Novel Analogic Algorithm for Finding the Shortest-Path Using Simple CNN Structure

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ABSTRACT: *In this paper we describe a new analogic algorithm for finding the shortest-path in planar graphs explicitly, based on the continuous-time CNN templates only. The constant-speed autovawe propagation phenomena combined with elements of analog logic are encapsulated in two layers of basic CNNs, with simple cell-circuit structure. Compared to the 2-D array of the coupled Chua's circuits, widely used for modeling the autowave propagation, our realization is found to be simpler and, thus, more efficient for the VLSI implementation. Despite its simple structure, this system exhibits complex behavior and performs sophisticated processing resulting in the steady-state output which determines all shortest routes from origin to any point in a given graph. Then, the extraction of the shortest path itself is accomplished using the simple single-layer CNN.*

1. Introduction

The Cellular Neural Networks (CNNs), introduced by Chua and Yang [1] have found a wide range of applications in solving 2D problems especially in static image treatment. Further investigations show that the CNN is an universal processing structure [2]. Also, having in view some similarity between the Hopfield network and the CNN, it was shown [3] that CNN can solve optimization problems, too. The natural symbioses of our interest in optimizations and the CNN's common information environment (as, for instance, the static and/or dynamic images) resulted in the ideas of solving some graph optimization problems. Subsequently, the shortest-path problem arises as interesting challenge. As much as we know, there are some pioneer steps toward solving that and related kind of problems using CNNs [4, 5]. The results are encouraging but not totally refined for the practical purposes. Namely, a way of finding the length of a shortest path in planar labyrinth using continuous-time CNN (2-D array of coupled Chua's circuits) [4] leads to the result, but needs to track the autowave propagation by an external digital computer. Also, it is not possible to obtain the preferred path directly from CNN's outputs. On the other hand, the piecewise-linear unity-gain discrete-time CNN [5] applied in the PCB layout design, implements the well-known Dijkstra algorithm and it is capable to find the shortest path but only in the case of orthogonal, four-connected environment. Despite the limited domain of application concerning the very specific topology, still the main disadvantage of this approach is the necessity of non-linear templates which are difficult to realize in VLSI.

Considering the best from both approaches [4, 5], we conclude that the wave propagation phenomena observed in continuous-time CNNs [6, 7] combined with the elements of logic implied in discrete-time CNNs can result in solving the shortest-path problem in more applicable way. Also, our final goal was not only to modify the existing methods but to define a totally new approach and a way of realization of the parallel analog CNN circuitry for solving a more general class of problems - trajectory optimization.

2. The Shortest-Path Analogic Algorithm

For the outset, let us precisely define the problem statement, the inputs and the desired output:

Problem statement:	Find the shortest path between two points in the planar graph.
Input elements:	Graph image (map) and positions of two points of interest (the origin, O, and the target, T).
Desired result:	Route of the shortest path between given points O and T.

The input image, graph itself, must comply with the following rules:

- every line (i.e., branch) must be one pixel tick,

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- in crosses (i.e., nodes), lines are connected in such a way that two lines have no adjacent pixels except the node itself, and
- two nodes must not be adjacent.

Note that these rules are general and invariant but their graphical interpretation changes if the different image geometry is chosen. The examples for some legal and illegal nodes in standard square (eight-connected) geometry are shown in Fig. 1. It is not hard to prove that those rules are not restrictive because any scanned graph image can be transformed to comply with these rules without changing the path-lengths.

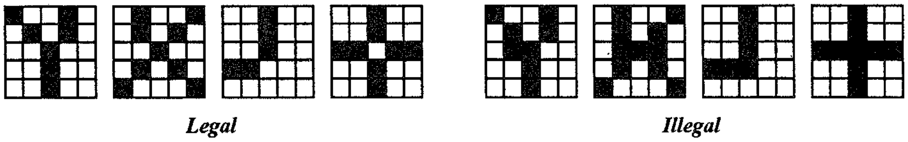

Figure 1: The examples of the legal and the illegal input graph nodes in square geometry.

