Study of Reentry Initiation in Coupled Parallel Fibers

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Abstract—Initiation of reentries in coupled parallel fibers is studied as a function of several control parameters intrinsic to those fibers. The influence of inhomogeneities in the fibers leading to drift of the vortices and the interaction between them is also analyzed numerically and experimentally.

I. INTRODUCTION

THE HYPOTHESIS that a circulating wave of excitation, usually referred to in the literature as a *reentry* phenomenon, could occur in a region of the myocardium of the heart was first documented by Allesie *et al.* [1]–[3], who described a type of reentry in which a wave front traveled around a functional obstacle, such as a region of blocked or prolonged action potential. In that case, the central vortex[1] of activity is out of phase with the rest of the tissue, and its features may change somewhat with each passage of the reentrant wave front. In a similar way, Spach *et al.* [4], [5] have demonstrated that reentrant activity can be initiated in a two-dimensional tissue with uniform membrane activity. This phenomenon is known as *vulnerability* and its properties have been analyzed recently in both continuous and discrete media [6]–[8]. The most dangerous disturbances of cardiac rhythm; namely, paroxysmal tachycardia and fibrillation, are associated with the appearance of vortices [9].

The simplest possible medium where vortices can occur is a system of two coupled fibers. In particular, such a system can be formed by the Purkinje fibers in the heart, or by the trabeculae in the myocardial tissue. The vortex in this system consists of a narrow pulse rotating along a closed path.

In the system of two coupled fibers, two basic types of inhomogeneities attract great interest among the scientists; inhomogeneities along the fibers and transverse to them. In the latter case, both fibers have equivalent properties but the coupling between them is not homogeneous, as for example, in the atrial tissue, where it is known that with age, there is a loss of side-to-side connections between myocardial cells

and more arrhythmias are possible under this condition. On the other hand, inhomogeneities along the length of the fibers have been shown to develop a drift of the vortex occurring in too-weakly coupled fibers [10]–[13].

Understanding the dynamics of these waves is a problem of fundamental interest, and could lead to new methods of treatment of cardiac disease by controlling the dynamics of rotating waves making them drift toward the inexcitable borders of the myocardium (auricle or ventricle) where they are annihilated. Such a drift has been observed in the myocardium [14], [15] or in chemical excitable media, such as the Belousov–Zhabotinsky reaction, where it has been shown that rotating waves can be made to drift under the action of a constant or alternating electric field [16]–[18].

Here, the authors provide a more exhaustive study of the reentry properties when intrinsic parameters such as the coupling strength along the fibers, the transversal coupling strength between fibers or, the excitability of the fibers, are changed. Drifting and interaction of vortices is also analyzed in this kind of system. The study has been numerically and experimentally performed with two one-dimensional arrays of Chua's oscillators previously used for the study of complex patterns arising in excitable media (see [19] for a review). These circuits provide an analogy of similar discretely coupled systems occurring in, for example, the cardiac muscle.

II. NUMERICAL MODEL

The basic unit (cell) of our one-dimensional array is a Chua's oscillator,[2] a simple electronic oscillator exhibiting a variety of bifurcation and chaotic phenomena [20].

The circuit dynamics can be modeled by a system of $3N$ first-order autonomous nonlinear differential equations. The explicit dimensionless form of the equations is [19], [21],

$$\dot{x}_i = \alpha(y_i - x_i - h(x_i)) + D(x_{i+1} + x_{i-1} - 2x_i)$$
$$\dot{y}_i = x_i - y_i + z_i$$
$$\dot{z}_i = -\beta y_i - \gamma z_i \qquad i = 1, \cdots, N \qquad (1)$$

where

$$\alpha = \frac{C_2}{C_1}, \quad \beta = \frac{C_2}{LG^2}, \quad \gamma = \frac{(C_2 r_0)}{(LG)}$$
$$m_0 = \frac{G_0}{G}, \quad m_1 = \frac{G_1}{G}, \quad m_2 = \frac{G_2}{G}$$
$$D = \frac{\alpha}{(GR)} \quad \text{and} \quad p = \frac{\alpha}{(GR_p)}.$$

[1] In the literature, the term vortex is used to describe the phenomenon of narrow pulses circulating periodically around a thin annulus in two-dimensional excitable media. In turn, this vortex excites its neighboring cells, giving rise to a spiral wave.

