Thus, if (4) is satisfied then

$$\Delta v(i,j) \leq 0 \tag{9}$$

where the equality sign holds only when $\mathbf{x}_{01}(i,j) = \mathbf{x}_{10}(i,j) = \mathbf{0}$. The remainder of the proof follows on similar lines as in [1].

It may be mentioned that the relation given by $\delta' = \mathbf{f}^T(\mathbf{y}(i,j))\mathbf{D}[\mathbf{y}(i,j) - \mathbf{f}(\mathbf{y}(i,j))] \geq 0$ is used in [1], which covers [2] not only the saturation arithmetic but also all other arithmetics (including zeroing arithmetic) belonging to the sector $[0,1]$[1]. By contrast, presently the nonnegativeness of δ [see (7)] is used, which exploits the structural properties of the multiple saturation nonlinearities in a greater detail (i.e., the so-called passivity property)[2] than merely the sector information. Observe that with $\lambda_{kl} = 0,\ k,\ l = 1, 2, \ldots, n\ (k \neq l)$, $\mathbf{C}$ [see (4b)–(4d)] is identified as the diagonal matrix $\mathbf{D} = [d_{kk}] = [\gamma_k]$ and consequently (4) reduces to (3). Thus, as compared to the approach in [1], these λ's are the additional degrees of freedom [subject to (4d), of course] which would result in an enhanced saturation overflow stability region in the parameter space [3]. □

III. Conclusion

A criterion for the asymptotic stability of the Fornasini–Marchesini second LSS model using saturation arithmetic is presented. The criterion is less stringent than that due to Hinamoto [1].

References

[1] T. Hinamoto, "Stability of 2-D discrete systems described by the Fornasini–Marchesini second model," *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 254–257, Mar. 1997.

[2] V. Singh, "A new realizability condition for limit cycle free state-space digital filters employing saturation arithmetic," *IEEE Trans. Circuits Syst.*, vol. CAS-32, pp. 1070–1071, Oct. 1985.

[3] H. Kar and V. Singh, "A new criterion for the overflow stability of second-order state-space digital filters using saturation arithmetic," *IEEE Trans. Circuits Syst. I*, vol. 45, pp. 311–313, Mar. 1998.

[4] V. Singh, "Elimination of overflow oscillations in fixed-point state-space digital filters using saturation arithmetic," *IEEE Trans. Circuits Syst.*, vol. 37, pp. 814–818, June 1990.

[5] V. Singh, "A generalized approach for the absolute stability of discrete-time systems utilizing the saturation nonlinearity, based on passivity properties," *IEEE Trans. Circuits Syst.*, vol. 37, pp. 444–447, Mar. 1990.

[6] I. W. Sandberg, "The zero-input response of digital filters using saturation arithmetic," *IEEE Trans. Circuits Syst.*, vol. CAS-26, pp. 911–915, Nov. 1979.

[7] D. Mitra, "The absolute stability of higher-order discrete-time systems utilizing the saturation nonlinearity," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 365–371, June 1978.

[1] It may be mentioned that the idea of characterizing saturation nonlinearities based on sector information was first reported in [2], in the context of one-dimensional (1-D) state-space digital filters.

[2] See, for instance, [3]–[7] regarding the use of the passivity property for deriving improved saturation overflow stability conditions for 1-D digital filters.

A New Technique for Chaos Prediction in RF Circuit Design Using Harmonic-Balance Commercial Simulators

Almudena Suárez and Juan-Mari Collantes

***Abstract*—In this paper a new method is presented for the detection of chaotic responses using harmonic balance commercial simulators. The method is applicable to circuits exhibiting homoclinic bifurcation routes. The successive period-doublings, often preceding the formation of homoclinic orbits, are predicted and calculated. This is done by means of a new probe technique that overcomes the harmonic balance difficulties when dealing with subharmonic frequencies. This calculation of the steady-state frequency divided responses has allowed an accurate prediction of the onset of chaos, through a homoclinic orbit, in the time-delayed Chua's circuit (TDCC). The method is specially suitable for circuits containing transmission lines that cannot be analyzed by standard time-domain simulations.**

***Index Terms*— Chaos, harmonic balance, homoclinic orbit, period-doubling, time-delayed Chua's circuit.**

I. Introduction

Circuits with chaotic behavior are at present finding many applications in secure communications, as the possibility of designing synchronizing systems driven by chaotic signals has been rigorously proved [1]. For other communication systems, this sort of response may be undesirable. In both cases, the availability of simulation tools enabling the chaos prediction is of a high interest for the designer.

