A SUBTHRESHOLD-CMOS CHAOTIC OSCILLATOR

John E. Neeley and John G. Harris
Computational Neuro-Engineering Laboratory
University of Florida
EE Dept., 405 CSE Bldg 42
Gainesville, FL 32611

ABSTRACT

This paper reports on the design of a subthreshold-CMOS chaotic oscillator based on Chua's circuit. The circuit is autonomous and generates chaotic signals from three state variables in a manner analogous to a Chua circuit. Previous CMOS IC versions of the Chua circuit operate above threshold and make use of capacitors over 200 times larger than the subthreshold design. Also, this design is based on fundamentally nonlinear subthreshold building blocks permitting flexible adjustment of bifurcation parameters for investigation of complex phenomena not present in a piecewise linear model. Finally, this design incorporates all necessary elements on chip, eliminating the need for an external resistor. The combination of reduced size and flexible nonlinearity make this design suitable for a single-chip VLSI synthesis of complex circuits making use of chaotic signals, including coupled maps generating spatio-temporal chaos and systems exploiting chaos synchronization.

1. INTRODUCTION

Electronic circuits exhibiting a well-defined bifurcation and chaotic behavior can be utilized in networks that produce spatial and temporal chaos and other systems exploiting their unique frequency spectrum. Chua's circuit [1] [4] [5] is the simplest autonomous circuit that can exhibit chaos and has become a research vehicle for studying chaos and a building block in systems exploiting its dynamics. The circuit has been studied extensively since its invention in 1983, and many interesting chaotic phenomena have been confirmed both by computer simulation and experimental observation (1/f phenomena, stochastic resonance, etc.) Chua's circuit has been implemented with discrete components [3] and as an integrated circuit [2]. Since not all phenomena of a real circuit may be explained by a piecewise-linear model [7], Chua's circuit has also been implemented with a smooth (cubic) nonlinearity in discrete components [9].

In this paper, we report the implementation details and simulation results of a subthreshold design exhibiting a predictable bifurcation sequence analogous to that of Chua's circuit. In Section II, we describe the design from a network element level. Then we examine the interactions of these elements from a state-variable approach. In Section III, we present some bifurcation sequences and chaotic attractors simulated via the circuit simulation package ANALOG.

2. INTERNAL STRUCTURE

A network schematic of Cruz's IC implementation of the Chua circuit [2] and the current subthreshold design are

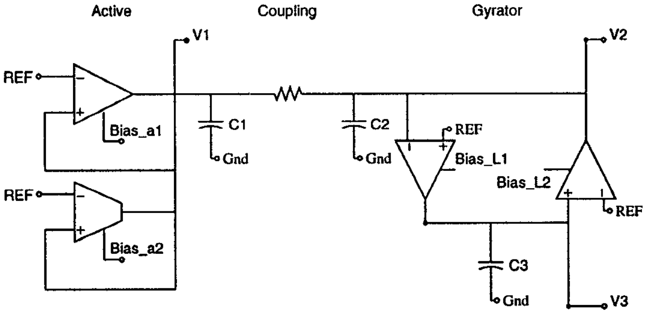

Figure 1. *Network schematic of Cruz's IC implementation of Chua's circuit.*

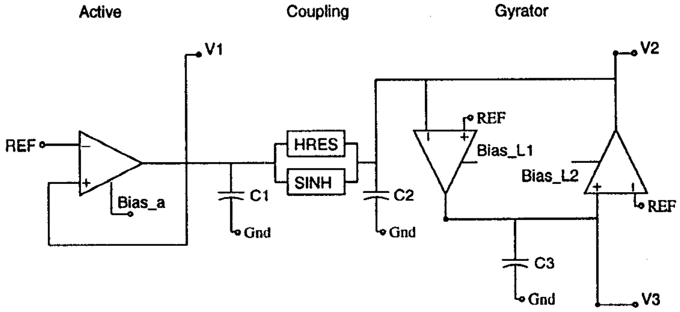

Figure 2. *Network schematic of subthreshold IC implementation.*

shown in Figure 1 and Figure 2 respectively.