[2] Chua's oscillator is now generally used in the literature [20] when a linear resistor is added in series with the inductor in the original Chua's circuit.

The dimensionless equation of the three-segment piecewise-linear characteristic of the nonlinear resistor (Chua's diode) is given by

$$h(x) = a_0 + a_1 x + b_1 |x - x_1| + b_2 |x - x_2| + \varepsilon \qquad (2)$$

where $a_1 = (m_1 + m_2)/2, b_1 = (m_0 - m_1)/2, b_2 = (m_2 - m_0)/2$ and $a_0 = b_1 x_1 - b_2 x_2, \varepsilon$ is a dc offset which is related to the excitability of the medium, and m_0, m_1 and m_2 denote the slopes of the middle, left and right segments, respectively, of $h(x)$. We will choose $x_1 = -1$ and $x_2 = (m_0 - m_1)/(m_0 - m_2)$ so that the classical symmetrical situation [19] is recovered when $m_1 = m_2$.

For a model consisting of two coupled fibers, we coupled two sets of (1) to obtain

$$\dot{x}_i^1 = \alpha(y_i^1 - x_i^1 - h(x_i^1)) + D_1(x_{i+1}^1 + x_{i-1}^1 - 2x_i^1)$$
$$+ p(x_i^2 - x_i^1)$$
$$\dot{y}_i^1 = x_i^1 - y_i^1 + z_i^1$$
$$\dot{z}_i^1 = -\beta y_i^1 - \gamma z_i^1$$
$$\dot{x}_i^2 = \alpha(y_i^2 - x_i^2 - h(x_i^2)) + D_2(x_{i+1}^2 + x_{i-1}^2 - 2x_i^2)$$
$$+ p(x_i^1 - x_i^2)$$
$$\dot{y}_i^2 = x_i^2 - y_i^2 + z_i^2$$
$$\dot{z}_i^2 = -\beta y_i^2 - \gamma z_i^2 \qquad i = 1, \cdots, N \qquad (3)$$

where p is the parameter determining the coupling between the fibers and the superscripts 1 and 2 denote the fiber number. We usually choose $D = D_1 = D_2$ for isotropic one-dimensional fibers.

Experimentally, (3) corresponds to connecting a linear resistor with conductance p between the floating nodes of capacitor C_1 in each corresponding Chua's oscillator in the two fibers.

The set of (3) together with zero-flux boundary conditions were integrated using an explicit Euler method with a uniform time step of 0.001. The set of fixed parameters used for numerical calculation throughout this paper is $\{\alpha, \beta, \gamma, m_0, m_1, m_2, G\}$ = $\{10, 0.3, 8.2, 10^{-3} - 1.26, 0.77, 55.8, 0.9, 10^{-3}\}$. The constants ε, D, and p were used as control parameters.

In all the simulations shown here, the vortex was generated as follows: an excitation wave was initiated to propagate along fiber 1 (Fig. 1(a)) by setting circuits number one and two to the initial value of $(x^1, y^1, z^1) = (-10, 0, 0)$, with $p = 0$. At a certain moment, both fibers are connected (i.e., $p \neq 0$) and a wave of excitation propagates to the second fiber, where two waves propagating in opposite directions are formed (Fig. 1(b)) in view of the symmetry of the line. Both waves (at fibers 1 and 2, respectively) traveling to the right of Fig. 1 form a collective wave (the second lags the first by a small time delay which becomes zero as the waves approach the boundary) while the single wave traveling to the left of fiber 2 remains alone. The left-moving wave in fiber 2 will excite corresponding cells in the first fiber (Fig. 1(c)) after they have recovered from refractoriness. After the first fiber is excited, two waves are formed traveling in opposite directions, repeating the scenario that took place moments earlier in fiber 2. In Fig. 1(d) the steady final situation is shown. This one corresponds to the excitation of the second fiber. Note during each excitation process, in either fiber 1 or 2, two collective
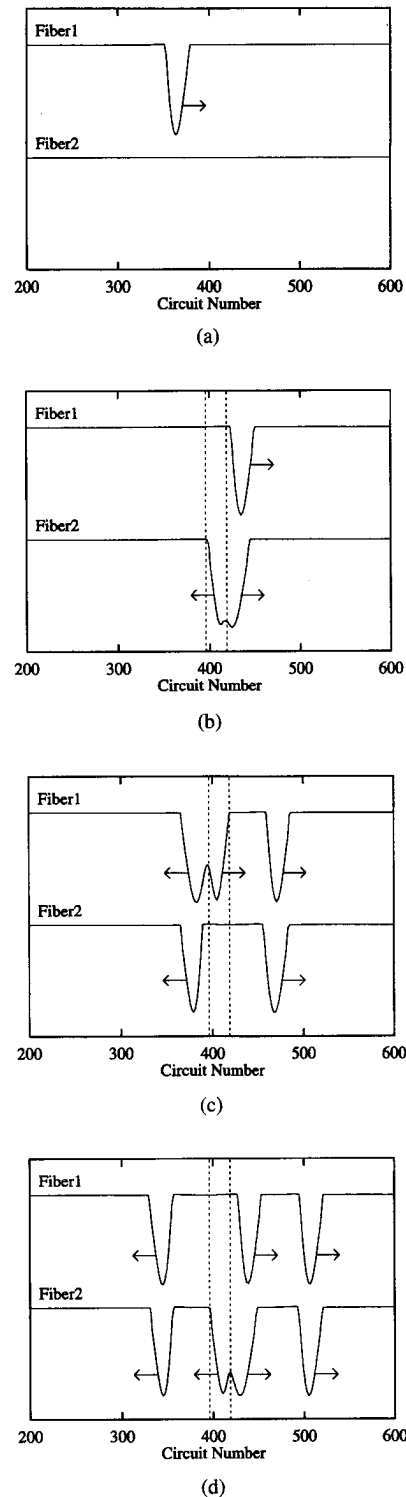

