International Journal of Bifurcation and Chaos, Vol. 8, No. 11 (1998) 2263–2278

A CNN APPROACH TO BRAIN-LIKE CHAOS-PERIODICITY TRANSITIONS

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Received April 15, 1997; Revised May 1, 1998

Chaos-periodicity transitions are studied within the framework of a one-dimensional CNN which are qualitatively similar to some brain-like transitions described in the literature. The nonlinearity of each cell is designed such that chaotic and periodic regimes coexist at the cell level. When a coupling between cells is considered, transition waves from chaotic behavior into a periodic one are observed to propagate along the one-dimensional array. The signals measured are qualitatively similar to some EEG measurements in biology. Here we present the study of these transitions for different types of couplings and parameters.

1. Introduction

Chaotic behavior and synchronization of chaotic cells have been observed during recognition processes at the cell level in the olfactory bulk [Skarda & Freeman, 1987; Freeman, 1987; Delaney *et al.*, 1994] or the visual cortex [Gray *et al.*, 1988] among others. Effectively, normal behavior in these cells is chaotic [Babloyantz & Destexhe, 1986; Yao & Freeman, 1990; Aihara *et al.*, 1990]. After stimulation (inhalation or visual stimuli) the system, composed of many coupled cells, bifurcates from a low-energy chaotic state to a high-energy state [Skarda & Freeman, 1987] producing a structured two-dimensional pattern characteristic of each stimuli. Here, chaos can be regarded as a reservoir of periodic behavior that can be activated in response to changing stimuli or external conditions [Matías *et al.*, 1997]. This system can be destabilized in some pathological cases (as epileptic seizures [Freeman, 1986, 1992]) when it is driven out of its normal range by excessive electrical stimulation and develops a dynamic asymmetry that carries the system temporarily into a periodic regime.

On the other hand, nonlinear electronic circuits have been used to model the dynamics of natural systems such as pulse propagation through cardiac tissue [Muñuzuri *et al.*, 1996a, 1996b; deCastro *et al.*, 1998] as well as to simulate biological retinal functions and functions that go beyond biological capabilities [Werblin *et al.*, 1995]. In this paper, we shall focus on the study of transitions from chaotic to periodic regime in an extended system after an excessive electrical stimulation is applied (qualitatively similar to the case described above [Freeman, 1992]). For this purpose, we considered a one-dimensional array of generalized Chua's circuits [Madan, 1993; Chua, 1994; Suykens *et al.*, 1997a]. A Chua cell is an electronic oscillator that exhibits a large variety of bifurcation and chaotic phenomena [Zhong &

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Ayrom, 1985; Chua, 1993a; Chua *et al.*, 1993b; Pérez-Muñuzuri *et al.*, 1995; Pérez-Villar *et al.*, 1993] and has been used in order to understand the underlying dynamics of biological processes (propagation of cardiac pulse [Muñuzuri *et al.*, 1996a, 1996b; deCastro *et al.*, 1998]). Chip implementations of cellular neural networks of such circuits have been already studied [Chua & Roska, 1993c; Chua *et al.*, 1995].

In Sec. 2, we present a description of the Chua's circuit used for the simulations. The results are presented in Sec. 3, considering two different couplings between circuits; first a homogeneous diffusive coupling via linear resistors (Sec. 3.1) and, second, a unidirectional coupling (Sec. 3.2). This last case is closer to real systems in nature (like the brain neurons or neuronal propagation in general). The other parameters of the system are also analyzed.

2. Model

The basic unit (cell) of our array is a generalized Chua's circuit [Chua *et al.*, 1986; Chua, 1992; Madan, 1993], a simple oscillator endowed with an extremely rich gamut of nonlinear dynamical phenomena [Muñuzuri *et al.*, 1993, 1995; Pérez-Muñuzuri *et al.*, 1995] that has also been used to understand the dynamics underlying biological processes (propagation of cardiac pulse [Muñuzuri *et al.*, 1996a, 1996b; deCastro *et al.*, 1998] among others). The circuit contains three linear energy-storage elements (an inductor with inductance L and two capacitors with capacitances C_1 and C_2), a linear resistor with resistance $1/G$ and a nonlinear resistor, N_R, called Chua's diode [Cruz, 1993]. We used the generalized version of the nonlinearity to a n-segment piecewise linear (e.g. [Suykens & Vandewalle, 1993; Arena *et al.*, 1996; Suykens *et al.*, 1997a; Suykens & Chua, 1997b]), and so much more rich dynamics can be expected. The equations of the one-dimensional array of cells are described, in the explicit rescaled dimensionless form, by

$$\begin{aligned} \frac{du_i}{dt} &= \alpha[v_i - h(u_i)] + \mathcal{C}_i(u_{i+1},\, u_i,\, u_{i-1}) \\ \frac{dv_i}{dt} &= u_i - v_i + w_i \\ \frac{dw_i}{dt} &= -\beta v_i - \gamma w_i \end{aligned} \tag{1}$$

with $i = 1, \ldots \mathrm{DIM}$, where DIM is the total number of circuits in the array, u, v and w are the dimensionless variables related with the physical variables by: $u = V_1/A$, $v = V_2/A$ and $w = I_L/(GA)$ (with A a scaling parameter equal to $A = 1$ V) and the dimensionless time $t = \tau G/C_2$ (τ is real time). α, β and γ are the parameters of the model related to the electronic components of the circuit [Muñuzuri *et al.*, 1995]. $\mathcal{C}_i$ is the coupling function that takes into account the influence of the neighboring circuits. Two different couplings are considered in this paper, the diffusive coupling,

$$\begin{aligned} \mathcal{C}_1 &= D_u(u_2 - u_1) && i = 1 \\ \mathcal{C}_i &= D_u(u_{i+1} + u_{i-1} - 2u_i) && 1 < i < \mathrm{DIM} \\ \mathcal{C}_{\mathrm{DIM}} &= D_u(u_{\mathrm{DIM}-1} - u_{\mathrm{DIM}}) && i = \mathrm{DIM} \end{aligned} \tag{2}$$