This design, like existing IC versions of Chua's circuit, is a third-order circuit. Both networks contain a locally active element coupled resistively to a linear oscillator. In the Chua circuit, the nonlinear driving point characteristics of the Chua diode provide three regions of linear dynamic evolution. The Jacobian or conductance matrix (shown below) specifies this evolution [5].

$$J_{Fb} = \begin{bmatrix} \frac{-G'}{C_1} & \frac{G_c}{C_1} & 0 \\ \frac{G_c}{C_2} & \frac{-G_c}{C_2} & \frac{1}{C2} \\ 0 & \frac{-G_{L1}G_{L2}}{C_3} & 0 \end{bmatrix} \tag{1}$$

where

$$G' = G_a + G_c \tag{2}$$

G_a and G_c are the transconductances of the active and coupling elements repectively. The associated eigenvalues

0-7803-3073-0/96/$5.00 ©1996 IEEE

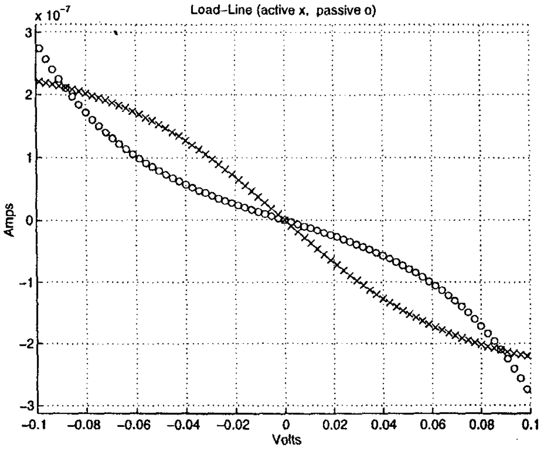

Figure 3. *Load line (i-v) plot for active/coupling elements.*

and eigenplanes of this matrix completely characterize the circuit's dynamics. When the magnitude of V_1 is greater than some energy level (E) the conductance matrix produces two unstable complex eigenvalues and one stable real eigenvalue, if the magnitude of V_1 is less than E the three eigenvalues are comprised of two stable complex and one unstable real eigenvalue.

In normal operation the conductance matrix remains invariant. However, to produce the aforementioned eigenvalues/eigenplanes we draw further attention to the relation between the conductance of the locally active element (G_a), the Chua diode and the conductance of the passive coupling (G_c), variable resistor, between the active element and the oscillator. When the magnitude of V_1 is greater than E, the conductance of the active element (G_a) is less than the coupling conductance (G_c) ($G_a < G_c, G' > 0$); conversely, if the magnitude of V_1 is less than E, the conductance of the active element is greater than the passive connection ($G_a > G_c, G' < 0$). In Chua's circuit G_c is constant while G_a varies. With appropriate scaling of the energy elements, these conditions produce the necessary eigenspace for chaos.

The proposed design utilizes nonlinear connections between a locally active element and a gyrator to provide the nonlinearity necessary for the required dynamics. Thus, G_c varies while G_a is approximately constant. The resistive nonlinearity is odd-symmetric and preserves the above qualitative ratios with conductance $G_a > G_c$ for $|V_1| > E$ and $G_a < G_c$ for $|V_1| < E$. The load-line for the active vs. coupling conductance in shown in Figure 3.

A simple transconductance amplifier provides the locally active element. Sinh and Hres resistive blocks (detailed in [6] and [8] respectively) provide nonlinear coupling between the active element and the oscillator. All elements of the circuit are implemented on chip and require bias voltages, permitting further modification of the standard Chua parameters a, b, and β [4] [5]. Specifically, the coupling transconductance (G_c) may be modified, affecting a and b, even in such a manner that G_c becomes nonsymmetric. Furthermore, the oscillator is implemented as a gyrator or second order transconductance matrix with inductance replaced with a capacitor. This permits individual variation of another bifurcation parameter β. We have simulated the complete design of this oscillator in the circuit simulation package ANALOG. The α bifurcation sequence is specified by the on-chip capacitors. For this circuit $\alpha = 10$ with the largest capacitor equal to 7.5pF (Previous designs required capacitances in the nF range). The β Bifurcation sequence from period doubling, spiral and scroll are shown in Figures 4, ?? and 5.

3. CONCLUSION

We have presented a design for a subthreshold microelectronic circuit that produces chaotic signals and whose simulated bifurcation sequence is analogous to that of a Chua's circuit implemented with a smooth nonlinearity. Our motivation for this work is to decrease the size of current chaotic microelectronic circuits that may serve as robust sources of chaotic signals while incorporating all necessary circuit elements on chip. In this way, we provide another circuit that can be used as self-contained building blocks for more complex systems that exploit desirable aspects of chaos.