The sketch of the new analogic algorithm for finding the shortest-path in planar graph is shown in Fig. 2. The proposed procedure is performed through two main steps. At the first step our algorithm find all shortest paths from the origin (the starting point) to any possible destination in the given input graph. This procedure is done using the two-layer CNN for eliminating all points from input graph that can be reached, starting from the given origin, through more than one minimal-length route. This network is labeled as SPF (Shortest-Path-Find) CNN in Fig. 2. The next step uses the target position to extrude the particular origin-to-target shortrest path from the SP-graph obtained as the output result from the SPF CNN. The realization of this procedure is performed using the simple single-layer CNN labelled as the SPE (Shortest-Path-Extrude) CNN in Fig. 2.

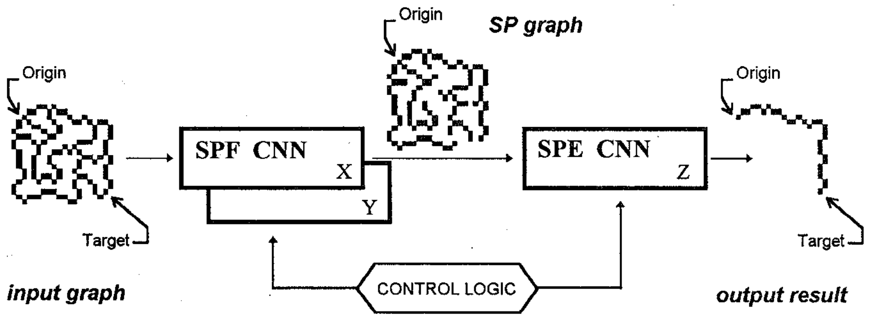

Figure 2: Analogic algorithm for extracting the shortest path in planar graph.

3. The CNN Realization of the Shortest-Path Algorithm

The SPF CNN contains two layers denoted as X and Y in Fig. 2. Each layer has (almost) standard CNN configuration with simple cell structure. The layer X is responsible for the autowave constant-speed propagation and controls the inhibition of wave-fronts in cases of colision. The layer Y contributes the resolving of wave-front colisions which cannot be detected by 3×3 templates of the cells in the layer X.

The state equations of cells in the first (X) and the second (Y) layer are as follows:

$$\tau_x \frac{dx_{i,j}}{dt} = -x_{i,j} + 2\hat{x}_{i,j} - \hat{y}_{i,j} + h\left(\sum_{k=-1}^{1} \sum_{l=-1}^{1} \hat{x}_{i+k,j+l} \right) + 2u_{i,j} - 2.5,$$

$$\tau_y \frac{dy_{i,j}}{dt} = -y_{i,j} + 2\hat{y}_{i,j} + \sum_{k=-1}^{1} \sum_{l=-1}^{1} \hat{x}_{i+k,j+l} - \sum_{k=-1}^{1} \sum_{l=-1}^{1} u_{i+k,j+l} + 0.75,$$

where $\hat{x}_{i,j}$ and $\hat{y}_{i,j}$ denote the outputs of cells $x_{i,j}$ and $y_{i,j}$, according to the output non-linearity function:

$$\hat{\lambda} = \tfrac{1}{2}\left[|\lambda| + 1 - |\lambda - 1| \right],$$

while $h(\cdot)$ is the unit-step (trasholding) function

$$h(x) = \begin{cases} 0\,, x < 0 \\ 1\,,\ x > 0 \end{cases}.$$

The initial conditions for both layers are:

$$y_{i,j}(t = 0) = 0$$

$$x_{i,j}(t = 0) = \begin{cases} 1\,, \text{origin} \\ 0\,, \text{all other points} \end{cases}$$

and common inputs $u_{i,j}$ for both layers are the pixel values of graph image defined by:

$$u_{i,j}(t) = \begin{cases} 0\,, \text{point is not on a graph image (points on the wall)} \\ 1\,, \text{point is on a graph image (points on the route)} \end{cases}.$$

Also, for correct operation of this two-layer system, the cell time-constants in the first and the second layers must satisfy $\tau_x >> \tau_y$. Practically, it is sufficient that $\tau_x \cong 10\tau_y$ is fulfilled.