Time-domain simulation techniques are ill-suited for studying the dynamics of circuits with elements described by their spectral responses. Such is the case with high frequency circuits, in which transmission lines are currently used. In addition, their typical long transients with respect to the signal period make time-domain simulation excessively time-consuming. The time-delayed Chuas circuit (TDCC), exhibiting a homoclinic route to chaos, is a particular example of a circuit whose dynamic at high frequencies is hard to investigate by means of time-domain simulations. Gilli and Maggio [2] proposed an attractive analytical approach, based on the work of Genesio and Tesi [3], to approximately predict chaos in the TDCC. In [2], the existence of limit cycles is analytically determined by means of the describing function technique. Then, by computing the distortion index, chaos can be approximately predicted. This analytical method turns out to be excessively cumbersome at high frequencies, due to the presence of parasitic elements. Harmonic balance (HB) is the usual tool when dealing with high-frequency circuits. The HB technique has been applied to the analysis of the most usual bifurcations from the periodic regime [4] and, in a recent work, it has been extended to the analysis of phase locking and other bifurcations from the quasiperiodic regime [5]. In this paper, HB standard simulations are applied to the prediction of chaos.

The HB technique has the drawback of badly dealing with subharmonic frequencies, as it converges by default to the mathematical solutions forced by the external generators, even when the frequency basis is properly determined [6]. This is a serious difficulty in the

Manuscript received March 11, 1998; revised October 5, 1998. This paper was recommended by Associate Editor C. W. Wu.

A. Suárez is with the Department of Ingeniería de Comunicaciones, Universidad de Cantabria, s/n 39005 Santander, Spain.

J.-M. Collantes is with the Electricity and Electronics Department, University of the Basque Country, Bilbao, Spain.

Publisher Item Identifier S 1057-7122(99)09250-8.

TABLE I
COMPARISON BETWEEN THE BIFURCATION PARAMETER VALUES OBTAINED WITH TIME-DOMAIN SIMULATIONS AND HB

	Free running oscillation start up	Period-two start up	Period-four start up	Onset of chaos
HB technique	τ = 405.2ns	τ = 464.0ns	τ = 472.0ns	τ = 473.8ns
SPICE	τ = 404.8ns	τ = 463.8ns	τ = 471.5ns	τ = 473.5ns

simulation of the period doublings usually preceding the formation of the homoclinic orbit. In this paper, a new method, based on the probe concept [6], is developed for overcoming the HB limitations in the analysis of these successive period doublings. When all the harmonics and subharmonics are taken into account, the onset of chaos is predicted with a high accuracy and there is no need for the calculation of the distortion index.

The new method has been applied to the TDCC described in [2], obtaining the successive period-two and period-four steady-state solutions and the parameter values for the onset of chaos through the detection of the homoclinic orbit. An excellent agreement has been obtained with SPICE simulations.

II. HB Analysis of the Homoclinic Route to Chaos

A. Homoclinic Route

A common cause of chaos in nonlinear systems is the formation and breakup of a homoclinic orbit $\phi(t)$ as a parameter is varied. This orbit satisfies $\phi(t) \rightarrow X_0$ when $t \rightarrow \pm\infty$, with X_0 being a saddle fixed point [7]. Then $\phi(t)$ must lie in both the stable $W^s(X_0)$ and unstable $W^u(X_0)$ manifolds of X_0 for all t. For a small perturbation ε of the homoclinic orbit $\phi(t)$, the manifolds may or not intersect, but if they do intersect they will, in general, do it transversally, obtaining what is known as a homoclinic tangle [8]. This gives rise to the stretching and folding of the iterates of the Poincaré map (Smale horseshoe), obtaining a chaotic behavior [7]–[8]. In circuits with more than one equilibrium point, the homoclinic orbit may form from the collision of a limit cycle, originated at an unstable focus, with a saddle equilibrium point as a parameter is modified. The formation of this orbit is often preceded by period doubling bifurcations [3], [9]. This is the behavior pattern followed by the TDCC.