Acknowledgements

This work was supported by an Office of Naval Research contract #N00014-94-1-0858 and an NSF CAREER award #MIP-9502307.

REFERENCES

[1] Leon O. Chua. A universal circuit for studying and generating chaos - part I: Routes to chaos. *IEEE Transactions on Circuits and Systems - I: Fundamental Theory and Applications*, 40:732–744, 1993.

[2] Jose M. Cruz. An IC chip of Chua's circuit. *IEEE Transactions on Circuits and Systems - II: Analog and Digital Signal Processing*, 40:614–625, 1993.

[3] M. P. Kennedy. Robust op amp realization of Chua's circuit. *Frequenz*, 46:66–80, 1992.

[4] M. P. Kennedy. Three steps to chaos - part I: Evolution. *IEEE Transactions on Circuits and Systems - I: Fundamental Theory and Applications*, 40:640–656, 1993.

[5] M. P. Kennedy. Three steps to chaos - part II: a Chua's circuit primer. *IEEE Transactions on Circuits and Systems - I: Fundamental Theory and Applications*, 40:657–674, 1993.

[6] D. Kerns. PhD thesis, California Institute of Technology, Pasadena, CA, 1991. Dept. of Electrical Engineering.

[7] A. I. Khibnik, D. Roose, and L. O. Chua. On periodic orbits and homoclinic bifurcations in Chua's circuit with a smooth nonlinearity. *International Journal of Bifurcation and Chaos*, 3:363–384, 1993.

[8] C. Mead. *Analog VLSI and Neural Systems*. Addison-Wesley, 1989.

[9] Guo-Qun Zhong. Implementation of Chua's circuit with a cubic nonlinearity. *IEEE Transactions on Circuits and Systems - I: Fundamental Theory and Applications*, 41:934–941, 1994.

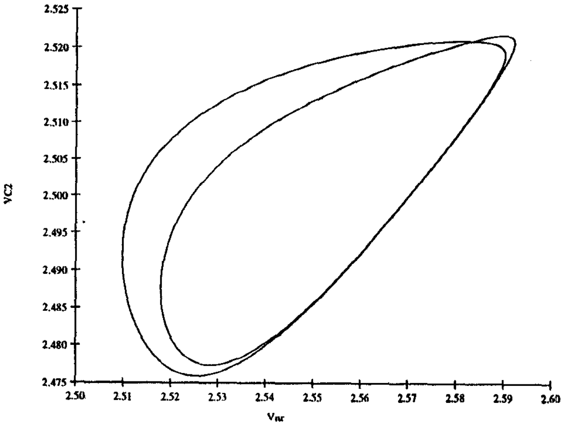

Figure 4. *β double.*

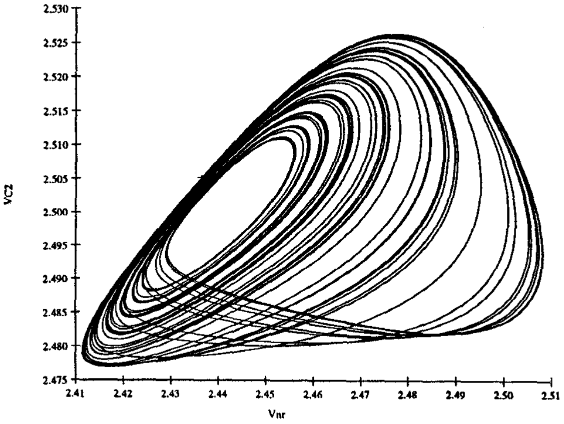

Figure 5. *β spiral.*

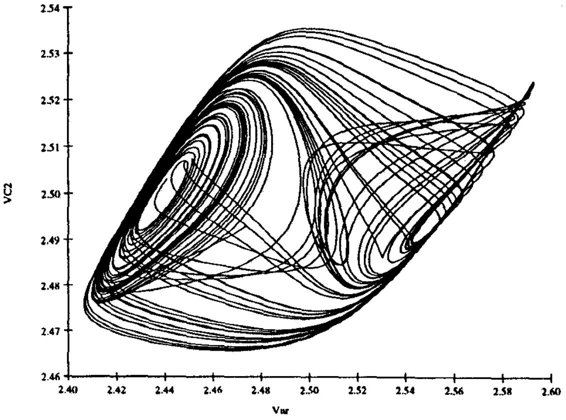

Figure 6. *β scroll.*