CNNA-94 Third IEEE International Workshop on Cellular Neural Networks and their Applications
Rome, Italy, December 18-21, 1994

Hyperchaos, Clustering and Cooperative Phenomena in CNN arrays Composed of Chaotic Circuits

Maciej J.Ogorzałek
Andrzej Dąbrowski
Władysław Dąbrowski
Department of Electrical Engineering
University of Mining and Metallurgy
al Mickiewicza 30
30-059 Kraków, Poland
tel. +48 12 33 91 00 ext. 3613; fax: +48 12 34 48 25
e-mail: maciej@zet.agh.edu.pl

Abstract– *This paper presents a study of complex dynamic phenomena in arrays composed of interacting chaotic circuits. Such arrays are thought of as a new paradigm for signal processing. Depending on the connection strength between the cells the array can show desorganised hyperchaotic behavior or organised in a specific way coherent behavior which might be useful in information processing. Patterns of behavior depending on coupling values are studied in this paper. Chua's circuits are taken as standard chaotic cells.*

1. Introduction

Recently there has been considerable interest in studies of dynamic properties of networks of chaotic elements [1–2], [4–8], [10–18]. Such networks are important as a model for physical systems with many degrees of freedom and biological signal processing. They offer also possible interesting engineering applications eg. in information processing [15]. Due to the high dimensionality of these systems most of the studies are based on simulation experiments alone.

There is a wide variety of phenomena encountered in chaotic oscillator arrays. Kaneko [7], [8] introduced a classification of these phenomena. Apart from apparently disordered behaviour characterised by one or more positive Lyapunov exponents, there are several orderly states. The simplest type of such an "organised" behavior is synchronisation or coherent behavior, when the variables in the i-th and j-th cell are identical in time. Oscillator arrays often show a universal behaviour [11] characterised by Feigenbaum constants. The most interesting types of dynamics encountered in large interconnections of chaotic oscillators are: the spatio-temporal chaos (sometimes referred to as turbulence [7], [8]) when the observed trajectories exhibit chaotic properties both in time and space, and recently discovered in arrays of electronic oscillators spiral waves [16] and travelling wavefronts [14–16]. There are very few tools for studying these types of behaviour. Building of experimental laboratory equipment is both expensive and extremely difficult. Thus electronic circuitry and available simulation tools offer versatility difficult to find in other areas of research.

2. Description of experiments

Our previous studies carried out in ladder and ring interconnections of Chua's circuits revealed interesting types of oscillatory phenomena [13]. In our paper we study the dynamic behaviors encountered in planar interconnections of identical simple electronic oscillators (Chua's circuits) - an extension of experiments carried out previously. Such interconnection can be considered as a generalised Cellular Neural Network with chaotic cells. The way the cells are interconnected is different to any of previously studied oscillator arrays - namely we treat each cell as an input-output block - and not as a resistive grid with oscillators connected to its nodes [14-16]. Inputs to each cell are via recistors connected to the capacitor C2 and outputs are via resistors connected to the capacitors C1 in each cell. In this study we consider square grids only. Using specifically developed software tools we were able to discover several types of behavior including synchronisation of oscillations sometimes referred to as "self-organisation". This last term is used often to describe the somewhat astonishing effect that a large number of disordered (chaotic) cells "adapting" interconnection weights can produce highly ordered "organised" behaviour.

We employ as a standard chaotic cell the canonical Chua's described in [3]. The unit cells are interconnected in such a way that only nearest neighbours are connected via a coupling resistance with a considered cell (CNN-like structure). Thus the cells at the corners have three neighbors, the cells at the edges have five neighbours and internal cells have eight neighbours each. The equations describing system dynamics can be written in a simplified way for each cell as follows:

$$\begin{aligned}
\dot{x}_i &= (A_1 + N_i A_7)x_i + A_2 y_i - A_1 z_i - A_7 \sum_{j \in N_i} z_j \\
\dot{y}_i &= A_3 x_i \\
\dot{z}_i &= -A_4 x_i + (A_4 + N_i A_6) z_i + A_5 f(z_i) - A_6 \sum_{j \in N_i} x_j
\end{aligned} \tag{1}$$

