route to A. LS by contrast, *localizes* this computation in that the d pieces of state are transferred unchanged to node A which then computes the route locally. RCP not only localizes, but *centralizes* this computation.

Our metric ranks distributed network computations as more complex than localized ones and hence DV as more complex than LS. Our metric ranks the complexity of LS as equal to that of the first centralized variant implying that a localized approach (*i.e.*, "flood everywhere then compute locally") is similar in complexity to a centralized one (*i.e.*, "flood to a central point, compute locally, then flood from central point"). This appears justified as both approaches are ultimately similar in the number and manner in which they accumulate dependencies. While the central server can ensure an update is consistently applied in computing routes for all nodes, it is still left with the problem of consistently propagating those routes to all nodes. LS must deal with the former issue but not the latter and is thus merely making the inverse tradeoff. These "simpler" approaches that localize or centralize computations might lead to greater message costs or reduced robustness and this tradeoff could be made apparent by simultaneously considering scalability, complexity and robustness metrics.

Introducing the use of source routing causes an $O(d)$ reduction in the complexity of the first RCP-inspired variant. Note too that introducing source routing to LS would result in a similar reduction. In some sense source routing localizes decision making for the *data* plane in much the same way as LS and RCP do for the control plane and hence the reduced complexity points again to the benefit of localized vs. distributed decision making. Finally, we note that, assuming $\varepsilon \to 0$, the combination of LS/RCP-inspired and source routing has $O(d)$ complexity which we conjecture might be optimal for directed routing over an arbitrary topology.

In terms of navigating simplicity and scalability we note that – unlike compact routing and intradomain ROFL – introducing hierarchy improves scalability without increasing complexity.

From our analysis we find that the complexity of compact routing is in large part because of the multiple passes needed to configure routing tables – a node must first build its vicinity ball (VB), then hear from nodes whose VBs it belongs to and finally explore the intersection of "adjoining" VBs. We found a similar source of complexity in our analysis of sensornet routing algorithms (presented in [9]) that use an initial configuration phase to elect landmark nodes and then proceed to construct "virtual" coordinate systems based on distances to these landmarks [33]. Such systems build up layers of depen-

Algorithm	State	Message	Complexity
ROWAA(read)	$O(1)$	$O(1)$	$O(1)$
ROWAA(write)	$O(1)$	$O(n)$	$O(1)$
Quorum(read)	$O(1)$	$O(k)$	$O(k)$
Quorum(write)	$O(1)$	$O(k)$	$O(k^2)$
2PC	$O(1)$	$O(n)$	$O(n^2)$
Paxos	$O(1)$	$O(n)$	$O(k^3)$
Multicast	$O(n)$	$O(n)$	$O(\log^3 n)$
Gossip	$O(n)$	$O(n \log n)$	$O(\log n)$
TTL-based	1	1	ε
Invalidation	1	1	2

Table 5: Evaluation of classical distributed system algorithms using different metrics.

dencies, leading to higher complexity.

Work on compact routing is typically cast as exploring the tradeoff between efficiency (path stretch) and scalability (table size). Throwing complexity into the ring enables discussing tradeoffs between simplicity, efficiency and scalability. For example, much of the complexity of `AG+_compact` stems from the additional mechanisms needed to bound the worst-case stretch when routing between nodes in adjoining vicinities (see [9]). Were we to instead reuse the same mechanism for nodes that are in adjoining vicinity balls as for those in distant vicinities, this would reduce the complexity of `AG+_compact` to O($\sqrt{n}d^2$) but weaken the worst-case stretch bound.

In summary, we show that our complexity metric can discriminate across a range of routing architectures, ranks solutions in a manner that is congruent with common design intuition and can point to alternate "simpler" design options and tradeoffs.

5.2 Classical Distributed Systems

In this section, we analyze the complexity of well-known classical distributed system algorithms: (1) shared read/write variables, (2) coordination/consensus, (3) update propagation, and (4) cache consistency. For each, we consider two solutions; one that offers inferior performance/correctness guarantees relative to the other but is typically viewed as being simpler. The algorithms we analyze operate under benign fault assumptions and we assume transport states have complexity 1. We denote by n the number of servers and denote by k ($> \frac{n}{2}$) the quorum size. The results are summarized in Table 5.

5.2.1 Shared Read/Write Variable

For availability or performance, applications frequently replicate the same data on multiple servers. The replicated data can be viewed as a shared, replicated read/write variable provided by a set of servers that allow multiple clients to read from, and write to, the variable. We compare a best-effort read-one/write-all-available (in short, ROWAA) that favors availability over consistency and quorum systems [28] used in cluster file systems such as GPFS [1]. Our analysis assumes a client knows the set of servers that participate in the algorithm.