and the unidirectional coupling,

$$\begin{aligned} \mathcal{C}_1 &= 0 && i = 1 \\ \mathcal{C}_i &= D_u(u_{i-1} - u_i) && 1 < i < \mathrm{DIM} \\ \mathcal{C}_{\mathrm{DIM}} &= D_u(u_{\mathrm{DIM}-1} - u_{\mathrm{DIM}}) && i = \mathrm{DIM} \end{aligned} \tag{3}$$

where D_u is the diffusion coefficient that takes into account the influence of the neighboring circuits through linear resistors. D_u is considered to be constant along the array. The n-segment piecewise linear characteristic is given by,

$$h(u_i) = \begin{cases} -m_0\, c_1 + m_1\,(c_1 - c_2) + m_2\,(c_2 - c_3) + m_3\,(u_i + c_3) & u_i \le -c_3 \\ -m_0\, c_1 + m_1\,(c_1 - c_2) + m_2\,(u_i + c_2) & -c_3 < u_i \le -c_2 \\ -m_0\, c_1 + m_1\,(u_i + c_1) & -c_2 < u_i \le -c_1 \\ m_0\, u_i & -c_1 < u_i \le c_1 \\ m_0\, c_1 + m_1'\,(u_i - c_1) & c_1 < u_i \le c_2 \\ m_0\, c_1 + m_1'\,(c_2 - c_1) + m_2'\,(u_i - c_2) & c_2 < u_i \le c_3 \\ m_0\, c_1 + m_1'\,(c_2 - c_1) + m_2'\,(c_3 - c_2) + m_3'\,(u_i - c_3) & c_3 < u_i \end{cases} \tag{4}$$

or, for the general nonsymmetric case, in a more compact form [Suykens & Vandewalle, 1993; Arena *et al.*, 1996; Suykens *et al.*, 1997a],

$$h(u_i) = m_0 u_i + \frac{1}{2}\sum_{j=1}^{N}\{[m_j - m'_j - (m_{j-1} - m'_{j-1})]c_j + [m_j + m'_j - (m_{j-1} + m'_{j-1})]u_i + (m_{j-1} - m_j)|u_i + c_j| - (m'_{j-1} - m'_j)|u_i - c_j|\} \quad (5)$$

where $(2N + 1)$ is the number of linear segments in the characteristic [for the present case $N = 3$ as explicitly indicated in Eq. (4)], c_1, c_2 and c_3 are the breakpoints and m_j (resp. m'_j) are the slopes at each of the negative (resp. positive) intervals of the nonlinear characteristic. The results showed in this paper were obtained for the symmetric form of the piecewise characteristic ($m_i = m'_i$). Then Eq. (5) becomes

$$h(u_i) = m_N\, u_i + \frac{1}{2}\sum_{j=1}^{N}\{(m_{j-1} - m_j)(|u_i + c_j| - |u_i - c_j|)\}. \quad (6)$$

The set of equations (1) was solved by using a fourth-order Runge–Kutta method with an accuracy of 10^{-5}.

Some useful information can be obtained from the stability analysis of the system Eq. (1). The equilibrium points can be calculated [Murray, 1989; Muñuzuri *et al.*, 1995], by imposing $\dot{u} = \dot{v} = \dot{w} = 0$, or

$$\begin{cases} v_i - h(u_i) = 0 \\ u_i - v_i + w_i = 0 \\ -\beta v_i - \gamma w_i = 0\,, \end{cases} \quad (7)$$

and the local stability of these points studied. Due to the n-segment piecewise linear characteristic of each of the cells, it is easier to choose the appropriate nonlinear characteristic in order to program the desired behavior at the cell level (equilibrium points and local stability analysis) and understand the relation between the behavior at the cell level and the emerging dynamics at the macroscopic level. One can take advantage of this fact to build up systems with the desired macroscopic behavior.

3. Results

The results presented in this paper are divided into two different groups depending on the type of coupling. First, we describe (Sec. 3.1) a one-dimensional array of Chua's cells coupled through linear resistors and, second (Sec. 3.2), the same system but with unidirectional couplings (as described in Sec. 2). This second part constitutes a more realistic approach to the neural cells case.

3.1. *Homogeneous diffusive coupling*

The system described in Sec. 2 was numerically integrated. As we are interested in transitions from chaotic behavior to periodic, we choose the nonlinear characteristic in Fig. 1(a) by setting the model parameters: $\alpha = 9$, $\beta = 14.3$, $\gamma = 0$, $m_0 = -0.14$, $m_1 = 0.28$, $m_2 = -0.21$, $m_3 = 0.1$, $c_1 = 1$, $c_2 = 3$, $c_3 = 5$ and DIM = 51. In this case, we have three equilibrium points,

$$\begin{matrix} u_- = -1.5 & v_- = 0 & w_- = 0 \\ u_0 = -0 & v_0 = 0 & w_0 = 0 \\ u_+ = 1.5 & v_+ = 0 & w_+ = 0 \end{matrix} \quad (8)$$

whose eigenvalues are given by,

$$\begin{matrix} \lambda^1_- = \lambda^1_+ = 2.1850 & \lambda^1_0 = -3.8907 \\ \lambda^2_- = \lambda^2_+ = -0.9625 + 2.7055\,\jmath & \lambda^2_0 = 0.1853 + 3.0377\,\jmath \\ \lambda^3_- = \lambda^3_+ = -0.9625 - 2.7055\,\jmath & \lambda^3_0 = 0.1853 - 3.0377\,\jmath \end{matrix} \quad (9)$$

characterizing the cells. For this set of parameters the system has a chaotic and a periodic attractor [Fig. 1(b)]. The initial values of the system determine the final attractor exhibited. The temporal evolution of the u-variable for each attractor (depending on the initial conditions of the circuit) is plotted in Figs. 1(c) and 1(d).

