Mutual synchronization of patterns and wave fronts in two coupled chains of Chua's circuits.

Vladimir I. Nekorkin, Victor B. Kazantsev, Dmitry V. Artyuhin
Dept. of Radiophysics, Nizhny–Novgorod State University,
23, Gagarin Avenue Nizhny–Novgorod 603600, Russia.
E-mail nekorkin@rf.unn.runnet.ru

Abstract

The interaction of two coupled chains of Chua's circuits is investigated. The estimated value of coupling between the chains above which mutual synchronization of all dynamic processes occurs is found. The examples of the synchronization of two steady patterns and traveling wave fronts are given.

1 Introduction

Synchronization of motions is one of the fundamental features of nonlinear systems. In recent years possible synchronization of not only periodic (regular) oscillations but chaotic (irregular) ones has been proved in [1] for identical coupled subsystems and in [2] for different systems. Such phenomenon has been found in a number of systems being extremely diverse in there physical nature [3, 4, 5, 6, 7]. It looks like complete or almost complete coincidence of time realizations generated by coupled identical or almost identical nonlinear systems.

From the other side there has been growing interest to studying the collective behavior of arrays consisting of a large number of interacting subsystems. The phenomena investigated in such systems include formation of patterns which can have both ordered and disordered spatial structure [9, 11, 14], synchronization of oscillations [7, 8, 10, 14], propagation of wave fronts and solitons [9, 11, 12, 13], spatio–temporal chaos and the others. It means that such arrays represent, in fact, active extended media where, as it is known, the problem of interaction of patterns and waves is of great importance both from the fundamental and applied points of view.

We study the problem of mutual interaction of two coupled one–dimensional arrays (chains) of Chua's circuits.We show that for certain conditions they synchronize and display the identical collective behavior both in time and in space. We also numerically illustrate how such synchronization leads to different effects in spatio–temporal dynamics of the two coupled chains.

2 Model

Consider the case when all elements of two independent arrays are identical and coupled diffusively in each of the arrays. The interaction between the arrays is provided by "point by point" coupling between the corresponding elements of two arrays. The equation describing such system are

$$\begin{aligned}
\dot{x}_j^1 = {} & \alpha(y_j^1 - x_j^1 - f(x_j^1)) + d^1(x_{j-1}^1 - 2x_j^1 + x_{j+1}^1) + \\
& + h(x_j^2 - x_j^1); \\
\dot{y}_j^1 = {} & x_j^1 - y_j^1 + z_j^1; \\
\dot{z}_j^1 = {} & -\beta y_j^1 - \gamma z_j^1; \\
\dot{x}_j^2 = {} & \alpha(x_j^2 - y_j^2 - f(x_j^2)) + d^2(x_{j-1}^2 - 2x_j^2 + x_{j+1}^2) + \\
& + h(x_j^1 - x_j^2); \\
\dot{y}_j^2 = {} & x_j^2 - y_j^2 + z_j^2; \\
\dot{z}_j^2 = {} & -\beta y_j^2 - \gamma z_j^2;
\end{aligned} \tag{1}$$

$$j = 1, 2, \ldots, N \ ,$$

where $\cdot^1$ and $\cdot^2$ denote the variables of the first and the second array respectively; d^1, d^2 are coefficients describing the strength of coupling between units in the arrays; h characterizes coupling between the arrays; the nonlinearity $f(x)$ is taken in the smooth form $f(x) = x(x-a)(x+b)$.

The dynamics of each chain taken independently($h = 0$) can be quite complex. The phenomena of spatial disorder [10], different nonlinear waves including solitons [13], pulses, fronts and chaotic pulse trains [14,16], different synchronous regimes [6] have been recently found in such a system. Studying the interaction of the two chains($h \neq 0$) we show the possibility of complete synchronization of all motions in the identical chains($d^1 = d^2 = d$) and provide a few examples of the interacting chains when their intra–array diffusions are not equal($d^1 \neq d^2$). In particular, it can be proved