ROWAA In ROWAA, a client issues a read request to any one of the replicas, but writes data to all available replicas in a best-effort manner. A replica that is unavailable at the time of the write is not updated and hence ROWAA can lead to inconsistency across replicas.

When a client reads a variable from a server, this fetched value (denoted by r) depends only on the current value at that server. Therefore, $c_r^{\mathrm{V}} = 1$. Reading involves a request from the client to a server and the response from the server; hence $c_r^{\mathrm{T}} = 2$. When a client writes a value to all available servers, it receives any acknowledgments from the servers in a best-effort manner; hence $c_w = O(1)$.

Quorum Quorum systems allow clients to tolerate some number of server faults while maintaining consistency although with lower read performance. To obtain this property, the client reads from and writes to multiple replicas, and the quorum protocol requires that there is at least one correct replica that intersects a write quorum and a read quorum thereby ensuring that the latest write is not missed by any client. For this purpose, each value stored is tagged with a timestamp.

To read a variable in a quorum system, a client sends requests to k servers and receives k (value, timestamp) pairs from a quorum. It chooses the value with the highest timestamp. Since reading a value depends on both k values and k timestamps, $c^{\mathrm{V}} = 2k$. Since there are k requests and k responses, $c^{\mathrm{T}} = 2k$.

A write operation requires two phases. In the first phase, a client sends a request to read the timestamp to each of the k servers. When it receives timestamps from k servers, it chooses the value with the highest timestamp t_{high} and computes a new timestamp t_{new} greater than t_{high}. t_{new} depends on k timestamps stored at servers and these timestamps are fetched via k requests and k responses. Therefore, $c_1^{\mathrm{V}} = k$ and $c_1^{\mathrm{T}} = 2k$.

In the second phase, the client sends write requests (value, t_{new}) to k servers and receives acknowledgments from k servers. When a server receives this request, it updates its local state s which depends on the value and t_{new}, and hence $c_s^{\mathrm{V}} = k+1$ and $c_s^{\mathrm{T}} = 2k+1$. The client finishes the second phase when it receives k acknowledgments from distinct servers. Therefore, $c_2^{\mathrm{V}} = k(k+1)$, $c_2^{\mathrm{T}} = k(2k+1)$ and hence overall complexity c is $O(k^2)$.

Observations Our complexity-based evaluation is in agreement with intuition and our survey. ROWAA has lower complexity but does not provide consistency; quorums have higher complexity but ensure consistency. This suggests that guaranteeing stronger properties (here, consistency) may require more complex algorithms.

5.2.2 Coordination

Two-phase commit (in short, 2PC) [14] and Paxos [26] coordinate a set of servers to implement a consensus service. Both protocols operate in two phases and require a coordinator that proposes a value and a set of acceptors, which are servers that accept coordinated results. 2PC is commonly used in distributed databases and Paxos is used for replicated state machines. 2PC requires that a coordinator communicate with n servers; on the other hand, Paxos requires that a coordinator (named as a proposer in Paxos) communicate with k servers, *i.e.*, a quorum of servers (named as acceptors in Paxos). Therefore, 2PC cannot tolerate a single server fault, but Paxos can tolerate $n-k$ server faults.

2PC In the first phase of 2PC, a coordinator multicasts to R (a set of acceptors) a $\langle prepare, T\rangle$ message where T is a transaction. When an acceptor receives the message, it makes a local decision on whether to accept the transaction. If the decision is to accept T, the acceptor sends a $\langle ready, T\rangle$ message to the coordinator. Otherwise, it sends a $\langle no, T\rangle$ message to the coordinator. The coordinator collects responses from acceptors. Since the acceptor's decision depends on its local state and T sent by the coordinator, the value dependency of the collection at the end of the first phase is $c_1^{\mathrm{V}} = n(1+1) = 2n$. Since there are n requests sent and n responses received, the transport dependency of the collection at the end of the first phase is $c_1^{\mathrm{T}} = n(1+1) = 2n$.

