CHAOS DETECTION IN MICROWAVE CIRCUITS USING HARMONIC BALANCE COMMERCIAL SIMULATORS

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ABSTRACT

In the present work a method has been developed for the detection of chaotic responses in microwave circuits. The new technique is based on the use of commercial harmonic balance software, which constitutes a major advantage for microwave circuit designers. In order to validate the new method, the bifurcation loci of Chua's chaotic circuit have been obtained and successfully compared with time domain simulations. The proposed method is readily applicable to IMPATT, Gunn and tunnel diode microwave oscillators.

INTRODUCTION

Chaos is an operating regime often found in microwave circuits of autonomous nature, such as oscillators and analog frequency dividers, which is characterized by a sensitive dependence on the initial conditions and a continuous spectrum [1]. Due to the latter characteristic, chaotic responses are associated to an anomalous increase of the noise level in the measured spectra. The prediction of these undesirable responses at the simulation step of the circuit design process is essential for MMIC design in order to reduce development cycles and lower manufacturing cost.

At the time to analyze the chaotic responses of microwave circuits, time domain simulations will not generally be possible, due to their long transients. On the other hand, the spectrum continuity prevents the use of frequency domain techniques, such as harmonic balance (HB), for the simulation of the steady chaotic solutions. When the main interest is the prediction of chaotic responses, a good strategy can be the detection of possible routes, or bifurcation sequences, leading to chaos.

In the present paper, a HB technique is proposed for the detection of chaotic behavior through the homoclinic route [2], which is a multi-harmonic generalization of the one presented by Genesio [3], and based on the use of the description function. A new specific technique has also been developed here in order to easily analyze this route to chaos using HB commercial software.

For a rigorous validation of the new simulation method, the well known Chua's circuit [4] has been chosen, as it represents a paradigm for the study of chaotic behavior. The classic two-dimensional bifurcation diagram has been traced from the HB commercial simulator HP-MDS. The use of discrete linear elements allowed a comparison with time domain simulations, obtaining an excellent agreement.

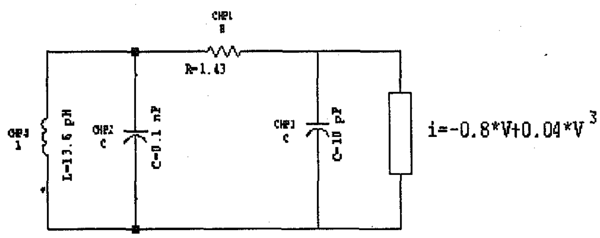

Fig.1 Chua's oscillator, based on a cubic nonlinearity. R=1.43Ω, L=13.6pH, C_1=10pF, C_2=0.1nF.

HOMOCLINIC ROUTE TO CHAOS

The constant (DC) solutions of a nonlinear circuit are given by the equilibrium points (EP) of the nonlinear differential equations, describing its behavior. For a saddle equilibrium point, the eigenvalues of the linearized circuit equations will have negative and positive real parts [2], respectively associated to its stable and unstable manifolds (nonlinear equivalents of the eigenspaces). Under some circumstances, the stable and unstable manifolds may intersect [2]. Then a trajectory leaving the equilibrium point through the unstable manifold will return to it through the stable one, giving rise to what is called a homoclinic orbit.

The theorem of Shilnikov establishes a relationship between the existence of homoclinic orbits and the presence near them of a Smale horseshoe [2] (stretching and folding of the Poincare mapping [2]), that will give rise to chaotic behavior. For a further modification of the parameter the homoclinic orbit will be destroyed, the solution period growing ad infinitum. In nonlinear circuits homoclinic orbits may form through the collision of a limit cycle with a saddle equilibrium point as a parameter is modified. The formation of this orbit is often preceded by period doubling bifurcations [3], [4].

CHAOS DETECTION THROUGH HARMONIC BALANCE

The new method is illustrated here by means of its application to a Chua's circuit, in which the nonlinear resistor has a van der Pol cubic characteristic (fig.1). In general, the method directly applies to the circuits in which the current-voltage characteristics are N-shaped like Gunn, and Tunnel diode oscillators [5]. Chua's circuit is commonly studied as a function of two parameters:

$$\alpha = \frac{C_2}{C_1} \qquad \beta = \frac{C_2}{LG^2}.$$

As these parameters vary different operating modes may be obtained, ranging from DC stable solutions to chaotic behavior.

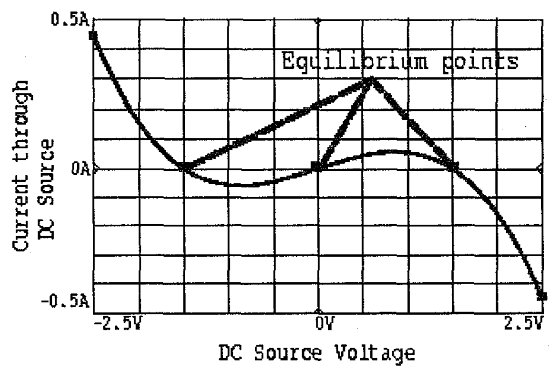

Fig.2 Determination of the autonomous circuit equilibrium points.

In order to determine the equilibrium points, a specific technique has been developed, based on the connection of an auxiliary DC source in parallel with the non-linearity. Then a DC sweep is performed over a wide range of voltage values. The equilibrium points will be given by the voltages values for which the current flowing through the DC source is equal to zero. In Chua's circuit three equilibrium points are found through this technique (fig.2):

V = -1.5 V, 0 V and 1.5 V, which agrees with the theoretical results [4].

