DYNAMIC BEHAVIORS IN TWO NOVEL AUTONOMOUS CIRCUITS WITH A PHYSICAL NEGATIVE RESISTOR CHARACTERIZED BY NON-SYMMETRIC FUNCTION *

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***Abstract*—In this paper we present two novel three-order autonomous circuits. The one of salient features of the circuits is that the nonlinear element in the circuits can be any negative resistor, and be not limited to the piecewise-linear. Using a type-S negative resistance element, we observed the double scroll attractors similar to that in Chua's circuit. Using an avalanche transistor, we observed more complicated and interested dynamic behaviors than that in the other autonomous circuits with piecewise-linear and symmetric element.**

I. INTRODUCTION

Recently more and more three-order autonomous circuits, such as the Chua's circuit [1], the torus circuit [2], the double hook circuit [3] and the canonical piecewise-linear circuit [4], etc., have been proposed and analyzed. Piecewise-linear circuits have emerged as a simple yet powerful experimental and analytical tool in studying bifurcation and chaos in nonlinear dynamics [5]. In order to obtain idealized symmetric characteristic the piecewise-linear devices are generally realized by Op Amps circuits . It is well know that many second-order nonautonomous circuits with a single physical nonlinear resistance element displayed the complicated chaotic and bifurcation phenomena [6-9]. But no such physical autonomous circuit with a negative resistor characterized by non-symmetric function has been reported to date.

In this paper, two novel three-order autonomous circuits are presented. In the circuits the constraint on nonlinear elements is only to have negative resistance characteristic. The negative resistance region may merely exist in the first quadrant at $u-i$ plane, and need not be symmetric about the origin. Using a type-S negative resistance element, we observed double scroll attractors similar to that in Chua's circuit. Using an avalanche transistor, we observed many interested attractors and more details on bifurcation.

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II. THE CIRCUITS AND THE STATE EQUATIONS

Two novel three-order autonomous circuits, denoted by the circuit (I) and the circuit (II) respectively, are shown in Fig. 1 and Fig. 2.

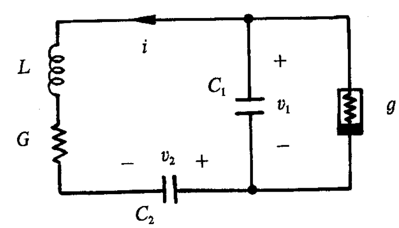

Fig. 1 The circuit (I) model

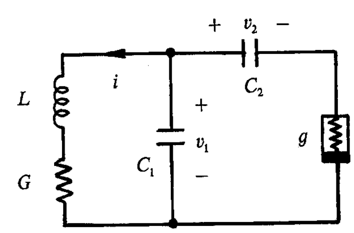

Fig. 2 The circuit (II) model

The dynamics of the circuit (I) is described by the following state equations:

$$C_1 \frac{dv_1}{dt} = -gv_1 - i$$
$$C_2 \frac{dv_2}{dt} = -i \qquad (1)$$
$$L \frac{di}{dt} = v_1 + v_2 - \frac{i}{G}$$

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The state equations of the circuit (II) are

$$C_1 \frac{dv_1}{dt} = -g(v_1 - v_2) - i$$

$$C_2 \frac{dv_2}{dt} = g(v_1 - v_2) \quad (2)$$

$$L \frac{di}{dt} = v_1 - \frac{i}{G}$$

In the following, our discuss is to pay particular attention to the circuit (I), as a representative example.

III. REALIZATION OF THE DOUBLE SCROLL

First we consider the case when the nonlinear element in the Fig. 1 is a type-S negative resistor, whose measured $u-i$ characteristic is shown in Fig. 3. This negative resistor is driven by a DC current source I_S, as shown in Fig. 4. The DC operating point is Q in Fig. 5. Since the DC source and the dynamic system are isolative

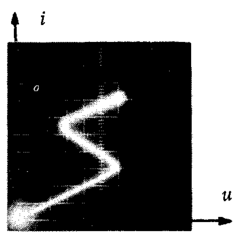


Fig. 3 Measured $u-i$ curve of the type-S negative resistor

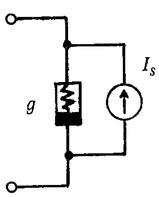


Fig. 4 The negative resistor driven by a source I_S

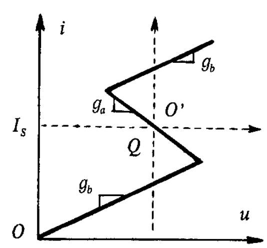


Fig. 5 The piecewise-linear curve

each other, after parallel-axis the quiescent point Q becomes the new origin O' for the dynamic circuit. From Fig. 5 we can see the characteristic is piecewise-linear and symmetric about the origin O'. Using the same analysis method as in [4], we obtained the characteristic equation of the equations (1) in the negative region (see Fig. 5) is

$$s^3 + \left(\frac{g_a}{C_1} + \frac{1}{LG}\right)s^2 + \left(\frac{1}{LC_1} + \frac{1}{LC_2} + \frac{g_a}{LGC_1}\right)s + \frac{g_a}{LC_1C_2} = 0 \quad (3)$$