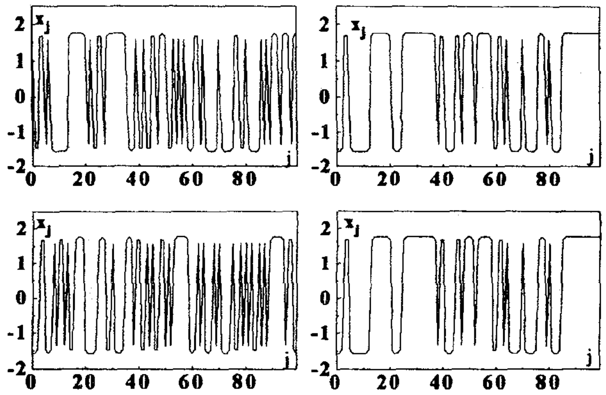

Figure 1: Synchronization of disordered steady patterns. Parameter values: $a = 2,\ b = 2.,\ \alpha = 1,\ \beta = 0.5,\ \gamma = 0.1,\ d = 0.2,\ h = 1.$

that increasing h above some critical value

$$h^* = \frac{\alpha(a^2 + ab + b^2)}{6}$$

yields the existence of the synchronization manifold M and its global stability in the phase space of the system (1). The manifold is

$$M : \{u_j = 0,\ \ v_j = 0\ \ w_j = 0\}, j = 1, 2, \dots, N$$

where $u_j = x_j^1 - x_j^2,\ \ v_j = y_j^1 - y_j^2,\ \ w_j = z_j^1 - z_j^2$. Thus, for $h > h^*$ all initial conditions after some transient process tend to the manifold where motions are governed by the system which is equivalent to the equations describing the independent array. It means that complete synchronization occurs for all processes in two coupled identical arrays of Chua's circuits. Now we give some examples of such synchronization obtained by numerical simulations of the system (1). Note, that h^* obtained theoretically gives only an upper estimated value of h enough for the synchronization. The real value of h^* depends on the types of synchronizing motions and usually appears to be smaller.

3 Synchronization of ordered and disordered patterns

It has been shown in a number of papers (see, for example, [9, 14]) that for $d < d^*$, where d^* is a some critical value, each independent array ($h = 0$) can have a very large number of steady patterns including ones with chaotic spatial structure. Let the initial conditions of the system (1) be two different patterns which are the stable equilibrium of the independent array. When the interaction starts and becomes strong enough ($h > h^*$) these distributions evolve to the synchronization manifold where the synchronized steady patterns are formed. The structure of the terminal patterns can be quite different depending on the system

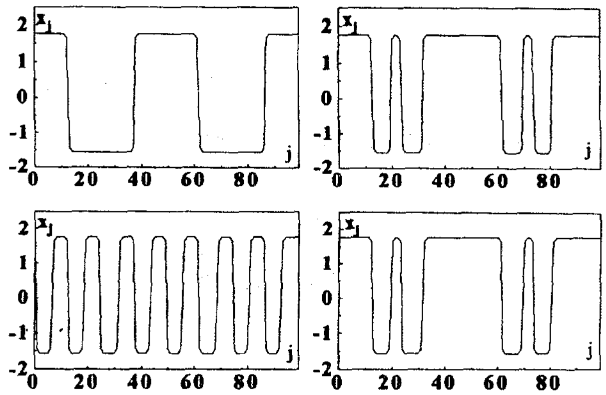

Figure 2: Synchronization of ordered steady patterns. Parameter values: $a = 2,\ b = 2.,\ \alpha = 1,\ \beta = 0.5,\ \gamma = 0.1,\ d = 0.2,\ h = 1.$