In a second step the periodic solution, i.e. the limit cycle, existing for some particular ranges, must be obtained. However, for circuits with multiple equilibrium points, the standard oscillation test in HB software can search for the oscillation conditions around an equilibrium point different from the one at which oscillations start up. Such is the case of the circuit in fig.1, for which the oscillation test failed to find a solution, since it was systematically carried out around the equilibrium point V = 0V. Actually the oscillation starts up at the equilibrium points V=1.5V or V=-1.5V, through a Hopf type bifurcation. This difficulty is sorted out here by means of a judicious voltage shift in the non-linear function. Then the periodic solution is correctly simulated (fig.3).

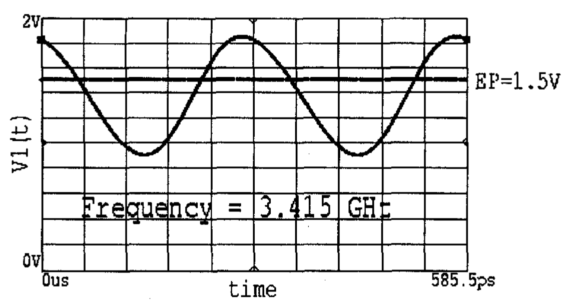

Fig.3 Free running oscillation around the equilibrium point V=1.5V, for α=7.4 and β=15.

The third step is the determination of the parameter values for frequency division by two. The conditions are checked out around the steady oscillation solution, obtained in the former step, making use of the so-called probe technique [6]. In the present case, two probes are connected in parallel with the non-linearity. The frequency and amplitude of probe 1 are respectively set equal to the frequency ω_0 and first harmonic amplitude V_1 of the steady oscillating solution, so this will not be perturbed. Probe 2 operates at $\omega_0/2$ having a neglecting amplitude, which represents a small signal perturbation at the divided frequency. Then the conditions for the start up of the period-two solution will be $\mathrm{Im}[Y(\omega_0/2)] = 0$ and $\mathrm{Re}[Y(\omega_0/2)] < 0$. To check for them, the phase of probe 2 is swept from 0° to 180°, representing the resulting admittance in a polar graph. In figure 4, the graphical prediction of a period-two solution is depicted.

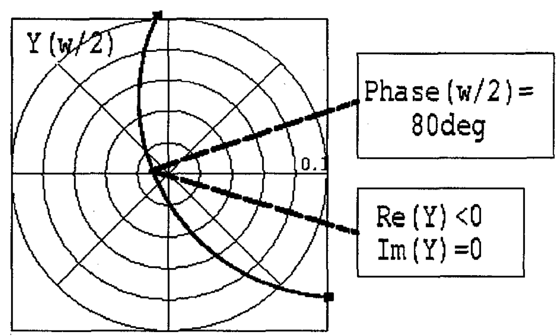

Fig.4 Prediction of a period doubled solution for α=9.09 and β=15.

Finally, the fourth step checks for the existence of a homoclinic orbit, due to the collision between the limit cycle and the equilibrium point. As an example, figure 5 shows the particular moment at which this interaction happens.

In fig.6 the complete bifurcation loci of Chua's circuit, as a function of α and β, have been obtained through the proposed technique, superimposing time domain simulations. The good agreement confirms the validity of the new method. The small discrepancies in the prediction of the chaotic operation border come from the fact that the limit cycle-equilibrium point interaction has been calculated neglecting the frequency division. To our knowledge this is the first time that the bifurcation loci of Chua´s circuit are obtained through harmonic balance and, furthermore, using commercial software.

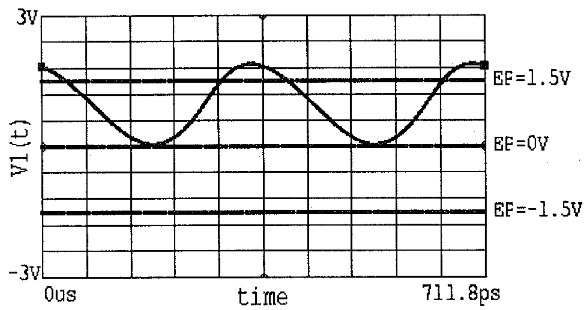

Fig.5 Interaction between the limit cycle and the equilibrium point V=0V, for α=9.7 and β=15

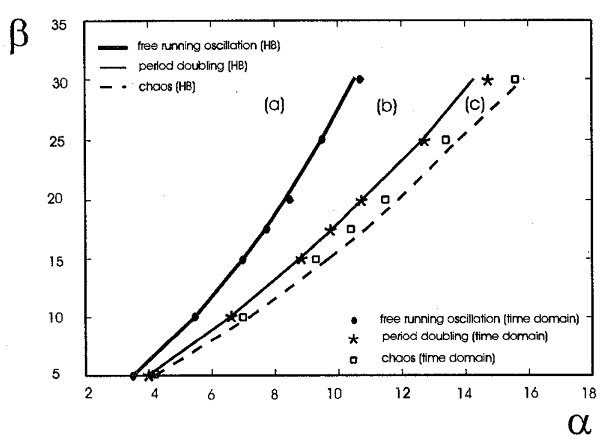

Fig.6 Bifurcation diagram. Region (a) DC stable solution. Region (b) free running oscillation. Region (c) period doubling.

CONCLUSION

In the present paper a new method is proposed for the detection of the onset of chaos through the harmonic balance technique. The method is thus specially suitable for microwaves frequencies, where time domain techniques are often inapplicable. In addition the new method makes use of HB commercial software, with the advantages of a great flexibility and an easy utilization by microwave circuit designers.

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