where: $f(z) = m_0 z + \frac{1}{2}(m_1 - m_0)(|z + b_p| - |z - b_p|)$, $m_0 = -0.5$, $m_1 = -0.8$, $b_p = 1$ [3]. In our following discussions we use the following notation: $A_1 = -\frac{G}{C_2}$, $A_2 = -\frac{1}{C_2}$, $A_3 = \frac{1}{L}$, $A_4 = -\frac{G}{C_1}$, $A_5 = -\frac{1}{C_1}$, $A_6 = -\frac{G_1}{C_1}$, $A_7 = -\frac{G_1}{C_2}$. N_i denotes here the set of neighbours of the i-th cells. In our studies we considered $i = 1...9$, $i = 1...16$, $i = 1...25$ and $i = 1...100$.

In simulation experiments we used typical parameter values for the canonical Chua's circuit given in Table III of [3] for which an isolated Chua's oscillator generates chaotic oscillations - the double scroll attractor ($L = 1/7$, $C_1 = 1/9$, $C_2 = 1$, $G = 0.7$). Depending on the actual coupling resistor values a variety of dynamic behaviors could be observed. Below we describe results of experiments carried out with grids of the size 5x5 only (25

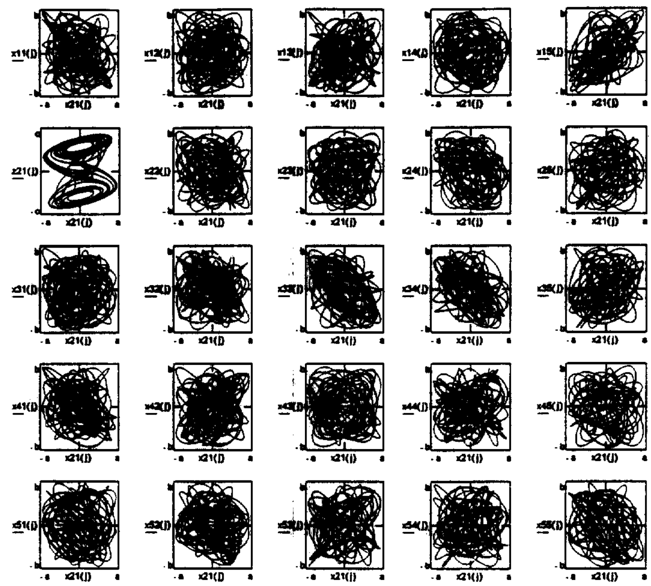

Figure 1: Hyper-chaotic solutions found in simulation experiments in the 5x5 grid of chaotic circuits ($G_1 = 0.001$, initial condition is applied only in the cell 21 marked by the double scroll attractor).

cells) - the cells are numbered using standard matrix notation eg. 21 - means second row, first collumn.

One common type of behavior is shown in Fig.1. This is a typical hyper-chaotic solution characterised by more then one positive Lyapunov exponent.

Fig.2 shows more orderly states. Here the calculation of Lyapunov exponents shows existence of quasiperiodicity characterised by trajectories winding around a higher - dimensional torus.

In some ranges of coupling resistance several cells in the system become synchronised. As the grid itself is symmetrical - in the case of a symmetrically applied initial condition eg. on the diagonal -the solutions show also spatial symmetry. On a phase plot synchronisation manifests itself in a line segment on the plot. This can be seen in the Figure 4 some of the cells are not synchronised while there exits some cells that synchronise. Such a coherent (synchronised) behavior is often called organised state. In some cases only some cells forming a neighborhood synchronise thus forming a cluster. Position and area of a cluster depend on the position of the cells where initial conditions (perturbation to the array) is applied.

We have also carried out several laboratory tests confirming existence of the phenomena discovered in simulations also in real circuits. In all laboratory tests we used the classical Chua's circuits (instead of the canonical ones) for unit cells. Kennedy's [9] op-amp implementations have been used for nonlinear resistors. Our laboratory setup described in [4] enables also experiments in array interconnections of Chua's ciruits. The ladder networks are constructed in chains of ten cells thus it is possible to introduce also parallel connections of the ten-cell chains. At the moment the laboratory experiments are carried out and the research is focused on construction of a display device allowing simultaneous insight into internal variables of all cells.