In the second phase, if the coordinator receives $\langle ready, T\rangle$ from all acceptors, it multicasts to R a $\langle commit, T\rangle$ message. Otherwise, it multicasts to R an $\langle abort, T\rangle$ message. When an acceptor receives a request for commit or abort, it executes the request and sends an $\langle ack, T\rangle$ back to the coordinator. When the coordinator receives acknowledgments from all acceptors, it knows that the transaction is completed. Let c_2^{V} and c_2^{T} be the value dependency and transport dependency at the completion of the second phase, respectively. Since the coordinator collects n acknowledgments, $c_2^{\mathrm{V}} = n(c_1^{\mathrm{V}}) = 2n^2$. When an acceptor receives a commit or abort message, the transport dependency of the message is $c_1^{\mathrm{T}}+1$. Since n acknowledgments are required at the coordinator, $c_2^{\mathrm{T}} = n(c_1^{\mathrm{T}}+1) = 2n^2+n$. Hence 2PC has an overall complexity of $O(n^2)$.

Paxos In Paxos, each acceptor maintains two important variables: s_m that denotes the highest proposal number the acceptor promised to accept and v_a that denotes an accepted value. A proposer multicasts to R a $\langle prepare, s\rangle$ message where s is a proposal number. When an acceptor receives this message, it compares s with s_m. If $s > s_m$, the acceptor sets s_m to s and returns a $\langle promise, s, s_a, v_a\rangle$ message where s_a is the proposal number for the accepted value v_a. Otherwise, it returns an $\langle error\rangle$ message.

When the proposer receives $\langle promise, s, s_a, v_a\rangle$ messages from k distinct acceptors, it chooses v_a with the highest s_a among k messages. Let v_c and s_c be the chosen value and proposal number, respectively. If v_a is not null, v_c is set to v_a; otherwise, v_c is set to a default value.

The proposer then multicasts to R an $\langle accept, s_c, v_c\rangle$ message. When an acceptor receives the accept message, it compares s_c with its local s_m. If $s_c \geq s_m$, s_m is set to s_c, s_a is set to s_c, and v_a is set to v_c. It then sends an $\langle ack, s_a, v_a\rangle$ message to the coordinator. Otherwise, it returns an $\langle error\rangle$ message. When the proposer receives $\langle ack, s_a, v_a\rangle$ messages from k distinct acceptors, it knows that the message is accepted by k acceptors and completes the consensus process.

Note that v_c depends on v_a's accepted by acceptors in the second phase. To account for this dependency, we use two *passes* to compute overall complexity. In the first pass, we compute the dependency of v_a without considering the dependency in the second phase. In the second pass, we use the dependency of v_a computed in the first pass to compute the dependency in the first phase and the total dependency of the algorithm.

In the first pass, $c_{v_c}^{\mathrm{V}} = k(2+1) = 3k$ since v_c depends on k s_m's and v_a's, each of which depends on s sent by the proposer. Also, $c_{v_a}^{\mathrm{V}} = c_{v_c}^{\mathrm{V}}+1 = 3k+1$ since v_a depends on v_c and a default value. $c_{v_a}^{\mathrm{T}} = k(1+1)+1$ since k prepare messages, k promise messages, and one accept message are required. In the second pass, $c_{v_c}^{\mathrm{V}} = k(c_{v_a}^{\mathrm{V}}+2)+1 = k(3k+3)+1$ and the final $c^{\mathrm{V}} = k(c_{v_c}^{\mathrm{V}}+1) = 3k^3+3k^2+2k$, and $c_{v_c}^{\mathrm{T}} = k(c_{v_a}^{\mathrm{T}}+1) = k(2k+2)$ and the final $c^{\mathrm{T}} = k(c_{v_c}^{\mathrm{T}}+2) = 2k^3+2k^2+2k$. Hence Paxos has an overall complexity of $O(k^3)$.

Observations Our complexity-based evaluation is in agreement with general intuition and our survey. Both 2PC and Paxos use $O(n)$ messages, maintain $O(1)$ state per node, and have the same operation time. However, Paxos is more complex than 2PC because of interdependencies between phases. At the same time, it is this additional dependency that enables Paxos to tolerate up to $n-k$ faults while 2PC becomes unavailable with even a single fault. Our results affirm once again that guaranteeing stronger properties (here, fault-tolerance) may require more complex system algorithms.

5.2.3 Update Propagation

Update propagation algorithms disseminate an update from a publisher to all nodes (e.g., publish-subscribe systems). We examine multicast (e.g., ESM [20]) using a constructed tree and Gossip [11] that exchanges updates with random nodes. To ease comparison, we assume each node in the system knows k random nodes in the system from a membership service.

Multicast In multicast, nodes run DV over a k-degree mesh to build a per-source tree over which messages are disseminated. Hence forwarding state has complexity $c_s = O(\log^2 n + \varepsilon \log n)$. A value received at a node depends only on the value published by the source and hence $c^V = 1$. On the other hand, if we assume the tree is balanced, $c^T = O(c_s \log n)$ and hence the overall complexity of multicast is $O(\log^3 n)$.