or

$$s^3 - p_1 s^2 + p_2 s - p_3 = 0 \quad (4)$$

where

$$\frac{g_a}{C_1} + \frac{1}{LG} = -p_1 \quad (5)$$

$$\frac{1}{LC_1} + \frac{1}{LC_2} + \frac{g_a}{LGC_1} = p_2 \quad (6)$$

$$\frac{g_a}{LC_1C_2} = -p_3 \quad (7)$$

The eigenvalue constraint on the circuit (I) is

$$(p_1 - q_1)(p_3 q_1 - p_1 q_3) = (p_2 - q_2)(p_3 - q_3) \quad (8)$$

where

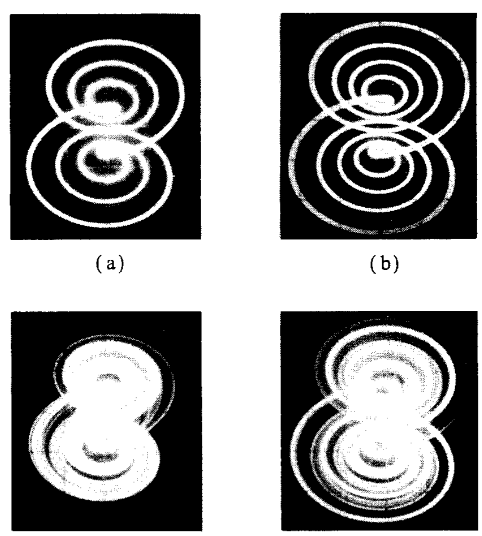
$$\frac{g_b}{C_1} + \frac{1}{LG} = -q_1 \quad (9)$$

$$\frac{1}{LC_1} + \frac{1}{LC_2} + \frac{g_b}{LGC_1} = q_2 \quad (10)$$

$$\frac{g_b}{LC_1C_2} = -q_3 \quad (11)$$

It is very interested that the equation (8) is in good agreement with the constraint on Chua's circuit [4], although their circuits and characteristic equations are obviously different. Some double scroll pectures of the chaotic attractors and the periodic attractors are shown in Fig. 6.

For the circuit (II), the results of analysis and observation are similar to above.



(c) (d)

Fig. 6 (a)(b) The periodic attractors. (c)(d) The chaotic attractors

IV. MORE COMPLICATED DYNAMIC BEHAVIORS

In this section, we will consider the case when the property of the nonlinear resistance element in Fig. 1 is not characterized by piecewise − linear function. Let us see a example. The measured $u-i$ curve of the physical negative resistor, as shown in Fig. 7, is avalanche characteristic of a transistor.

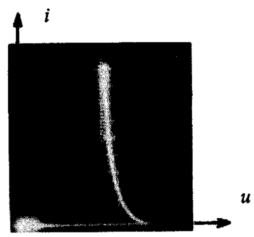


Fig. 7 The measured avalanche characteristic

Transform the equations (1) into the following form

$$\begin{bmatrix} \frac{dx}{d\tau} \\ \frac{dy}{d\tau} \\ \frac{dz}{d\tau} \end{bmatrix} = \begin{bmatrix} -\alpha & 0 & -1 \\ 0 & 0 & -1 \\ 1 & \beta & -\gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (12)$$

where

$$x=\frac{v_1}{I}\sqrt{\frac{C_1}{L}}\ ,\ y=\frac{v_2 C_2}{I\sqrt{LC_1}}\ ,\ z=\frac{i}{I}\ ,$$

$$\tau=\frac{t}{\sqrt{LC_1}}\ ,\ \alpha=g\sqrt{\frac{L}{C_1}}\ ,\ \beta=\frac{C_1}{C_2}\ ,\ \gamma=\frac{G_0}{G}\ . \qquad (13)$$

The characteristic equation is

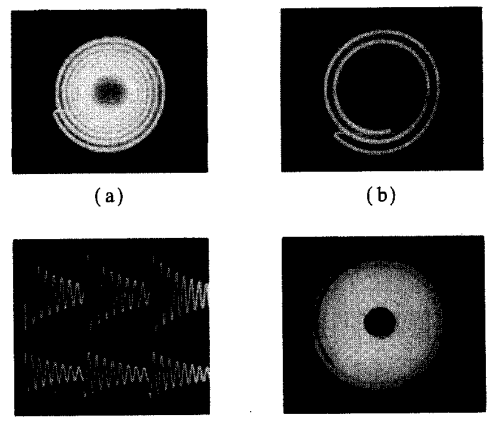
$$s^3+(\alpha+\gamma)s^2+(\alpha\gamma+\beta+1)s+\alpha\beta=0 \qquad (14)$$

Fixing the values of parameters G, L, C_1 and C_2, the eigenvalues of the equation (14) are only dependent on dynamic conductance g ($g<0$). From Fig. 7 we see the curve is monotonically decreasing when $i>0$. The negative resistor is also driven by a DC current source I_S, so the value of g is dependent on I_S. As the I_S is varied, the eigenvalues present various patterns. Physically, it causes chaos and bifurcation phenomena to appear alternatively. Through a lot of experiments and analyses the following results have been obtained:

1. We use a smooth variable DC current sorce I_S for driving the negative resistor. As we adjust coarsely the values of I_S from small to large, with L, G, C_1 and C_2 held fixed and $C_1>C_2$, we can easily observe periodic attractors and chaotic attractors appear alternatively, that is

$$\cdots,\ 12\ ,\ \text{chaos}\ ,\ 11\ ,\ \text{chaos}\ ,\ \cdots,\ 3\ ,\ \text{chaos}\ ,\ 2\ ,\ \cdots$$

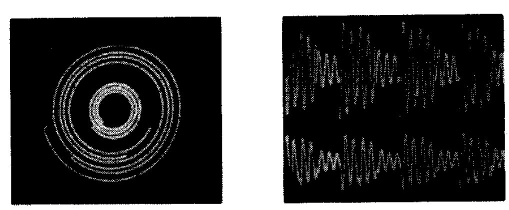
Fig. 8 (a) and (b) give two different period − attractors, Fig. 8 (c) is the time waveform corresponding to Fig. 8 (a). Fig. 8 (d) shows a chaotic attractor.



(a) (b) (c) (d)

Fig. 8 A group of the pictures when $C_1>C_2$

With fine adjustment for I_S, we can observe minute bifurcations (as the devil's staircase). By means of experimental verifications we discover these minute periodic subsequences are the same to that in nonautonomous systems reported by [10]. For example, between 1/3 (period − 3) and 1/2 (period − 2) we observed 6/17, 4/11, 3/8 , 2/5 , 5/12 , 3/7 , 5/11. Fig. 9 (a) and Fig. 9 (b) show the attractor and the time waveform corresponding to 3/7.



(a) (b)

Fig. 9 The attractor and the time waveform for 3/7

2. When L, G, C_1 and C_2 held fixed and $C_1<C_2$, repeating above process, the forms of the attractors are different from that when $C_1>C_2$, but the alternation sequences of chaos and bifurcation are alike. Fig. 10 give some periodic and chaotic attractors at $C_1<C_2$.

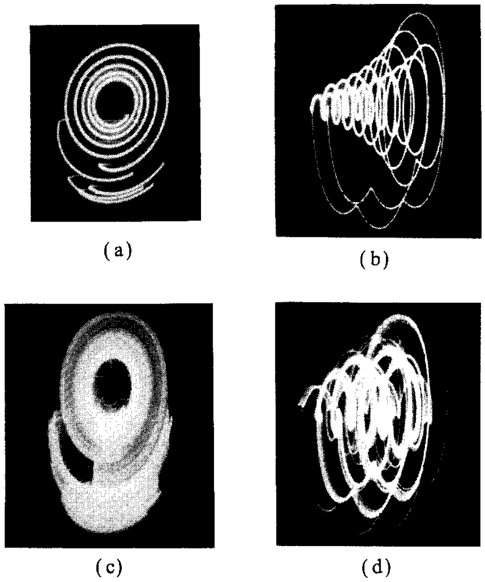


Fig. 10 Some new attractors when $C_1 < C_2$

3. While I_S is constant, with varying the value of L, C_1 or C_2 respectively, chaos and bifurcations are similar to above. The shapes of the attractors mainly depend on the parameter β, that is, when $\beta > 1$ ($C_1 > C_2$), they resemble the result 1, and when $\beta > 1$ ($C_1 < C_2$), they resemble the result 2.

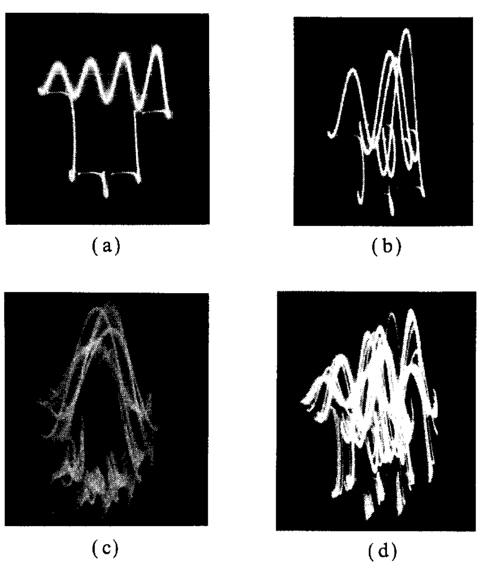


Fig. 11 Some new attractors for the circuit (II)

For the circuit (II), the same analysis and experiment have been performed. This circuit also exhibited rather complicated dynamic behaviors. Here, we only give some pictures of the attractors, as shown in Fig. 11.

V. CONCLUSIONS

Two novel three-order autonomous circuits have been presented in this paper. The nonlinear element in the circuits may be any physical negative resistor, either piecewise-linear or irregular. This is very easy to realize, as in nonautonomous systems. Besides the double scroll attractors, bifurcation and chaos behaviors in the circuits are more complicated and interested than that in the other autonomous circuits with a piecewise-linear element, when the nonlinear resistor is irregular. This work provided a new way for the study of dynamic behaviors in autonomous circuits.

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