Note that the cells in the first layer (X-cells) have slightly different structure than the original cells proposed by Chua and Yang in [1]. The original cell involves only the output non-linearity while the X-layer cells are modified by introducing also the input non-linearity through the $h(\cdot)$ term in the state equation. This modification has the crucial role for the constant-speed wave-front propagation in all directions.

After a sufficiently long time, which can be estimated based on the constant wave-front propagation speed and the network size, the SPF CNN output reaches its equilibrium state. As pointed before, the SPF CNN output image contains the shortest paths from the origin to all other points in the given graph. Now, the elimination of all other but the shortest path to the specified target point is necessary. For this purpose we use the 'Shortest-Path-Extrude' (SPE) CNN. This network is simple one-layer CNN described by the state equation:

$$\frac{dz_{i,j}}{dt} = -z_{i,j} + 2\hat{z}_{i,j} + \sum_{k=-1}^{1}\sum_{l=-1}^{1}\hat{z}_{i+k,j+l} + 4v_{i,j} - 6.5\,.$$

The initial states of these cells are described as the outputs of the preceding SPF CNN:

$$z_{i,j}(t_o > T_{stop}) = \hat{x}_{i,j}(T_{stop})\,,$$

where T_{stop} denotes the working-time of the SPF CNN, when its output is 'transferred' to the SPE CNN (see Fig. 2). Inputs $v_{i,j}$ are defined as:

$$v_{i,j}(t) = \begin{cases} 0\,, \text{point } \textit{is not} \text{ on a graph image (is not passable - the wall)} \\ 1\,, \text{point } \textit{is on} \text{ a graph image (is passable - the route)} \\ 2\,, \text{point } \textit{is origin or target} \end{cases}.$$

4. Simulation Example

In order to prove the functionality and the applicability of the described CNN analogic algorithm, we performed a series of simulations on digital computer using numerical integration of state equations. One example is shown in Fig. 3. The graph image (Fig. 3a) is obtained by post-processing (skeletonization) of the scanned labyrinth sketch. The next three figures (Figs 3b-d) demonstrate the propagation of the wave-front starting from the origin and spreading with constant speed through the graph. Notice the inhibition of the pixels where two or more wave-fronts collide. Finally, in the SPF CNN steady-state output (Fig. 3e), only the shortest paths from the origin to all other points exist.

When the SPF CNN reaches its equilibrium state, the output image (Fig. 3e) is 'copied' to initial state of the SPE CNN and the desired target is specified (at the lower-right corner in this example). Figures 3f-h illustrate the process of extruding the shortest path. Notice the marked spot in the final output image in Fig. 3h (SPE steady-state output). Obviously, there are two different routes having identical (minimal) lengths. Thus, we can conclude that the proposed algorithm extrudes not only one but all paths with (the same) minimal lengths, during the single pass.