(a)

(b)

(c)

(d)

Fig. 1. A vortex in a homogeneous system of two coupled parallel fibers. First, (a) with $p = 0$ in (3), a wave initiated at fiber 1, moves with constant velocity to the right of the array. At some time, (b) fibers 1 and 2 are coupled by setting $p \neq 0$ in (3), thereby inducing a wave in fiber 2. The single wave moving to the left in fiber 2 (c) excites the opposite fiber after the passage of the refractory tail of the first wave in fiber 1. (d) The vortex remains rotating around the core marked by two dashed lines which covers a size of 23 circuits. The parameters are: $\varepsilon_1 = \varepsilon_2 = -0.02, D = 0.6, p = 0.06$, and $N = 1000$ circuits. Iteration times: (a) 110 t.u., (b) 130 t.u., (c) 140 t.u., and (d) 150 t.u.

waves travel to the right, while only one travels to the left. A single wave travels to the left through the second fiber for a time equal to the refractory period of the cell and
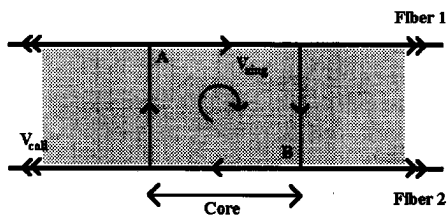
Fig. 2. Sketch of vortex rotation in a system of two homogeneous coupled fibers. The core (corrsponding with the number of circuits in between the dashed lines in Fig. 1) corresponds to the length between points A and B. The gray shadow between fibers shows the cell-to-cell coupling. Collective waves are emitted by the reentry (vortex), rotating around the core with velocity V_{sing}, and they propagate in opposite directions to the ends of the fibers with velocity V_{coll}.

velocity V_{sing}, until it will be able to excite the first fiber. The positions at which a single wave traveling to the right in fiber 1 (to the left in fiber 2) can excite the opposite fiber are marked by dashed lines in Fig. 1. The number of circuits located between the two lines is called the size of the core of the vortex. Note that periodic pulses originate from the boundaries of the core and propagate in opposite directions toward the ends of the fibers with velocity V_{coll}. The two velocities V_{sing} and V_{coll} are different because the coupling p mainly affects the dynamics (3) of those cells within the core as it will be shown below. Fig. 2 shows a sketch of the vortex rotation around a core in a system of two homogeneous coupled fibers. This pulse generation mechanism is called the reentry phenomenon because a pulse remains steadily rotating around a core. This is intrinsically different from a pulse rotating in a ring, since in the former case the vortex emits waves to the ends of the fibers moving in opposite directions. Observe any information (e.g., a pulse or a sequence of pulses) that could be propagating through the fibers before the reentry initiation, will be destroyed if its frequency is lower than the vortex one.