B. Analysis Method

The successive period doublings preceding the formation of the homoclinic orbit are analyzed here through HB by means of a two-stage method: period doubling bifurcation prediction, from a period-2^{n-1} state, and determination of the steady-state period-2^n solution. The lack of generators at the autonomous and frequency divided components is overcome by making use of probes at the corresponding frequencies. Probes are virtual generators, having no influence at frequencies different from their own [6]. Two different types will be distinguished: the perturbation probes, introducing small amplitude perturbations into the circuit, which will be used for the stability analysis and the nonperturbation probes, satisfying a nonperturbation condition of the steady state.

Let ω_0 be the fundamental frequency of a free-running oscillating solution, determined through the standard oscillation analysis tools of the HB commercial simulator, and V_0, the corresponding fundamental amplitude, at a sensitive observation port. Suitable ones are those defined between the nonlinear device terminals. In order to check for the stability of the V_0, ω_0 solution, two probes will be inserted in parallel at the observation port: a nonperturbation probe, having a voltage value $V_{p1} = V_0$ and frequency $\omega_{p1} = \omega_0$, and a perturbation

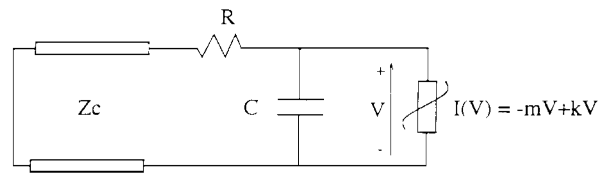


Fig. 1. Time-delayed Chua's circuit. Parameter values: $m = 4/5$, $k = 2/45$, $R = 1.43\ \Omega, C = 0.109\ \mu$F and $Z_c = 1/\sqrt{7}\ \Omega$.

probe, of neglecting amplitude $V_{p2} = \varepsilon$ and frequency $\omega_{p2} = \omega_{p1}/2$. The nonperturbation probe is needed in order to ensure the HB convergence toward the autonomous solution as, in the presence of the perturbation probe, it will be no longer possible to perform a standard oscillation analysis in the commercial simulator. In order to detect a possible period doubling, an HB sweep will be carried out in the perturbation probe phase φ_{p2}, evaluating the total input admittance Y, including linear and nonlinear contributions, at the observation port. If the conditions $\mathrm{Re}[Y(\omega_0/2)] < 0$, $\mathrm{Im}[Y(\omega_0/2)] = 0$ are fulfilled for a certain phase value, a period-two solution will generally exist [6].

Once the period-doubled solution has been predicted, the corresponding steady state will be obtained through a standard optimization on the variables $V_{p1}, V_{p2}, \omega_{p1}, \varphi_{p2}$, with the constraint $\omega_{p2} = \omega_{p1}/2$. The goal will be the zero value of the admittance function at the observation port when evaluated at the two probe frequencies ω_{p1} and ω_{p2}. When this condition is satisfied, both probes will become nonperturbation probes.

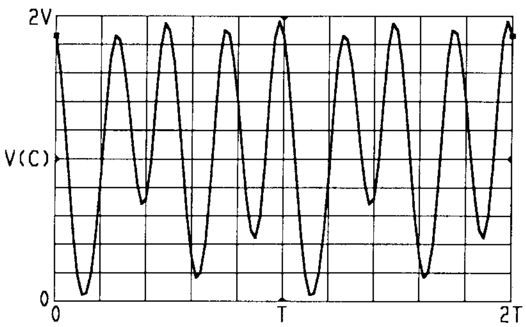
The prediction of a possible frequency division by four will be carried out by keeping both nonperturbation probes at the values resulting from the former optimization and introducing a third perturbation probe at $\omega_{p3} = \omega_{p1}/4$. The latter will be used for the next period-doubling prediction through a phase sweep, as formerly described. The period-four steady-state will be obtained by optimization of the probe values $V_{p1}, V_{p2}, V_{p3}, \omega_{p1}, \varphi_{p2}, \varphi_{p3}$, with the constraints $\omega_{p2} = \omega_{p1}/2$ and $\omega_{p3} = \omega_{p1}/4$. The optimization goal is now provided by the three probe admittance functions equal to zero. Optimization times in a workstation HP-720 of the 9000 series are of about 2 min.

If further frequency divisions are present, they can be obtained by recursively applying the previous technique. However, their neglecting influence on the waveform often make this unnecessary.