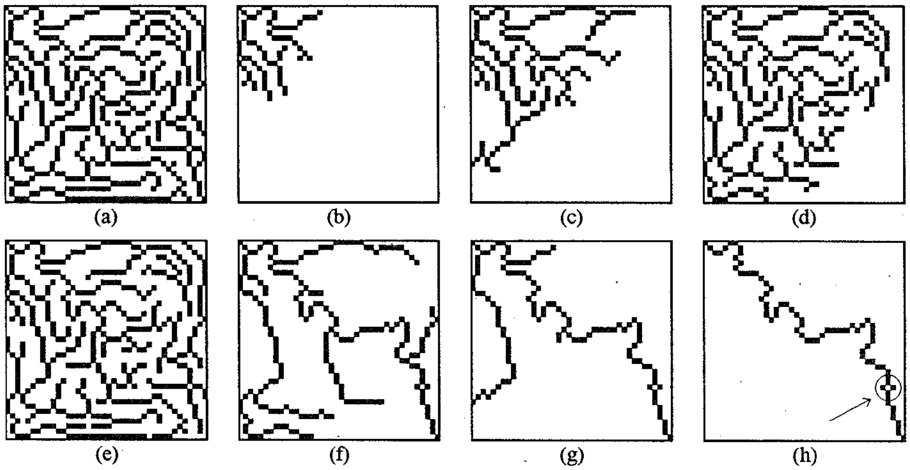

Figure 3: (a) The input graph (the planar labyrinth): the origin and the target points are in the upper-left and in the lower-right corner, respectively. (b-d) The snapshots of the SPF CNN output at few distinct moments during the transience. (e) The steady-state output of the SPF CNN (this is the initial state of the SPE CNN). (f-g) The snapshots of the SPE CNN output during the transience. (h) Final output of the SPE CNN showing two (equal-valued) shortest paths from the origin to the target.

4. Conclusions

In this paper we propose a novel analogic CNN algorithm for finding a shortest-path in a planar graphs. The algorithm is based on the autowave propagation phenomena combined with the analog logic in the two-layer CNN. The simple cell-circuit structure makes this network suitable for the efficient VLSI implementation [8], [9]. The proposed algorithm, slightly modified, can be used in a variety of applications, having in view that many optimization problems can be lead-down to the shortest-path problem.

5. References

[1] L. O. Chua and L. Yang, "Cellular Neural Networks: Fundamentals and Applications", IEEE Trans. Circuits and Systems, Vol. 35, No. 5, 1988.

[2] T. Roska and L. O. Chua, "The CNN Universal Machine: An Analogic Array Computer", IEEE Trans. Circuits and Systems, Vol. 40, No. 3, 1993.

[3] B. Reljin and P. Kostić, "Solving some optimization problems using Celular Neural Networks", Journal of Automatic Control, Univ. of Belgrade, Fac. of Electrical Eng., pp. 73-82, Belgrade, 1995.

[4] V. Pérez-Munuzuri, V. Pérez-Villar and L.O.Chua, "Autowaves for Image Processing on a Two-Dimensional CNN Array of Excitable Nonlinear Circuits: Flat and Wrinkled Labyrinths", IEEE Trans. Circuits and Systems, Vol. 40, No. 3, March 1993

[5] P.L. Venetianer, P. Szolgay, K.R. Crounse, T. Roska and L.O. Chua, "Analog Combinatorics and Cellular Automata - Key Algorithms and Layout Design", in Proc. CNNA-94, pp. 249-254, Dec. 1994, Rome, Italy

[6] T. Matsumoto, L.O. Chua and R. Furukama, "CNN cloning template: hole-filler", IEEE Trans. Circuits and Systems, Vol. 37, 1990.

[7] V.I. Krinsky, V.I. Bikatshev and I.R. Efimov, "Autowaves principles for parallel image processing", Physica, 49D, 1991.

[8] B. Reljin, T. Serdar, P. Kostić, A. Pavasović, "Analog CMOS Realization of Programmable Cellular Neural Network Cell with Reduced Number of Active Components", in Proc. 20th Int. Conf. on Microelectronics (MIEL-95), IEEE Catalog No. 95TH8108, Vol. 2, pp. 497-502, Sept. 1995, Niš, Yugoslavia

[9] B. Reljin, P. Kostić, T. Serdar, A. Pavasović, "Single Amplifier CNN Cell Suitable for VLSI CMOS Implementation", Int. J. Circ. Th. & Appl., 1996 (in print).

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