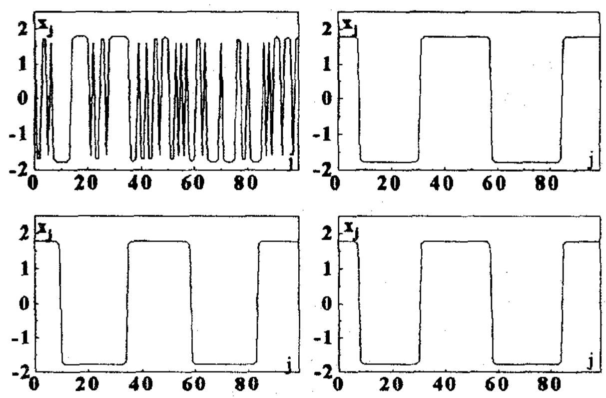

Figure 3: Synchronization of two different steady patterns. The left pair of pictures is the initial distribution and the right pair is identical terminal patterns in two coupled chains. Parameter values: $a = 2,\ b = 1.8,\ \alpha = 1,\ \beta = 0.5,\ \gamma = 0.1,\ d = 0.15,\ h = 1.$

parameters and the form of initial patterns. In the Figs. 1,2 a few examples of such interaction are displayed. Figure 1 illustrates the interaction of two initially disordered patterns (the left pair of the pictures is initial conditions, the right pair is terminal patterns).The synchronized patterns are also disordered (Here we can speak about the synchronization of spatial disorder.).

Another example (Fig. 2) is the interaction of two regular patterns with different spatial structure. The synchronized patterns are also regular but have another form "mixed" from two initial states.

Finally, let us illustrate the interaction of ordered and disordered patterns (Fig. 3). For parameter values given in the figure the initially disordered chain "stimulated" by the regular pattern of the second chain evolves to the pattern with the spatial structure almost identical to the regular initial pattern. This case

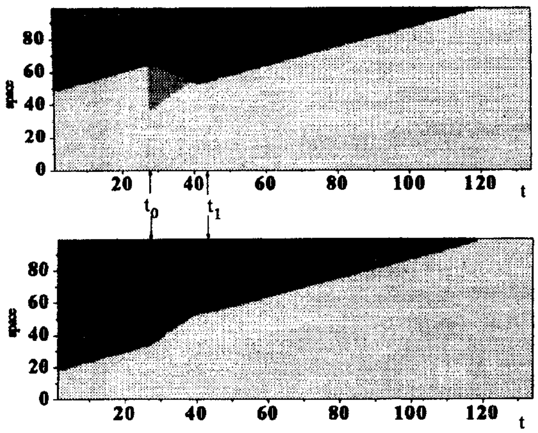

Figure 4: Mutual synchronization of two chains with traveling wave fronts. Parameter values: $a = 2$, $b = 1.8$, $\alpha = 1$, $\beta = 0.5$, $\gamma = 0.1$, $d = 2$. $t < t_0 = 29$ – $h = 0$ the chains are independent $h = 0$, t_0 – the moment when interaction switches on, $t > t_0$ – $h = 0.5$, time interval $t_0 < t < t_1 \approx 44$ corresponds to transient process.

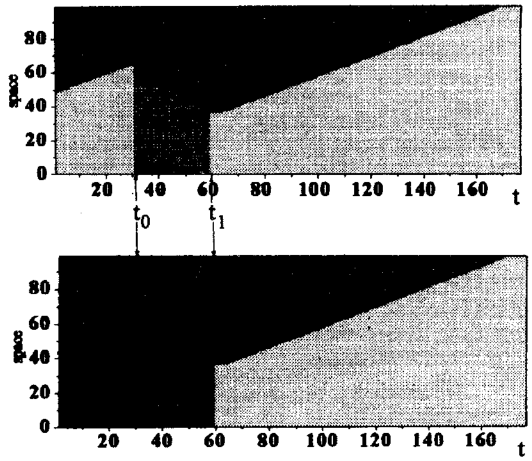

Figure 5: Mutual synchronization of two chains with traveling wave fronts. Parameter values: $a = 2$, $b = 1.8$, $\alpha = 1$, $\beta = 0.5$, $\gamma = 0.1$, $d = 2$.Initiation of the traveling wave front through mutual synchronization. $t < t_0 = 31$ – the chains are independent $h = 0$, $t_0 < t < t_1 = 59$ – the interaction is not enough for synchronization $h = 0.5$, $t > t_1$ – $h = 1$ – complete synchronization.

may be interpreted as the disordered chain *copies* the main features of the regular pattern in the results of mutual interaction.