The interesting behavior appears when a small linear resistance is considered to couple neighboring cells [as described in Sec. 2, Eq. (2)]. These

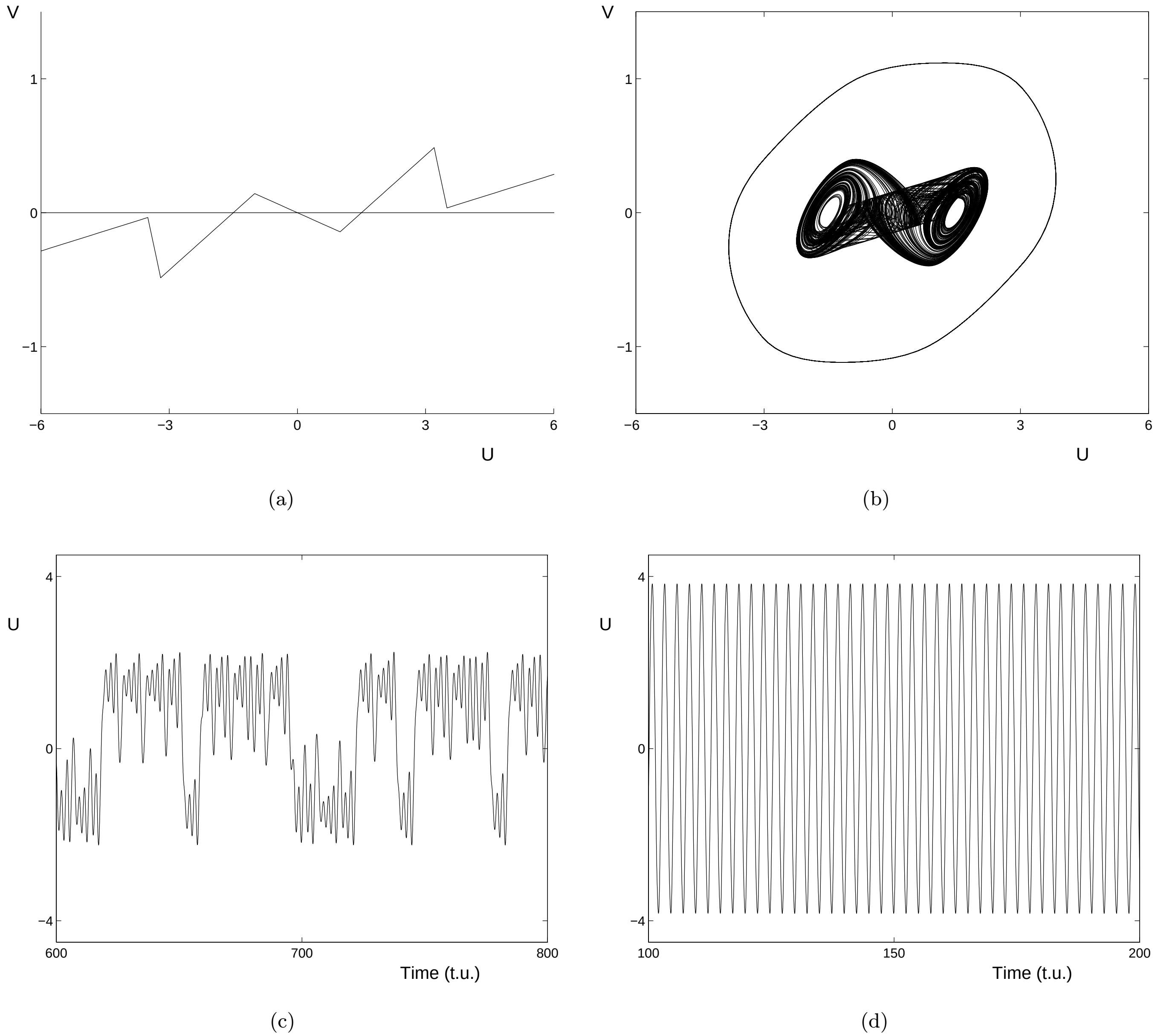
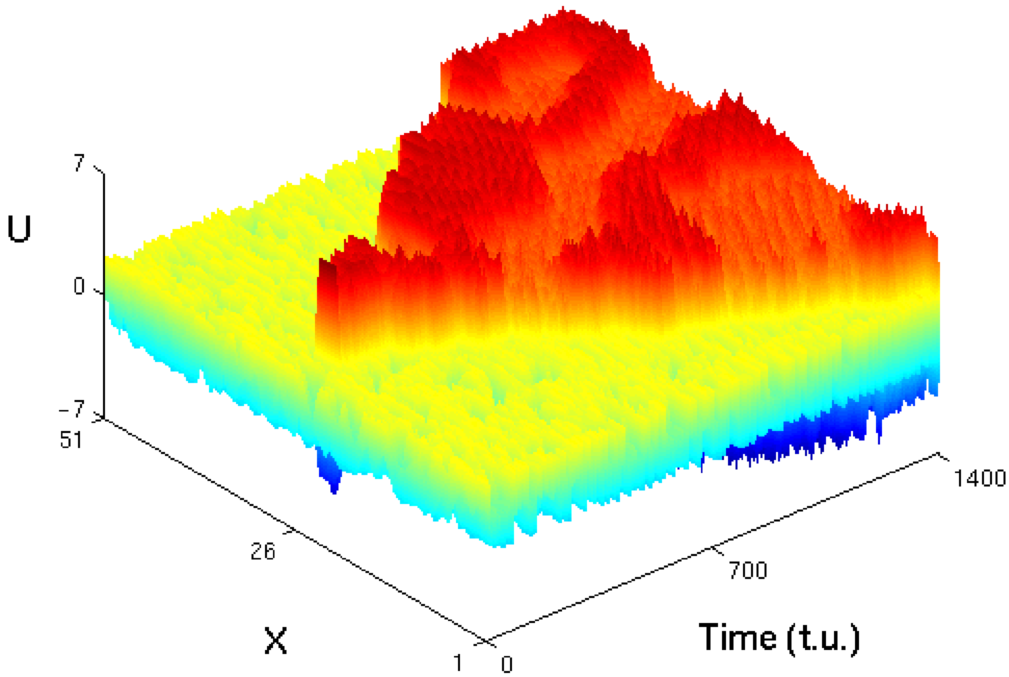
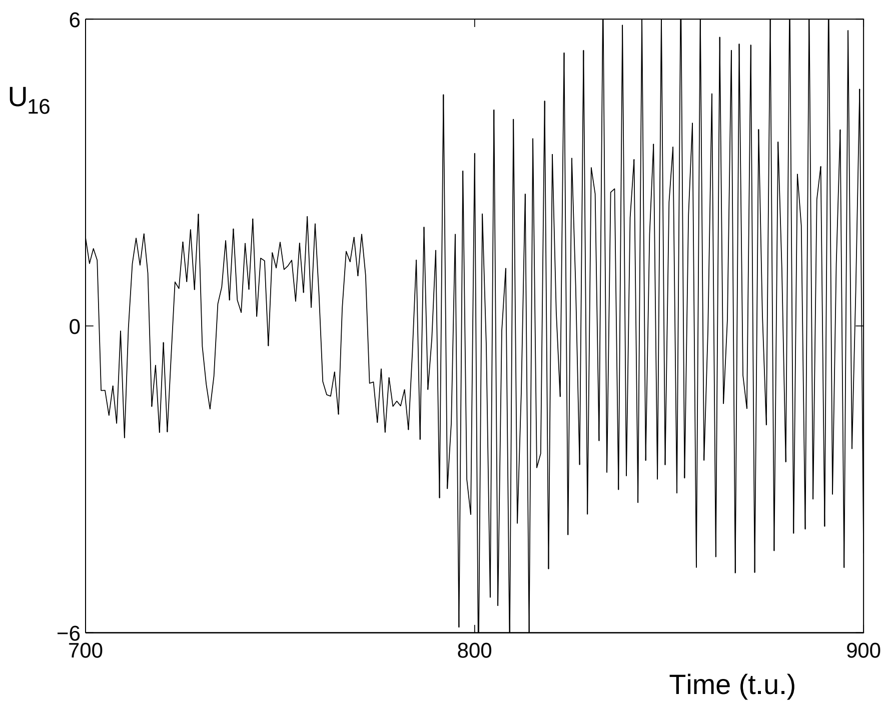