III. EXPERIMENTAL SETUP

Experiments were carried out with two linear arrays, each one consisting of 30 resistively coupled Chua's oscillators, with each cell operating in an excitable (monostable) state. Initially, all cells were adjusted to have the same initial stable state within the tolerances allowable for commercially available discrete components used in each unit (10% for inductance, 5% for capacitance, and 1% for resistance). In order to study the wave propagation and vortex initiation process, a single pulse was applied to a cell of the first array. The pulse was triggered by a waveform generator (Hewlett–Packard 33120A) with a constant amplitude of 3.2 V, and a pulsewidth of 20 μs. In order to prevent the pulse from initiating two waves propagating in opposite directions and finally disappearing at the boundaries, a diode was first inserted in series with the coupling between the triggered cell and one of the adjacent cells in the first array. This allows us to select the direction of the vortex rotation.

Once the wave is initiated, the diode is automatically short circuited by a PC so as not to disturb the subsequent wave propagation process.

Trying to experimentally reproduce the same method to initiate a reentry as described in the previous section will mean initiating a wave in the first fiber when $p = 0$, and all 30 corresponding Chua's oscillators are connected side-to-side in both fibers simultaneously when the wave at the first fiber passes through the central cell, for example. This procedure should be automatically controled by a PC with an acquisition board with a trigger time of the order of microseconds or even less. Unfortunately, this setup requires high-precision instrumentation to synchronize the PC and the electronic devices connecting all the cells, which is presently not available in our laboratory.

On the other hand, it is well known in cardiology that vortices can be anchored to some obstacle occurring in a tissue, as for example veins. In order to experimentally initiate a reentry by this method, both fibers are initially connected by means of resistances $R_p = 5.6$ kΩ cell-to-cell, except five of them, to the left of the diode (cell 21). Those are connected by means of high resistances ($R_p^* = 22$ kΩ), above the value for propagation failure [19]. By doing so, a pulse initiated in fiber 1 at cell 21, will follow the same procedure as described in the previous section, except the wave moving to the left in fiber 2 will now necessarily reenter in fiber 1, just after the obstacle in cell 16, which was previously calculated to the left behind the refractory tail caused by the first wave at fiber 1.

After several clockwise rotations, around the obstacle, the period, velocity and shape of the emitted collective waves stabilize and remain invariant thereafter ($T = 103 \pm 2$ μs and $V_{coll} = 95 \pm 5$ circuits/ms). The minimum size of the obstacle for vortex initiation to be possible is 4 circuits. Once the reentry has been initiated, and in order to obtain a homogeneous transversal coupling between fibers, the high values of the resistances R_p^* between cells at the obstacle were smoothly decreased to the same value as the adjacent cells, $R_p = 5.6$ kΩ. However, for values of R_p^* below 10 kΩ, this procedure led to the annihilation of the vortex. Probably, because the cells are not identical, and the fibers are not homogeneous, this nonuniformity induces a drift of the reentry as $R_p^* \rightarrow R_p$ because the obstacle dissappears and the vortex cannot remain anchored any longer. Nevertheless, we were unable to prove this hypothesis due to the high wave velocities and the small length of the arrays.

The circuits were sampled with a digital oscilloscope (Hewlett-Packard 54601) with a maximum sample rate of 20 MSa/s, an 8-b A/D resolution, and a record length of 4000 points.

The electronic cell components were fixed throughout this paper as follows: $C_1 = 1$ nF, $C_2 = 12$ nF, $L = 10$ mH, $r_0 = 10$ Ω, $G = 3.7 \ 10^{-3}$ Ω^{-1}, $G_0 = -4.56 \ 10^{-3}$ Ω^{-1}, $G_1 = 3.81 \ 10^{-3}$ Ω^{-1}, $G_2 = 3.7 \ 10^{-3}$ Ω^{-1} and $R = 4.7$ kΩ. The parameters ε and R_p were used as control parameters of the experiments described below.