Gossip In Gossip, when a node receives a message, it chooses a random node and forwards the message to the selected node. This process continues until all nodes in the system receive the new update. Hence $c^V = 1$ as before. Each transport depends on a single hop from a forwarding node to a randomly chosen node, and in average $\log n$ such hops are required. Hence $c^T = O(\log n)$ and the overall complexity of Gossip is $O(\log n)$.

Observations Our metric ranks multicast as more complex than Gossip which matches our survey. However, multicast offers a deterministic guarantee of $O(\log n)$ delivery time and does so using an optimal $O(n)$ number of messages. Once again, our results convey that efficiency need not be congruent with complexity.

5.2.4 Cache Consistency

When mutable data are replicated across multiple servers, a cache consistency algorithm provides consistency across replicas. We compare TTL-based caching to invalidation-based approaches.

TTL-based caching In TTL-based caching, a cache server that receives a request first checks whether the requested data item is locally available. If so, it serves the client's request directly. Otherwise, it fetches the item from the corresponding origin server and stores the data item for its associated time-to-live (TTL). After the TTL expires, the item is evicted from the cache. Once a data item is cached, it does not depend on the item value stored at the origin server and hence a cached data item has $c = \varepsilon$.

Invalidation With approaches based on invalidation, the origin server tracks which caches have copies of each data item. When a data item changes, the origin server sends an invalidation to all caches storing that item. Since a cached item depends on the master copy of the origin server, $c^V = 1$, $c^T = 1$, and $c = 2$.

Observations TTL-based caching is a soft-state technique while invalidations are a hard-state technique. Soft-state is typically viewed as simpler than hard-state because of the lack of explicit state set-up and tear-down mechanisms and our metric supports this valuation.

5.3 Other systems

Resource discovery is a fundamental problem in networked systems where information is distributed across nodes in the network. We subjected a number of well-known approaches to this problem to our complexity based analysis. Due to space constraints, because these solutions are well known in the community and our results are (we hope) fairly intuitive, we only present the final ranks of our analysis: centralized directory (*e.g.*, Napster) < (DNS, flooding-based (*e.g.*, Gnutella)) < DHT. The derivation of the complexities and discussion of the results are described in [9].

We also analyzed several wireless routing solutions including GPSR [21] (a scalable geo routing algorithm), noGeo [33] (a scalable, but more complex solution that constructs "virtual" geographic coordinates) and AODV [31] (a less scalable but widely deployed approach). At a high level, our results (described in [9]) reflect a similar intuition as our analysis from Section 5.1 and hence we do not discuss them here.

6 Discussion

Defining a metric involves walking the line between the discriminating power of the metric (*i.e.*, the level of detail in system behavior that it can differentiate across) and the simplicity of the metric itself. Our prototype metric represents a particular point in that tradeoff. We discuss some of the implications of this choice in this section.

6.1 Limitations and possible refinements

Weighting value *vs.* transport dependencies Our metric assigns equal importance to value and transport dependencies. However, depending on the system environment, this may not be the best choice and a more general form of the complexity equation might be to assign:

$$c_{s\leftarrow x} = w_v u_{s\leftarrow x} + w_t \sum_{y \in T_{s\leftarrow x}} \max(c_y, \varepsilon) + c_x$$

For example, a system wherein the transport state is known to be very stable while the data value of inputs change frequently might choose $w_v \gg w_t$, thus favoring system designs that incur simpler value dependencies.

Weighting dependencies Our metric treats all input or transport states as equally important. However, sometime certain input or transport states are more important (for correctness, robustness, *etc.*) than others. For example, DHTs maintain multiple routing entries but only the immediate "successor" entry ensures routing progress hence one might emphasize the complexity due to successor. Again, this might be achieved by weighting states based on system-specific knowledge of their importance.

Correlated inputs Our metric treats all inputs as independent which might result in over-counting dependencies from correlated inputs. This could be avoided by maintaining the set identifying the actual dependencies associated with each piece of state rather than just count their number although this requires significantly more fine-grained tracking of dependencies.

Capturing dependencies in time In our counting-based approach we only consider the inputs and transport states by which state was ultimately derived without worrying about the precise temporal sequence of events that led to the eventual value of state. While a time-based analysis might enable a more fine-grained view of dependencies this would also seem more complicated since it requires incorporating a temporal model that captures the evolution of state over time.

6.2 Scope

Scalability *vs.* **Complexity** As seen in the previous sections, our complexity metric complements traditional scalability metrics. As an example of their complementary nature: our metric would not penalize system A that has the same per-state or per-operation complexity as system B but constructs more state in total than B.