Fig. 1. Uncoupled system. (a) Piecewise linear characteristic function of the unit cell used (generalized Chua's circuit). (b) Trajectories followed by the system for different initial states: For small initial values of the cell variables, the full trajectory of the system lies in a double-scroll chaotic attractor (central part of the figure) while for larger initial values the behavior becomes periodic. (c) Temporal evolution of one circuit in the chaotic regime. (d) Temporal evolution of one circuit in the periodic regime. (Model parameters: $\alpha = 9$, $\beta = 14.3$, $\gamma = 0$, $m_0 = -0.14$, $m_1 = 0.28$, $m_2 = -0.21$, $m_3 = 0.1$, $c_1 = 1$, $c_2 = 3$ and $c_3 = 5$.)

results are plotted in Fig. 2. Figure 2(a) presents values of the u-variables of all circuits as a function of time for a value of $D_u = 0.12$. At early stages, all the cells are in chaos; at $t = 100$ time units (t.u.) the central cells (numbers 25, 26 and 27) are perturbed by a pulse of $u_{25} = u_{26} = u_{27} = -5$, leaving unperturbed all other variables of the system. This perturbation drives these cells out of the chaotic attractor into a periodic behavior. Due to the diffusive term, the periodic behavior propagates to the neighboring cells until the whole system oscillates periodically at $t = 1400$ t.u. In this case, a transition wave can be clearly observed to propagate in both directions with nonconstant velocity. Figure 2(b) shows the temporal evolution of one cell (circuit number 16) before, during and after



(a)

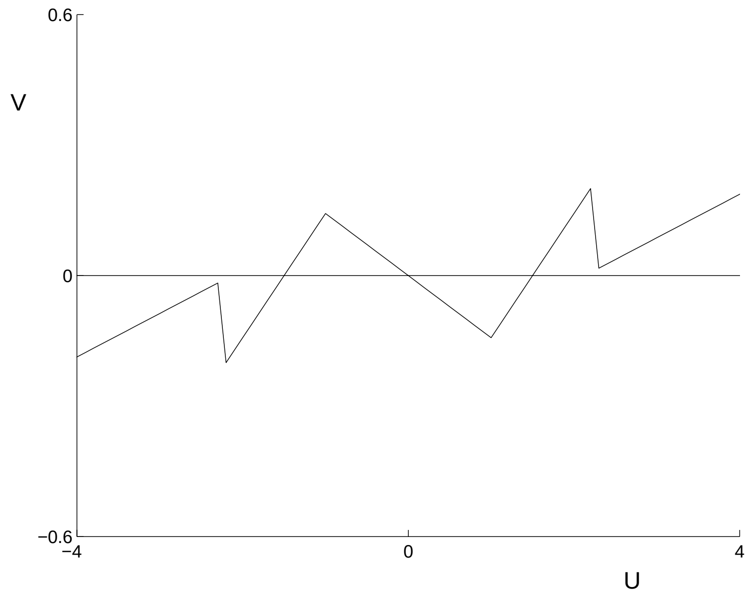


(b)