IV. RESULTS

Consider what happens to a vortex in a system of two coupled fibers, proceeding from its simplest properties. As it follows from Fig. 1, such a system is characterized by the following basic parameters: the velocity V_{sing} of a single wave,
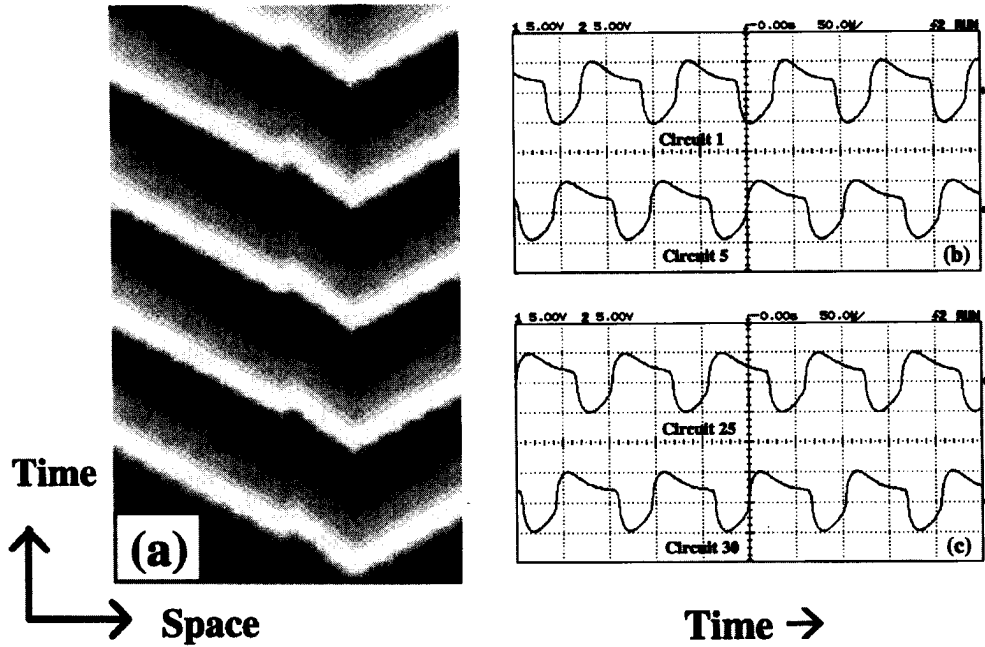
Fig. 3. Experimental reentry in two parallel arrays of Chua's oscillators. (a), Collective waves propagating toward the boundaries can be seen in the space–time plot where time is vertical and space is horizontal. Core is seen as the space between discontinuities at the bands of equal amplitude. Note that the velocity at the core (between circuits 16 and 21) is smaller than the collective one ($V_{\text{sing}} = 70 \pm 5$ circuits/ms and $V_{\text{coll}} = 95 \pm 5$ circuits/ms). (b) and (c), Collective waves traveling to the boundaries excite circuits 5 and 25 first, and then circuits 1 and 30, respectively,. The parameters are: $\varepsilon_1 = \varepsilon_2 = 0$ μA and $R_p = 5.6$ kΩ. Gray scale color codes (a): the white color corresponds to the maximum voltage, while the black color correspond to the minimum voltage at node 1 in Chua's oscillators [20].

the velocity V_{coll} of a collective wave, and the refractoriness R of the medium. The constants ε, D and p control the values of these parameters. In particular, the ratio ε/p controls the number of cycles of reentry a homogeneous system can develop [12].

Fig. 3 shows an experimentally obtained reentry for two arrays consisting of 30 circuits as described in the previous section. In the space–time plot of Fig. 3(a) a vortex is located between circuits 16 and 21. The distance between the discontinuities at the bands of equal amplitude (equal gray scale color in Fig. 3(a)) in the horizontal axis determines the effective size of the wave source (that is, the core). Note the velocity at the core, V_{sing}, is lower than the collective one. The temporal behavior of voltage at node 1 in Chua's oscillator [20] near the boundaries is shown in Fig. 3(b) and (c). Note circuits 5 and 25 are excited before circuits 1 and 30, respectively, showing the effect of the collective waves when moving to the boundaries. In this case, because the core is located nearest the right boundary, collective waves moving to the right have a period smaller than those propagating to the left as it can be observed in Fig. 3(b) and (c).

Thus, in a homogeneous system of two coupled fibers, a vortex is a single wave rotating around a core (between both dashed lines in Fig. 1) with velocity V_{sing} and a period T equal to the medium refractory period R. If the fibers are not homogeneous (i.e., some of the parameters defining the waves are different in both fibers) the steady state circulation of the vortex becomes impossible and the vortex drifts with velocity V_{dr}.