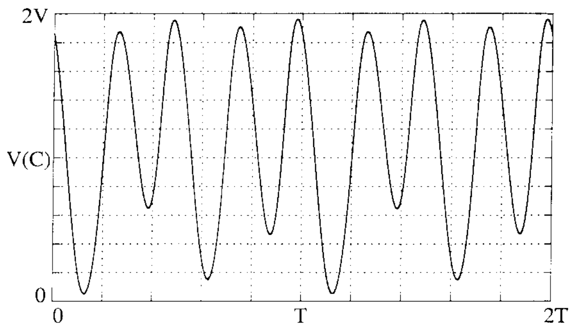
Once the waveform has been accurately obtained, the parameter values for the onset of chaotic behavior will be determined from the collision between the limit cycle and the saddle-type equilibrium point [3].

C. Application to the TDCC

The method presented above has been applied to the TDCC (Fig. 1). The Chua's diode has been approximated by the van der Pol function. For the sake of consistency with a previous work [2], the transmission line is ideal, having a characteristic impedance $Z_c = 1/\sqrt{7}\ \Omega$ and a time delay τ. The circuit dynamics, as a function



(a)



(b)

Fig. 2. Period-four waveform for $\tau = 473.0$ ns. (a) From the new HB technique, the obtained signal period is $T = 12.7$ μs. (b) From time domain simulations, the obtained signal period is $T = 12.6$ μs.

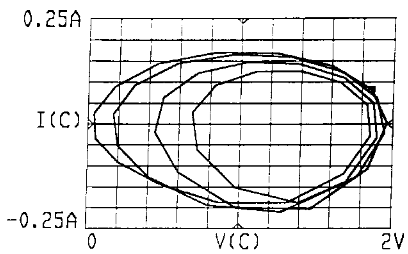


Fig. 3. HB prediction of the homoclinic orbit. Collision between the period-four cycle and the saddle-type equilibrium point for $\tau = 473.8$ ns.

of τ, have been studied through the proposed technique by making use of the HB commercial simulator HP-MDS. The bifurcation parameter values are shown in Table I, where they can be compared with the time-domain results from SPICE. The excellent agreement can be noted.

For $\tau = 473.0$ ns, the period-four waveforms resulting from the proposed HB technique and from time domain simulations are respectively shown in Fig. 2(a) and (b). The excellent agreement confirms the validity of the new technique. The limit cycle at the moment of the collision with the saddle-type equilibrium point, located at the origin [3], is depicted in Fig. 3. To our knowledge, it is the first time that such a limit cycle exhibiting period-four is obtained from a HB simulator.

III. Conclusions

A new method is proposed here for the detection of chaotic behavior from HB commercial simulators. It is especially suited for high-frequency circuits and circuits containing elements described in the frequency domain, for which time domain simulations would be excessively time consuming. The method applicability to commercial software has the advantages of a great flexibility and an easy utilization by RF circuit designers. The onset of chaos through a homoclinic route in the TDCC has been accurately predicted through the proposed method.

References

[1] L. M. Pecora and T. L. Carroll, "Driving systems with chaotic signals," *Phys. Rev. A*, vol. 44, no. 4, pp. 2374–2383, Aug. 1991.

[2] M. Gilli and G. M. Maggio, "Predicting chaos through an harmonic balance technique: An application to the time-delayed Chua's circuit," *IEEE Trans. Circuits Syst.*, vol. 43, pp. 872–874, Oct. 1996.

[3] R. Genesio and A. Tesi, "A harmonic balance approach for chaos prediction: Chua's Circuit," *Int. J. Bifurcation Chaos*, vol. 2, no. 1, pp. 61–79, Jan. 1992.

[4] V. Rizzoli and A. Neri, "State of the art and present trends in nonlinear microwave CAD techniques," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 343–356, Feb. 1988.

[5] A. Suárez, J. Morales, and R. Quéré, "Synchronization analysis of autonomous microwave circuits using new global stability analysis tools," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 494–504, May 1998.

[6] R. Quéré, E. Ngoya, M. Camiade, A. Suárez, M. Hessane, and J. Obregón, "Large signal design of broadband monolithic frequency dividers and phase-locked oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1928–1938, Nov. 1993.

[7] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*, 3rd ed. New York: Springer-Verlag, 1990.

[8] S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*. New York: Springer-Verlag, 1990.

[9] T. Matsumoto, "Chaos in electronic circuits," *IEEE Trans. Circuits Syst.* (*Special Issue on Chaotic Systems*), pp. 1033–1057, Aug. 1987.