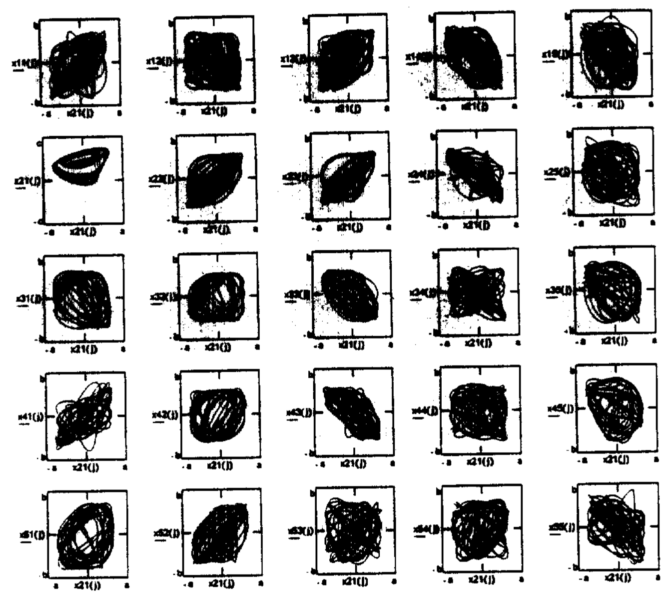

Figure 2: Quasiperiodic solutions found in simulation experiments in the 5x5 grid of chaotic circuits ($G_1 = 0.05$, initial condition applied on C1 capacitor in the cell no. 21). The trajectory winds up a torus-like object. Note that due to the coupling the attractor in cell 21 is now a spiral one.

3. Conclusions.

The main results of our study can be summarised as follows:

- Our study reveals extremely interesting phenomena not described so far in the literature. Several interesting types od dynamic behaviors have been confirmed to exist in the considered arrays of Chua's circuits these include hyperchaos, synchronised states, clustering. We expect to be able to construct a sort of bifurcation diagram - showing the dynamic phenomena displayed as a function of the bifurcation parameter - the coupling resistance.
- For the same values of coupling resistances the system behavior and in particular synchronisation properties change depending on the initial conditions. For unsymmetrically applied initial conditions clusters of synchronised cells are formed.

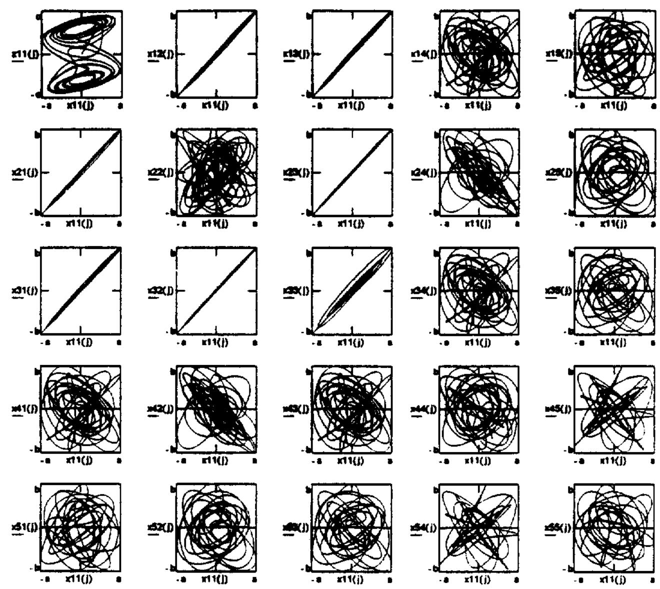

Figure 3: Clustering of cells observed in simulation experiments ($G_1 = 10^{-9}$, initialisation - cell No. 22).

Formation of such clusters is extremely sensitive to initial condition changes and is not fully understood so far.

- Synchronisation offers possible interesting applications in signal and image processing. Synchronised states can be also used for coding and storing information - some developments towards constructing a chaotic associative memory are under way.
- For arrays of larger sizes the observed phenomena can be considered also on the spatial scale - behavior changes chaotically not only in time but also in space.
- Changes of the coupling resistances offer a possibility to controlling the dynamic behavior. One can choose between different types of dynamics. To be able to take advantage of this kind of control, a thorough study of dynamic phenomena must be carried out.