For example, by changing the relative excitability $\varepsilon_r = (\varepsilon_2 - \varepsilon_1)/\varepsilon_1$ between the fibers, the refractoriness at both fibers differs. Fig. 4 shows the behavior of both fibers in a space–time diagram. Note that only after a certain time do collective waves at the second fiber arise and move in both directions with constant velocity V_{coll}. On the other hand, because of the difference between excitabilities in both fibers, the positions at which the single wave excites fibers 1 and 2 no longer remain constant in time, but they move with velocity V_{dr}, satisfying

$$V_{dr} < V_{\text{coll}} \quad \text{for} \quad \varepsilon_r \neq 0. \tag{4}$$

Once the reentry reaches the right boundary and disappears, no excitation takes place along either fiber.

Numerically the following properties have been measured.

1) For $\varepsilon_r \neq 0$, the vortex drifts with a direction determined by the following vector relation:

$$\overline{V_{dr}} = \overline{\omega} \times \overline{\nabla}\varepsilon \tag{5}$$

where $\overline{\omega}$ is the angular vector velocity of the vortex rotation and $\overline{\nabla}\varepsilon$ is the vector orthogonal to the fiber and directed toward the greater excitability.

2) The drift velocity, V_{dr}, increases with the relative excitability until it tends towards a near constant saturation value equal to the single wave velocity V_{sing} (Fig. 5). This saturation level, as well as the drift velocity, increases with a decreasing value of the coupling coefficient p between fibers. Note that this saturation level is obtained for some value of ε_r which increases with
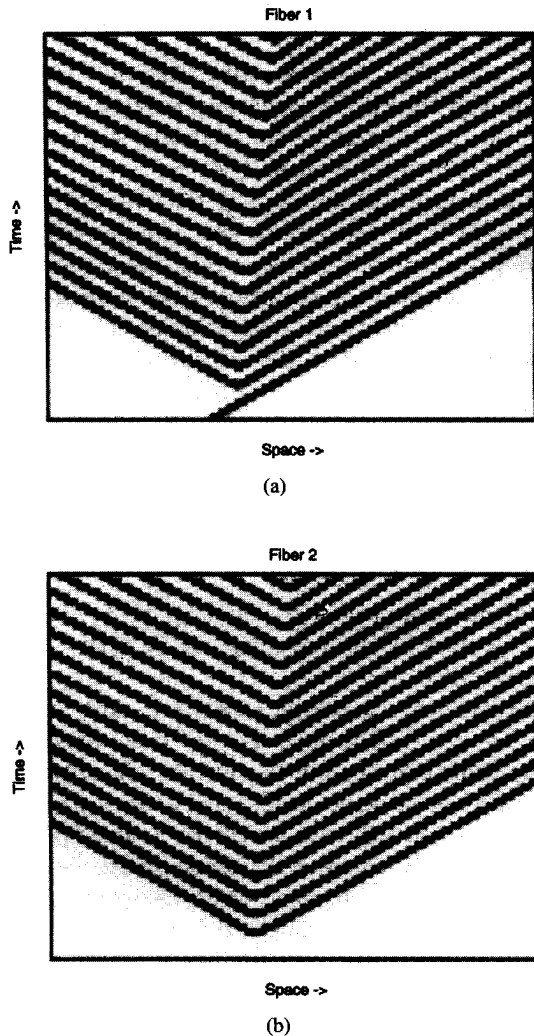
Fiber 1

(a)

Fiber 2

(b)

Fig. 4. Space–time diagrams for (a) fiber 1 and (b) fiber 2 showing the drifting of a vortex. Time is shown in the vertical direction, while the number of circuits is along the horizontal. Lines represent the consecutive positions of the peak of the wave propagating through the fibers 1 and 2. After coupling both fibers ($t = 125$ t.u.) collective waves spread out to the boundaries with constant velocity, $V_{coll} = 3.54$ circuits/t.u. The positions at which each fiber is excited by the other one are shown as a vertex in the space–time plot. Note their position moves with time (drifting of the reentry) at constant velocity, $V_{dr} = 0.22$ circuits/t.u. In both diagrams, as $t \rightarrow \infty$, the number of fronts arriving to the right boundary is greater than those arriving to the left, according to Doppler effect. The parameters are: $\varepsilon_1 = -0.01, \varepsilon_2 = -0.02, D = 0.6, p = 0.06$, and $N = 1000$ circuits. Gray scale color codes as in Fig. 3.

