CS 188 Fall 2005 — Introduction to AI — Stuart Russell

Final Solutions

1. (12 pts.) Some Easy Questions to Start With

(a) (2) True. This follows from the property that each variable is independent of its predecessors given its parents. Since $X_1, \ldots, X_k$ have no parents, they are absolutely independent.

(b) (2) True. Since both players are perfectly rational, each player receives the minimax value of the game from their point of view. The other player has a choice of any optimal strategy (there may be several), but they all have the same value.

(c) (2) True. This follows from monotonicity.

(d) (2) False. Consider $\alpha \equiv A$, $\beta \equiv B$, $\gamma \equiv (A \wedge B)$.

(e) (2) False. They may contain identical ground literals.

(f) (2) False. New elements may be added to categories, but the set of categories almost never changes.

2. (15 pts.) Search

(a) (4) (iii) n^{2n}. There are n vehicles in n^2 locations, so roughly (ignoring the one-per-square constraint) $(n^2)^n = n^{2n}$ states.

(b) (3) (iii) 5^n.

(c) (2) Manhattan distance, i.e., $|(n-i+1)-x_i| + |n-y_i|$. This is exact for a lone vehicle.

(d) (2) Only (iii) $\min\{h_1, \ldots, h_n\}$.

(e) (4) The explanation is nontrivial as it requires two observations: first, the total work required to move all n vehicles is $\geq n min\{h_1, ..., h_n\}$; second, the total work we can get done per step is $\leq n$. Hence, completing all the work requires at least $n min\{h_1, ..., h_n\}/n = min\{h_1, ..., h_n\}$ steps.

3. (16 pts.) Propositional Logic

(a) (4) $S^{t+1} \Leftrightarrow [(S^t \wedge a^t) \vee (\neg S^t \wedge b^t)]$.

(b) (4) Because the agent can do exactly one action, we know that $b^t \equiv \neg a^t$ so we replace b^t throughout. We obtain four clauses:
1: $(\neg S^{t+1} \vee S^t \vee \neg a^t)$
2: $(\neg S^{t+1} \vee \neg S^t \vee a^t)$
3: $(S^{t+1} \vee \neg S^t \vee \neg a^t)$
4: $(S^{t+1} \vee S^t \vee a^t)$

(c) (8) The goal is $(\neg S^t \wedge a^t) \Rightarrow \neg S^{t+1}$. Negated, this becomes three clauses: 5: $\neg S^t$; 6: a^t; 7: S^{t+1}. Resolving 5, 6, 7 against 1, we obtain the empty clause.

4. (15 pts.) Pruning in search trees

(a) (2) No pruning. In a max tree, the value of the root is the value of the best leaf. Any unseen leaf might be the best, so we have to see them all.

(b) (2) No pruning. An unseen leaf might have a value arbitrarily higher or lower than any other leaf, which (assuming non-zero outcome probabilities) means that there is no bound on the value of any incompletely expanded chance or max node.

(c) (2) No pruning. Same argument as in (a).

(d) (2) No pruning. Nonnegative values allow *lower* bounds on the values of chance nodes, but a lower bound does not allow any pruning.

(e) (2) Yes. If the first successor has value 1, the root has value 1 and all remaining successors can be pruned.

(f) (2) Yes. Suppose the first action at the root has value 0.6, and the first outcome of the second action has probability 0.5 and value 0; then all other outcomes of the second action can be pruned.

(g) (3) (ii) Highest probability first. This gives the strongest bound on the value of the node, all other things being equal.

5. (8 pts.) MDPs

	π^0	V^{π^0}	π^1	V^{π^1}	π^2
S	a	6	a	6	a
$\neg S$	a	4	b	5	b

6. (16 pts.) Probabilistic inference

(a) (3) (ii) and (iii). (For (iii), consider the Markov blanket of M.)

(b) (2) $P(b, i, \neg m, g, j) = P(b)P(\neg m)P(i|b, \neg m)P(g|b, i, \neg m)P(j|g)$
$= .9 \times .9 \times .5 \times .8 \times .9 = .2916$

(c) (4) Since B, I, M are fixed true in the evidence, we can treat G as having a prior of 0.9 and just look at the submodel with G, J:
$\mathbf{P}(J|b, i, m) = \alpha \sum_g \mathbf{P}(J, g) = \alpha[\mathbf{P}(J, g) + \mathbf{P}(J, \neg g)]$
$= \alpha[\langle P(j, g), P(\neg j, g)\rangle + \langle P(j, \neg g), P(\neg j, \neg g)\rangle$
$= \alpha[\langle .81, .09\rangle + \langle 0, 0.1\rangle] = \langle .81, .19\rangle$
That is, the probability of going to jail is 0.81.

(d) (2) Intuitively, a person cannot be found guilty if not indicted, regardless of whether they broke the law and regardless of the prosecutor. This is what the CPT for G says; so G is context-specifically independent of B and M given $I = false$.

(e) (5) A pardon is unnecessary if the person is not indicted or not found guilty; so I and G are parents of P. One could also add B and M as parents of P, since a pardon is more likely if the person is actually innocent and if the prosecutor is politically motivated. (There are other causes of $Pardon$, such as $LargeDonationToPresidentsParty$, but such variables are not currently in the model.) The pardon (presumably) is a get-out-of-jail-free card, so P is a parent of J.

7. (18 pts.) Language and statistical learning

(a) (3) (i).

(b) (5) This has two parses. The first uses $VP \rightarrow VP\ Adverb$, $VP \rightarrow Copula\ Adjective$, $Copula \rightarrow is$, $Adjective \rightarrow$ **well**, $Adverb \rightarrow$ **well**. Its probability is $0.2 \times 0.2 \times 0.8 \times 0.5 \times 0.5 = 0.008$. The second uses $VP \rightarrow VP\ Adverb$ twice, $VP \rightarrow Verb$, $Verb \rightarrow is$, and $Adverb \rightarrow$ **well** twice. Its probability is $0.2 \times 0.2 \times 0.1 \times 0.5 \times 0.5 \times 0.5 = 0.0005$. The total probability is 0.0085.

(c) (2) (i) (ii).

(d) (2) True. There can only be finitely many ways to generate the finitely many strings of 10 words.

(e) (1) (ii) MAP learning. It cannot be Bayesian learning because it outputs only one hypothesis; it cannot be maximum likelihood because it takes complexity into account. If $C(h)$ is linearly related to $\log P(h)$ then the algorithm is doing MAP learning.

(f) (5) The prior is represented by rules such as

$$P(N_0 = A): \qquad S \rightarrow A\ S_A$$

where S_A means "rest of sentence after an A." Transitions are represented as, for example,

$$P(N_{t+1} = B|N_t = A): \qquad S_A \rightarrow B\ S_B$$

and the sensor model is just the lexical rules such as

$$P(W_t = \text{is}|N_t = A): \qquad A \rightarrow \text{is}\ .$$