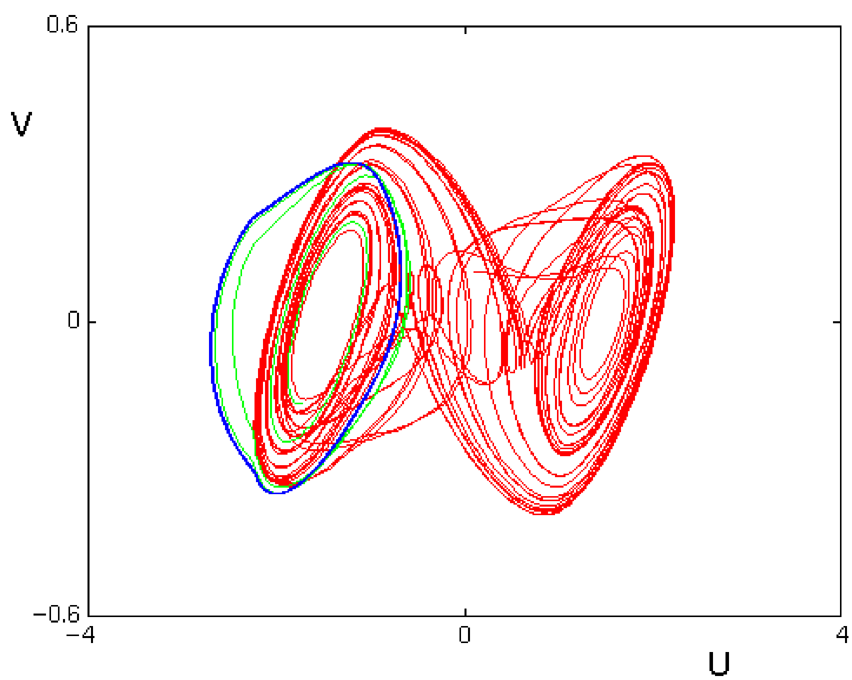
Fig. 2. One-dimensional array of Chua's circuits with homogeneous diffusive coupling. (a) Space-time plot showing a wave of periodicity propagating along the one-dimensional array in both directions. At any time, the value of the u-variable is plotted for all the cells in the array. At $t = 100$ t.u. the central cells of the array are perturbed and this perturbation starts propagating to the neighboring cells until the whole system oscillates periodically (the periodic behavior can be clearly seen because of the large values of the u-variable — red and dark blue colors). (b) Temporal evolution of the u-variable for circuit number 16. (Same parameters as in Fig. 1 and $D_u = 0.12$ and DIM $= 51$.)

the transition wave reaches the circuit. Note that the transition is smooth, first the chaotic behavior is perturbed and some time is required to drive the system completely into periodic behavior.

For smaller values of the diffusion coefficient, D_u, the periodic wave cannot propagate to every cell and some regions of periodic behavior can be observed embedded in chaotic regions. For larger

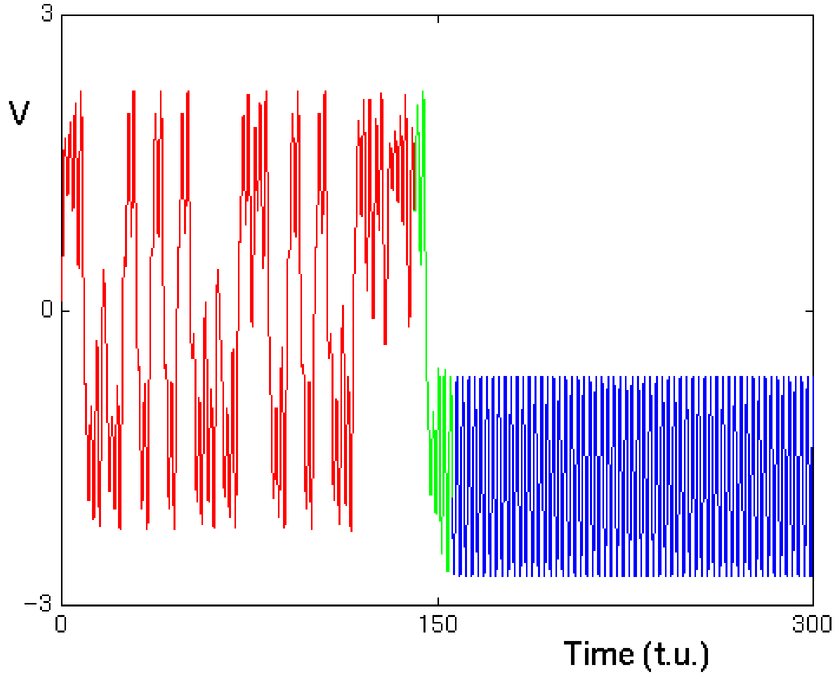


(a)



(b)

Fig. 3. Uncoupled system. (a) Piecewise linear characteristic function of the unit cell used (generalized Chua's circuit). (b) Typical trajectory followed by the system (red line corresponds with the chaotic regime, blue line with periodic regime and green one with the transitory period between both states). (c) Temporal evolution of such circuit (same color codes as in the previous figure). (Model parameters: $\alpha = 9$, $\beta = 14.3$, $\gamma = 0$, $m_0 = -0.14$, $m_1 = 0.28$, $m_2 = -50.0$, $m_3 = 0.1$, $c_1 = 1$, $c_2 = 2.24$ and $c_3 = 2.244$.)



(c)

Fig. 3. (*Continued*)

values of D_u the periodic wave does not need to be initiated but rather small perturbations intrinsic to the system are able to drive some of the circuits spontaneously out of the chaotic regime into the periodic one.

More interesting behaviors are observed when the breakpoints in the circuit characteristic are displaced. When the breakpoints are separated [i.e. c_3 is larger than the previous case (Fig. 2)] the transition reported becomes more and more complicated; while for smaller values of c_3, the periodic and the chaotic attractors become a single one such that any cell may present sudden transitions to periodicity during its evolution. Such a behavior is shown in Fig. 3 by setting the model parameters: $\alpha = 9$, $\beta = 14.3$, $\gamma = 0$, $m_0 = -0.14$, $m_1 = 0.28$, $m_2 = -50.0$, $m_3 = 0.1$, $c_1 = 1$, $c_2 = 2.24$, $c_3 = 2.244$ and $DIM = 51$. In Fig. 3(a) Eqs. (7) are plotted. The observed behavior is shown in Fig. 3(b) and the temporal evolution of the u-variable is plotted in Fig. 3(c). Notice that in this case the equilibrium points and their local stability is the same as in Eqs. (8) and (9).

When a coupling resistor is considered ($D_u = 0.025$) the result is shown in the space-time plot of Fig. 4(a). We impose some random initial conditions for each circuit and let the system evolve. As time increases, each circuit undergoes a transition form chaotic behavior to periodic and vice-versa. The small coupling between the cells destabilizes the periodic regime but it is not strong enough as to avoid the formation of clusters of cells that remain for a certain time in periodicity. The temporal evolution of the circuit number 25 is plotted in Fig. 4(b). This presents a similarity with the situation observed for neurons in the brain, where some of the neurons are more sensitive to external perturbations that may induce transitions to periodicity for a period of time (epileptic episodes [Freeman, 1979]), while some others are very stable and only rarely or under very strong perturbations exhibit this transition.