Fig. 5. Dependence of the drift velocity on the relative excitability between fibers for different values of the coupling parameter p. Note that for high values of ε_r, a saturation level is obtained equal to V_{sing}. The parameters for the numerical simulation are: $D = 0.6$ and $N = 1000$ circuits.

Fig. 6. Dependence of the core size on the relative excitability between fibers for different values of the coupling parameter p. Note that for high values of ε_r, a saturation level is obtained. The parameters for the numerical simulation are: $D = 0.6$ and $N = 1000$ circuits.

decreasing value of p. For $\varepsilon_r = 0$, both fibers are homogeneous and the vortex rotates steadily and does not drift, $V_{dr} = 0$.

3) The core size decreases while increasing ε_r until it reaches a saturation value (Fig. 6). Note that increasing the coupling p between the fibers leads to a decrease of the core size.

The interaction between two reentries moving in opposite directions was also numerically investigated, although only preliminary results are shown here. Two vortices initiated at the same time from opposite sides of the same fiber rotate and drift in opposite directions leading to their annihilation, for example, when both fibers have different excitability. Fig. 7 shows a sequence of space–time diagrams for fiber 2. First,
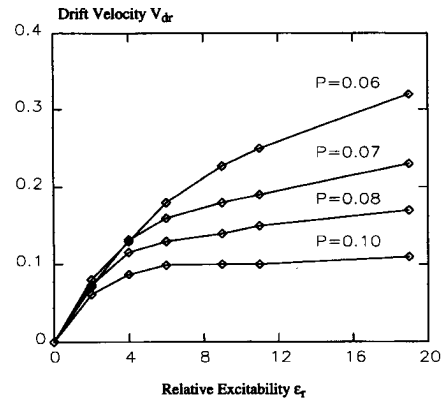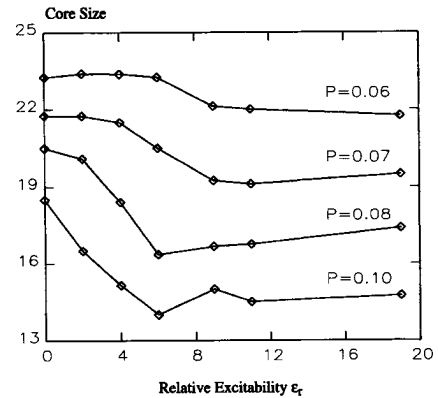
two reentries are initiated at opposite sides, π out of phase, and drifting towards the center of the array as shown in Fig. 7(a). After several iterations, both reentries diminish their drifting velocities and remain rotating close enough while the periods of rotation synchronize and the phase difference between both vortices becomes zero, finally annihilating both vortices and disappearing as shown in Fig. 7(c). We were not able to find any set of suitable parameters where both reentries could remain synchronized while rotating around a single core.

V. DISCUSSION

The influence of the coupling strength between fibers and their relative excitability has been studied in discretely-coupled fibers by means of one-dimensional arrays of Chua's oscillators.