References

[1]V.S. Aframovich, V.I. Nekorkin, G.A. Osipov, V.D. Shalfeev, *"Stability, Structures and Chaos in Nonlinear Synchronization Arrays"*. Eds. A.V.Gaponov- Grekhov, M.I.Rabinovich, IAP, USSR Acad.Sc., 1989 (in Russian).

[2]V.N. Belykh, N.N. Verichev, Lj. Kocarev, L.O. Chua *"On chaotic synchronization in a linear array of Chua's circuits"*. J. of Circuits, Systems and Computers, vol.3, No.2, June 1993, pp.579-589.

[3]L.O. Chua, G.-N. Lin, *"Canonical realisation of Chua's circuit family"*. IEEE Trans. Circuits Systems, Vol. CAS-37, No.7, pp.885-902, 1990.

[4]A.M. Dąbrowski, W.R. Dąbrowski, M.J. Ogorzalek, *"Dynamic Phenomena in Chain Interconnections of Chua's Circuits"*. IEEE Trans. Circuits Systems, vol.CAS-40, No.11, pp.868-871, 1993.

[5]A.D. Defontaines, Y. Pomeau, *"Chain of coupled bistable oscillators: a model"*. Physica D 46,p.201-216,1990.

[6]L. Fabiny, K. Wiesenfeld, *"Clustering behavior of oscillator arrays"*. Physical Review A, vol.43, no.6, 15 March 1991, pp.2640–2648.

[7]K. Kaneko, *" Pattern dynamics in spatio-temporal chaos"*. Physica D 34, p.1-41, 1989.

[8]K. Kaneko, *"Clustering,coding,switching,hierarchical,ordering and control in a network of chaotic elements"*. Physica D 41, p.137-172, 1990.

[9]M.P. Kennedy, *"Robust Op-Amp Implementation of Chua's circuit"*. Frequenz, vol.46, No.3–4, pp.66–80, 1992.

[10]Lj. Kocarev, Lj. Karadzinov, L.O. Chua, *"N-Dimensional canonical Chua's Circuit"*. J.of Circuits, Systems and Computers, vol.3, no.1, 1993, pp.239-258.

[11]H. Kook, F.H. Ling, G. Schmidt, *"Universal behavior of coupled nonlinear systems"*. Physical Review A, vol.43, no.6, 15 march 1991, pp.2700–2708.

[12]V.I. Nekorkin, *"Spatial chaos in the discrete model of the radio- technical medium"*. Radiotechnika i Elektronika, 4, p.651-660, 1992 (in Russian).

[13]M.J. Ogorzalek, A.M. Dąbrowski *"Theoretical and experimental studies of oscillations in simple CNN structures"*. Proc. 2nd International Workshop CNNA'92, Munich 1992, pp.123-128.

[14]V. Perez-Munuzuri, V. Perez-Villar, L.O. Chua, *"Traveling wave front and its failure in a one dimensional array of Chua's circuits"*. J. of Circuits, Systems and Computers, vol.3, No.1, pp.215–229, 1993.

[15]V. Perez-Munuzuri, V. Perez-Villar, L.O. Chua, *"Autowaves for image processing on a two dimensional CNN array of Chua's circuits: flat and wrinkled labyrinths"*. IEEE Trans. Circuits and Systems, vol.CAS-40, March 1993, pp.174–181

[16]A. Perez-Menuzuri, V. Perez-Munuzuri, V. Perez-Villar, L.O. Chua, *"Spiral Waves on a 2-D Array of Nonlinear Ciruits"*, IEEE Trans. Circuits Systems, vol.CAS-40, No.11, pp.872-877, 1993.

[17]K.Y. Tsang, I.B. Schwartz *"Recurring anti-phase signals in coupled nonlinear oscillators: Chaotic or random time series?"*. Int. J. Bifurcation and Chaos, Vol.3, No.3, pp.773-778, 1993.

[18]A.L. Zheleznyak and L.O. Chua *"Coexistence of low- and high-dimensional spatiotemporal chaos in a chain of dissipatively coupled Chua's circuits"*. Int. J. Bif. Chaos, Vol.4, No.3, pp.639-674, 1994.