When the coefficient is decreased ($D_u = 0.01$), the influence of the neighboring cells is smaller and the periodic behavior becomes more stable. This is shown in Fig. 5(a), the system spontaneously jumps into the periodic behavior and remains there as the influence of the neighboring cells is not strong enough to destabilize it. The temporal evolution of the circuit number 8 is plotted in Fig. 5(b), after a transient where the cell jumps from chaos to periodicity several times, the circuit finally behaves periodically and remains there.

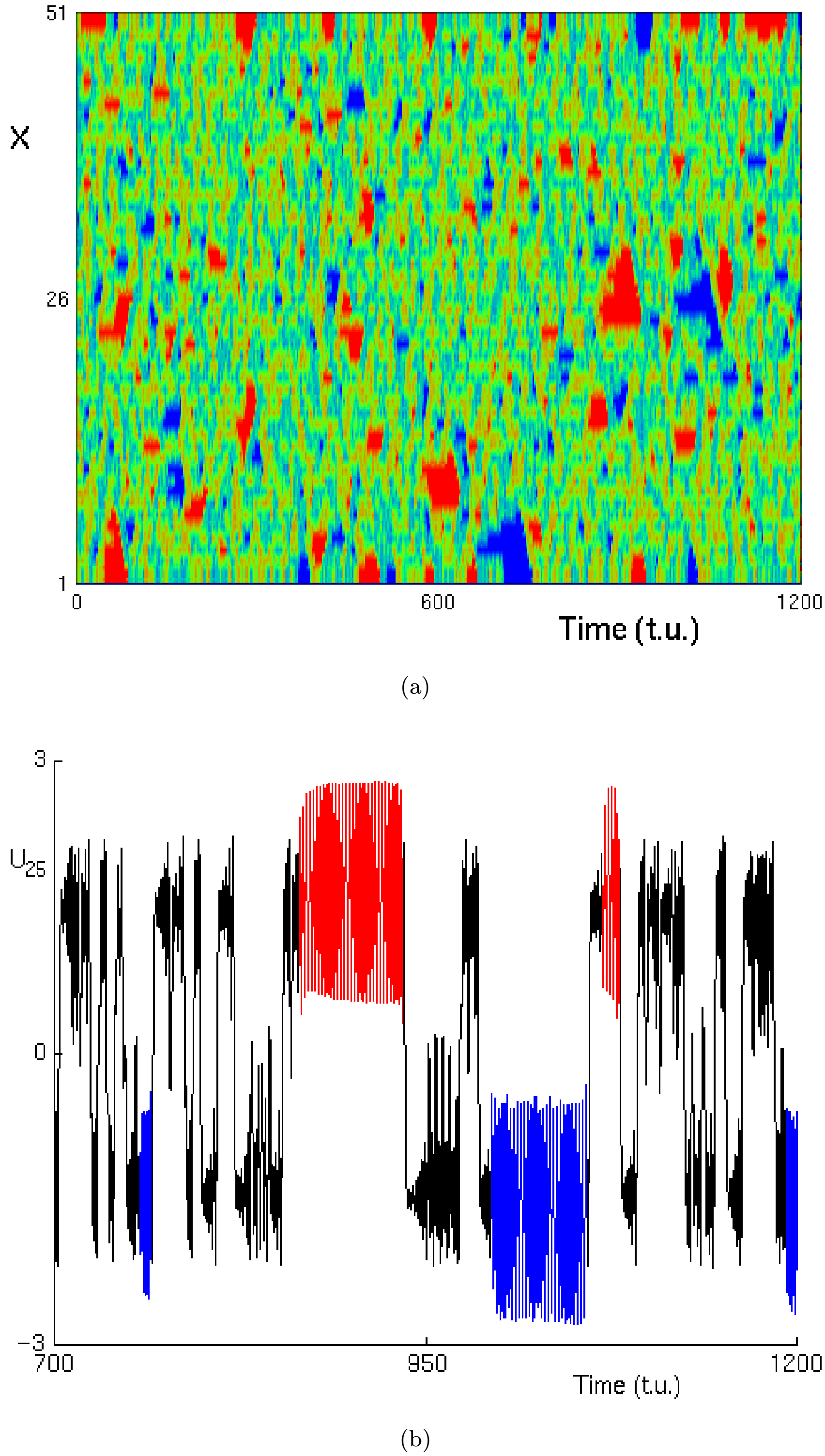
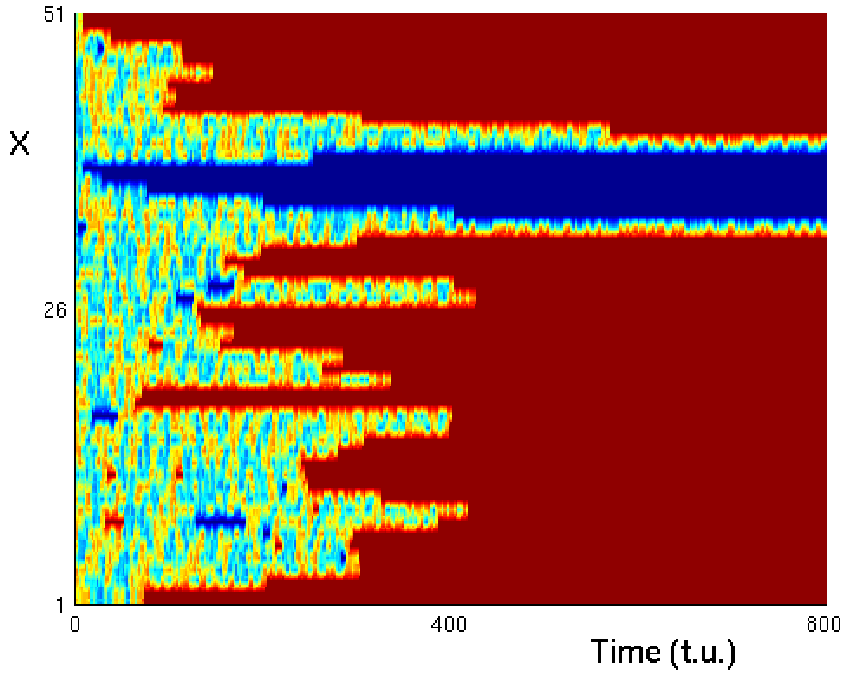
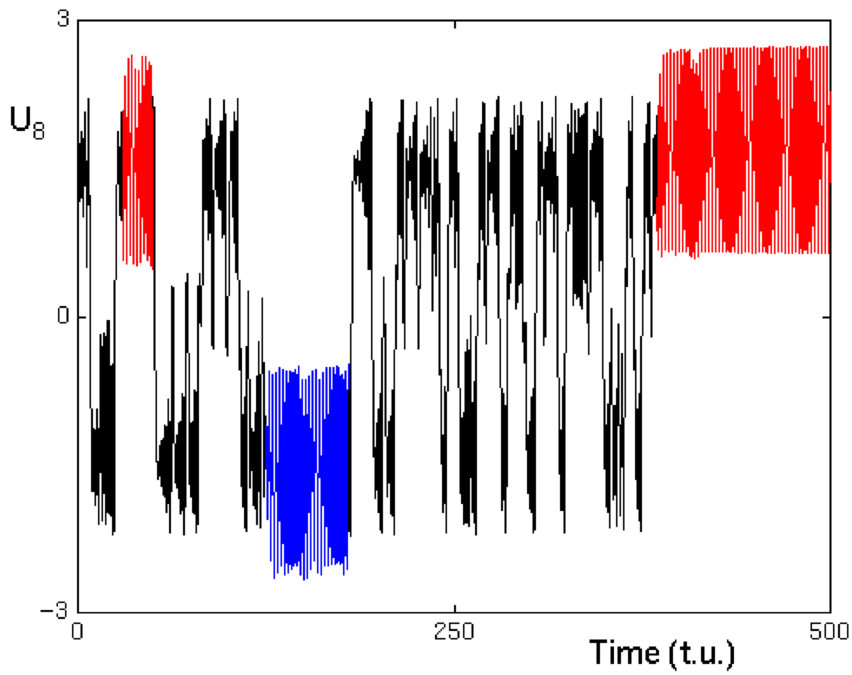