Under some circumstances, vortex initiation (also called reentry) occurs in two coupled fibers. When both fibers are not homogeneous, it has been shown the vortex does not rotate steadily but drifts. Let us suppose the excitability of fiber 1 is greater than the excitability of fiber 2 (i.e., $\varepsilon_1 > \varepsilon_2, \varepsilon_r > 0$). In this case, wave velocity at fiber 1 is greater than in fiber 2 and the single wave moving around the core in Fig. 2 should need
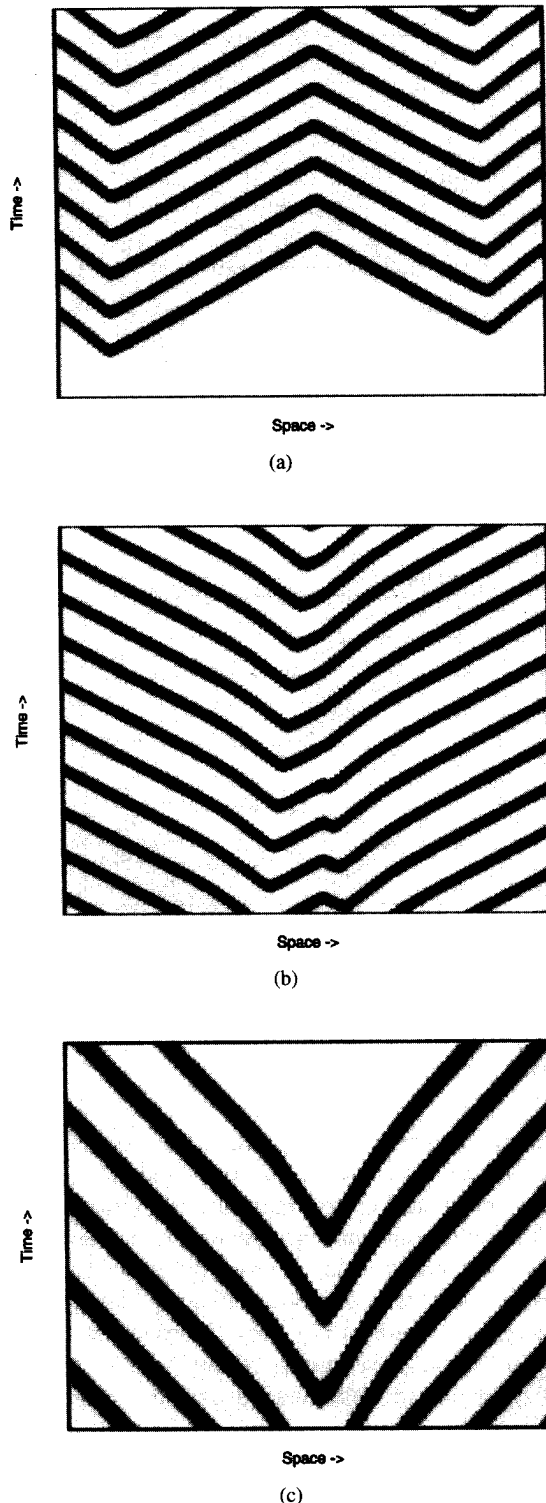
Space ->

(a)

Space ->

(b)

Space ->

(c)

Fig. 7.   Sequence of space–time plots for fiber 2 when (a), (b), two reentries moving in opposite directions interact and (c) finally annihilate each other. Two reentries are initiated in opposite sides of fiber 1 and move to the center of the array with constant drift velocity. Once they interact, (b) the periods of rotation and excitation times of the different fibers synchronize, finally leading to (c) the annihilation of both vortices. Set of parameters as in Fig. 2. Iteration times: (a), from 0 to 200 t.u., (b), from 801 to 999 t.u., and (c), from 990 to 1090 t.u. Gray scale color codes as in Fig. 3.

to move further along fiber 1 in order to be possible to excite fiber 2 (i.e., it recovers from the previous excitation wave). By this procedure, the path traveled by the single wave along

fiber 1 is greater than in fiber 2 for the same period of time, and because this process recurs in each turn-over of the single wave, it causes the vortex to drift. Thus, the drift velocity increases with the relative excitability between fibers, but the size of the core decreases with increasing ε_r, both tending toward a saturated value when $\varepsilon_r \to \infty$.

On the other hand, the coupling terms $+p(x_i^2 - x_i^1)$ and $+p(x_i^1 - x_i^2)$ in (3) for fibers 1 and 2, respectively, will be different from zero only for those cells where a single wave moves and zero otherwise (i.e., those parts of the fibers where collective waves spread out are synchronized). Therefore, by increasing the value of p, the velocity of the single wave diminishes and the drift velocity also decreases with increasing p. So, while V_{coll} is not affected by p, V_{sing} becomes smaller than V_{coll} when p is increased.

Since, decreasing the coupling p between the fibers, leads to an increase in the single wave velocity, V_{sing}, the size of the core increases (i.e., the single wave travels further along the fiber for small values of the coupling than for high values of p, for the same period of time) for $\varepsilon_r = 0(\varepsilon_1 = \varepsilon_2)$. When $\varepsilon_r \neq 0$, a similar reasoning can be made except now the vortex drifts.

Interaction between reentries and their drift towards the boundaries is an important matter for controlling abnormal activities in the heart muscle. In particular, the influence of an external field on the vortex drift should be studied in detail for discretely–coupled cells as in the cardiac tissue [15]. Although we have studied the influence of internal parameters relative to each fiber, transversal inhomogeneities between them should be studied in detail, for example, the influence of age in the side-by-side connections between fibers and its importance for reentry initiation. The problem concerning the interaction among vortices with different and equal topological charges [22] remains to be investigated.

ACKNOWLEDGMENT

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