4 Synchronization of wave fronts

Consider now the situation when the parameters of the independent array are taken in the region of possible propagation of the wave fronts. It occurs, for example, with increasing the coupling coefficient d above the critical value d^*. Let the initial conditions be two fronts propagating in the same direction with a finite delay (Fig. 4, ($t < t_0$)). By the mutual interaction of the chains switched on at the time moment $t = t_0$ the delay is eliminated through rather short transient process (Fig. 4 ($t_0 < t < t_1$)), the fronts become synchronized and propagate together (Fig. 4 ($t > t_1$)). The level of grey color corresponds to the value of the variables x_j in the junctions of the chain.

Another initial conditions are the wave front in the one chain and a stable homogeneous state in the other. Such situation is of interest, for example, when studying the reentry phenomenon in parallel neural fibres [6]. When the interaction starts but is not strong enough to synchronize the chains the front can not "excite" another "fibre" but changes its own direction of propagation and starts to travel to the opposite side (see Fig. 5 ($t_0 < t < t_1$)). Increasing the value of h at the moment t_1 we provide the condition of mutual synchronization and the front completely identical to the original one is excited in the fibre which has been initially homogeneous (Fig. 5 ($t > t_1$)).

5 Control of wave fronts

In summary, let us illustrate how the mutual interaction of two chains with different diffusion coefficients ($d^1 \neq d^2$) allows to control the wave front behavior. In particular, let a stable wave front in a one chain propagates in the presence of a stable steady pattern in the other chain. Figures 6,7 show that either the front can stay traveling when the characteristic scale of the pattern ($T < T^*$) is small enough (Fig.6) or it can fail to propagate being "locked" within a half–period of the pattern when $T > T^*$ as shown in Fig.7.

Conclusion

We have investigated the interaction of two one–dimensional arrays of Chua's circuits and showed that the arrays are able to synchronize. Taking as initial conditions two different steady patterns and two wave fronts traveled with a finite delay we illustrate that their synchronization may lead to non–trivial effects in the spatio–temporal dynamics of such system (synchronization of the spatial disorder, appearance of new spatial forms by the synchronization, coordination of wave fronts). Although the examples are quite simple we hope that the *phenomenon* of mutual synchronization we proved will be helpful in understanding dynamic processes in nonlinear systems of different physical nature.

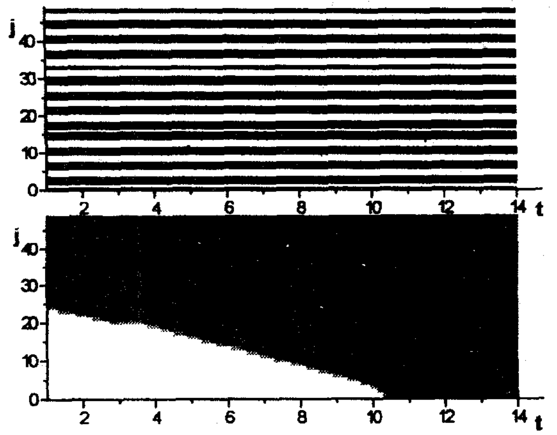

Figure 6: Wave front stays to travel interacting with the steady pattern. Parameter values: $a = 2$, $b = 1.8$, $\alpha = 1$, $\beta = 0.5$, $\gamma = 0.01$, $d^1 = 0.01$, $d^2 = 20$, $T = 13$, $h = 0.1$.

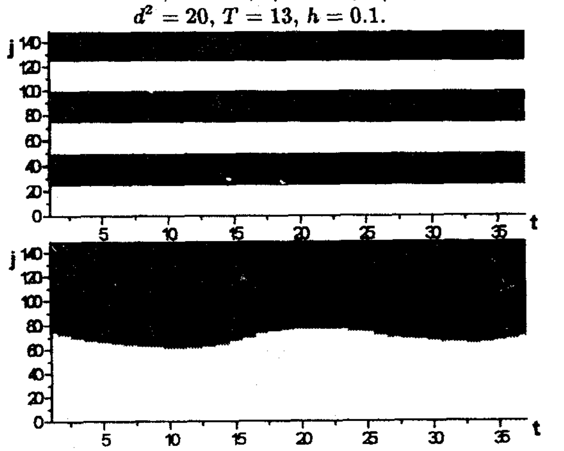

Figure 7: Wave front fails to propagate. Parameter values: $a = 2$, $b = 1.8$, $\alpha = 1$, $\beta = 0.5$, $\gamma = 0.01$, $d^1 = 0.01$, $d^2 = 20$, $T = 3$, $h = 0.1$.