Fig. 4. One-dimensional array of Chua's circuits with homogeneous diffusive coupling. (a) Space-time plot showing the appearance of clusters of periodicity (areas colored in red or dark blue) embedded in chaotic areas (colored with intermediate colors). (b) Temporal evolution of the u-variable for circuit number 25 (periodic episodes are marked in red and blue colors as in the previous figure). (Same parameters as in Fig. 3 and $D_u = 0.025$ and DIM $= 51$.)



(a)



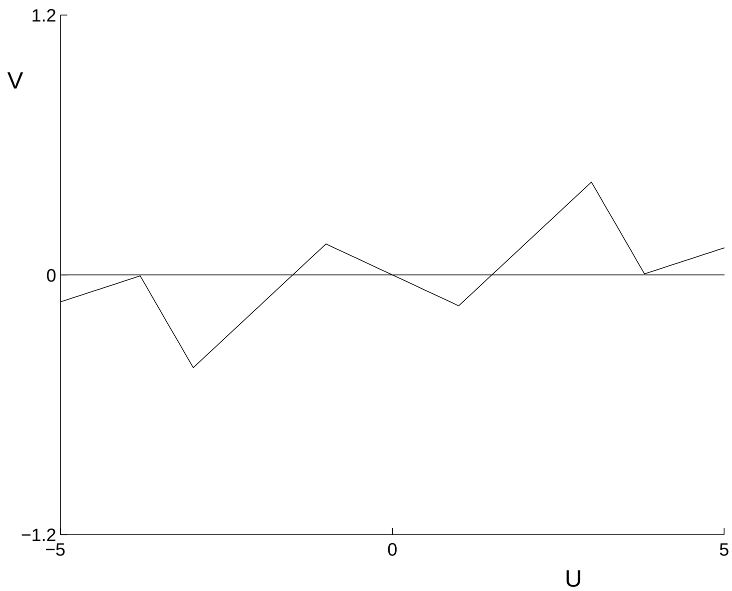
(b)

Fig. 5. One-dimensional array of Chua's circuits with homogeneous diffusive coupling. (a) Space-time plot showing the spontaneous self-organization of the system into periodic areas (colored in red or dark blue) separated by a transition chaotic circuit. (b) Temporal evolution of the u-variable for circuit number 8 (periodic episodes are marked in red and blue colors). (Same parameters as in Fig. 3 and $D_u = 0.01$ and $\mathrm{DIM} = 51$.)