Correctness *vs.* **Complexity** Our metric does little to validate the assumptions, correctness or quality of a solution. For example, our metric might capture the complexity of route construction but says little about the quality or availability of the source-to-destination path. Likewise, our metric is oblivious to undesirable assumptions that might underlie a design. For example, our metric ranks hierarchical routing favorably and cannot capture the loss in flexibility due to its requirement of aggregatable addresses (section 5.1). Similarly, our metric ranks traditional geo routing as simple despite its problematic assumption of "uniform disc" connectivity [9].

Robustness *vs.* **Complexity** Perhaps less obvious is the relationship between our complexity metric and robustness. In some sense, our metric does relate to robustness since a more complex scaffolding of dependencies does imply greater opportunities for failure. However, this relation is indirect and does not always translate to robustness. For example, consider a system where state at n nodes is derived from state at a central server. Our complexity metric would assign a low complexity to such a system, while, in terms of robustness, such a system is vulnerable to the failure of the central server.

However, we conjecture that our dependency-centric viewpoint might also apply to measuring robustness and this is something we intend to explore in future work. In particular, there are two aspects to dependencies that appear important to robustness. The first is the *vulnerability* of the system which could be captured by counting the "reverse" dependencies of a state s as the number of output states that derive *from* s. The second aspect is the *extent* to which a piece of state is affected by its various dependencies and this is a function of both the importance of that dependency (*e.g.*, the address of a server vs. estimated latency to the server as a hint for better performance) and the degree to which redundancy makes the dependency less critical (*i.e.*, deriving a piece of state from any k of m inputs with $k \ll m$ is likely more robust that one derived from k specific inputs). The former consideration (importance) can be captured by weighting dependencies as proposed above. A fairly straightforward extension to capture the effect of redundancy would be to further weight complexity by the fraction of states required; *i.e.*, a weighted metric r_s of state s defined as: $r_s = \frac{r}{m} c_s$ where r and m are the required and available number of inputs, respectively.

7 Related Work

There is much work – particularly in software engineering – on measuring the complexity of a software *program*. For example, Halstead's measures [18] capture programming effort derived from a program's source code. Cyclomatic complexity [30], simply put, measures the number of decision statements. Fan in-fan out complexity [19] is a metric that measures coupling between program components as the length of code times the square of fan in times fan out. Kolmogorov complexity is measured as the length of the program's shortest description in a description language (e.g., Turing machine). These metrics work at the level of system implementation rather than design, focus on a standalone program and do not consider the distributed dependencies of components that are networked. We believe the latter are key to capturing complexity in networked systems and both viewpoints are valuable.

Similarly, there is much work on improved approaches to system *specification* with recent efforts that focus on network contexts [22]. Metrics are complementary to system specification and cleaner specifications would make it easier to apply metrics for analysis. An interesting question for future work is whether the computation of network complexity (as we define it here) can be derived from a system specification (or even code) in an automated manner. This appears non-trivial as the *accumulation* of distributed dependencies is typically not obvious at the program or specification level.

While we derive our dependency-based metric from a system design, there have been many recent efforts at *inferring* dependencies or causality graphs from a *running* system for use in network management, troubleshooting, and performance debugging [4, 12, 16].

Finally, this paper builds on an earlier paper that articulated the need for improved complexity metrics [34].

8 Conclusions

This paper takes a first step towards quantifying the intuition for design simplicity that often guides choices for practical systems. We presented a metric that measures the impact of the ensemble of distributed dependencies for an individual piece of state and apply this metric to the evaluation of several networked system designs. While our metric is but a first step, we believe the eventual ability to more rigorously quantify design complexity would serve not only to improve our own design methodologies but also to better articulate our design aesthetic to the many communities that design for real-world networked contexts (*e.g.*, algorithms, formal distributed systems, graph theory).

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Notes

[1] We thank Paul Francis and Robert Kleinberg for discussion on this.

[2] We include this since the time complexity of distributed algorithms is commonly used in the theory community [24, 27]. Time complexity is the maximum number of message-exchange rounds needed to complete the required computation.

[3] This can be inferred by noting that route construction is similar to the canonical "m inputs in series" scenario from the previous section.

[4] This is quickly inferred by noting the similarity to the "m inputs in parallel" scenario with $m = d$ inputs relayed along a path of O(d) hops and transport state of complexity ε at each hop.

[5] This use of source routing is the key difference relative to RCP which uses the underlying intra-domain routes for the same purpose.

[6] We do not consider name-dependent algorithms [10, 29] as these require an additional name translation service for IP routing.