Acknowledgments

This research has been supported by Russian Foundation for Basic Research (grant 97-02-16550).

References

[1] H. Fujisaka, T. Yamada, Stability theory of synchronized motions in coupled oscillator systems, Progr. of Theor. Phys. **69**, 1983, 32–47.

[2] V.S. Afraimovich, N.N. Verichev, M.I. Rabinovich, Stochastic synchronization of oscillations in dissipative systems, Izv. Vys. Uch. Zav., Radiofizika, **29**, 1986, 1050–1060.

[3] T.L. Carrol, L.M. Pecora, Synchronization in nonautonomous chaotic circuits, IEEE Trays. Circuits and Systems, **38**, 1991, 453–456.

[4] V. Perez-Villar, A.P. Munuzuri, V. Perez-Munuzuri, L.O. Chua, Chaotic synchronization of a one-dimensional array of nonlinear active systems, Int. J. Bifurcation and Chaos, **3**, 1993, 1067–1074.

[5] V.N. Belykh, N.N. Verichev, L. Kocarev, L.O. Chua, On chaotic synchronization in a linear array of Chua's circuits, J. Circuits Syst. Comput., **3**, 1993, 579–589.

[6] I. Perez Marino, M. de Castro, V. Perez-Munuzuri, M. Gomez-Gesteira, L. O. Chua and V. Perez-Villar, Study of Reentry Initiation in Coupled Parallel Fibers, IEEE Trays. Circuits and Systems, **42**, 1995, 665–672.

[7] J.F. Heagy, T.L. Carroll, L.M. Pecora, Synchronous chaos in coupled oscillator systems, Physical Review E, **50**, No. 3, 1994, 1874–1885.

[8] V. S. Afraimovich, S-N. Chow, J.K. Hale, Synchronization in lattices of coupled oscillators, Preprint CDSNS95-208, 1995, Georgia Tech.

[9] V.I. Nekorkin and L.O. Chua, Spatial disorder and wave fronts in a chain of coupled Chua's circuits, Int. J. Bifurcation and Chaos,**3**, 1993, 1281–1291.

[10] V.I. Nekorkin, V.A. Makarov, Spatial chaos in a chain of coupled bistable oscillators, Phys. Rev. Lett., **74**, 1995, 4819–4822.

[11] V.I. Nekorkin, V.A. Makarov, M.G. Velarde, Spatial disorder and waves in a ring chain of bistable oscillators, Int. J. Bifurc. and Chaos, **6**, 1996.

[12] V.I. Nekorkin, V.B. Kazantsev, N.F. Rulkov, M.G. Velarde and L.O. Chua, Homoclinic orbits and solitary waves in a one-dimensional array of Chua's circuits, IEEE Trans. Circuits Syst.-I: Fundamental Theory and Applications, **42**, N 10, 1995, 785–801.

[13] V.I. Nekorkin, V.B. Kazantsev and L.O. Chua Chaotic attractors and waves in a one-dimensional array of modified Chua's circuits, Int. J. Bifurcation and Chaos, **6**, 1996, 1295–1317.

[14] V.I. Nekorkin, V.A. Makarov, V.B. Kazantsev and V.G. Velarde Spatial disorder and pattern formation in lattices of coupled bistable systems Physica D 100,1997,330-342.

[15] V.B. Kazantsev, V.I. Nekorkin, M.G. Velarde Pulses, fronts and chaotic wave trains in a one-dimensional Chua's lattice, Int.J.Bifurcation and Chaos,1997(to appear).