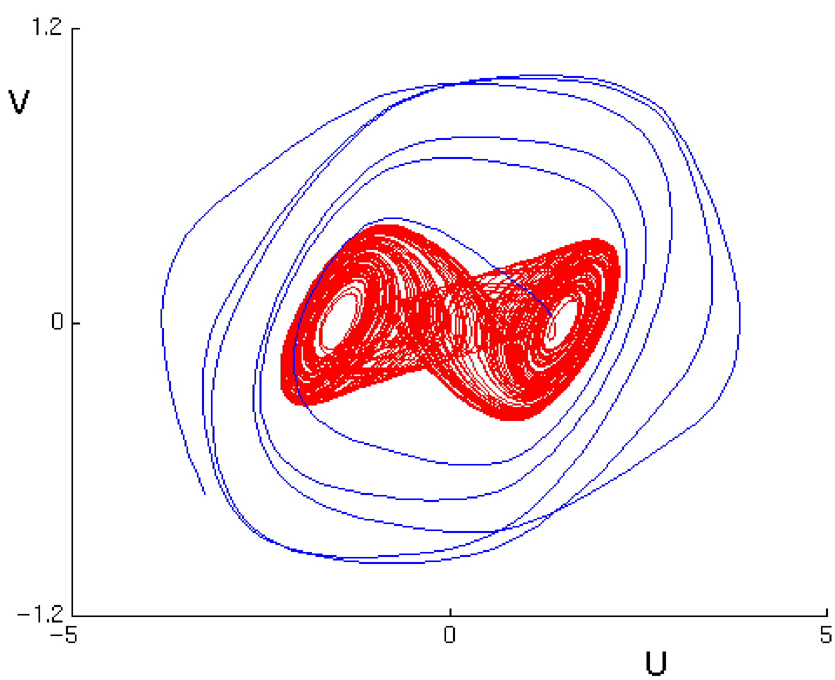
3.2. *Unidirectional coupling*

In this section we considered the one-dimensional array of Chua's circuits as described in Sec. 2 with a unidirectional coupling, each circuit receives only the influence of the previous circuit and influences only the next one. This type of coupling is more related to the type of couplings between neurons in biology.

We integrated this system for the set of parameters: $\alpha = 9$, $\beta = 14.3$, $\gamma = 0$, $m_0 = -0.14$, $m_1 = 0.28$, $m_2 = -0.53$, $m_3 = 0.1$, $c_1 = 1$, $c_2 = 3$,

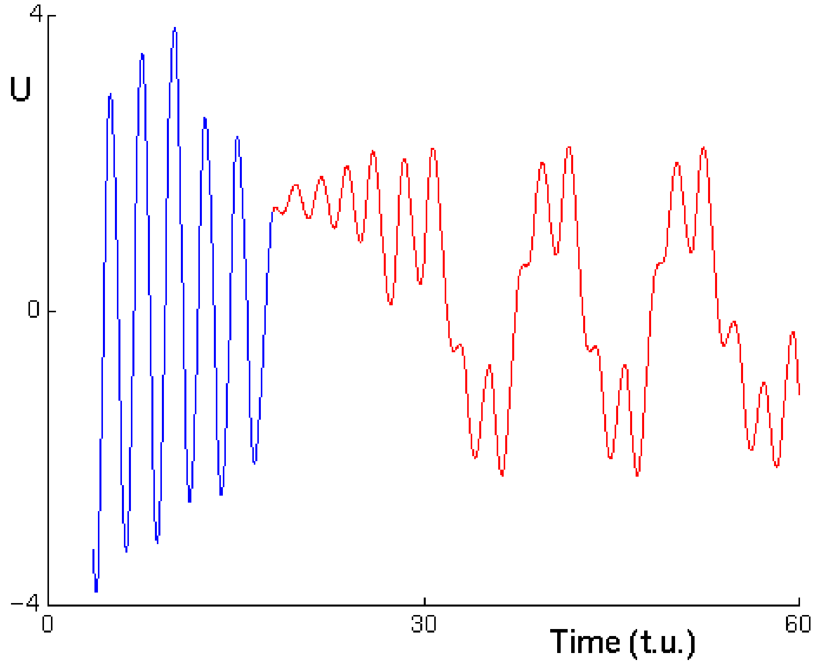


(a)



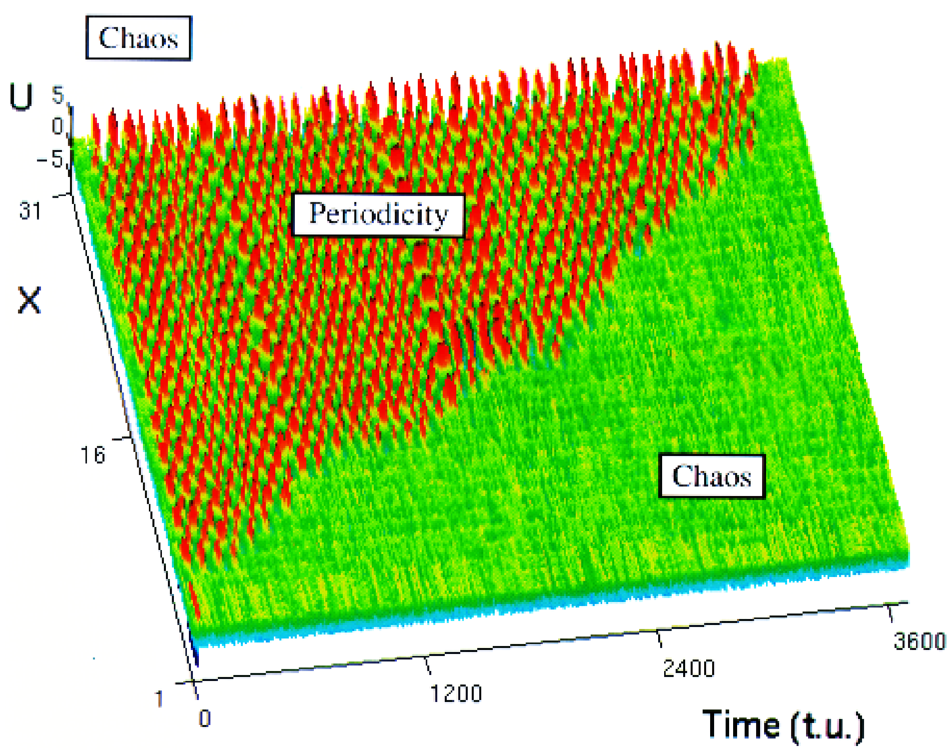
(b)

Fig. 6. Uncoupled system. (a) Piecewise linear characteristic function of the unit cell used (generalized Chua's circuit). (b) Typical trajectory followed by the system (red line corresponds with the chaotic regime, blue line with transitory periodic regime). (c) Temporal evolution of such circuit (same color codes as in the previous figure). (Model parameters: $\alpha = 9$, $\beta = 14.3$, $\gamma = 0$, $m_0 = -0.14$, $m_1 = 0.28$, $m_2 = -0.53$, $m_3 = 0.1$, $c_1 = 1$, $c_2 = 3$ and $c_3 = 3.8$.)



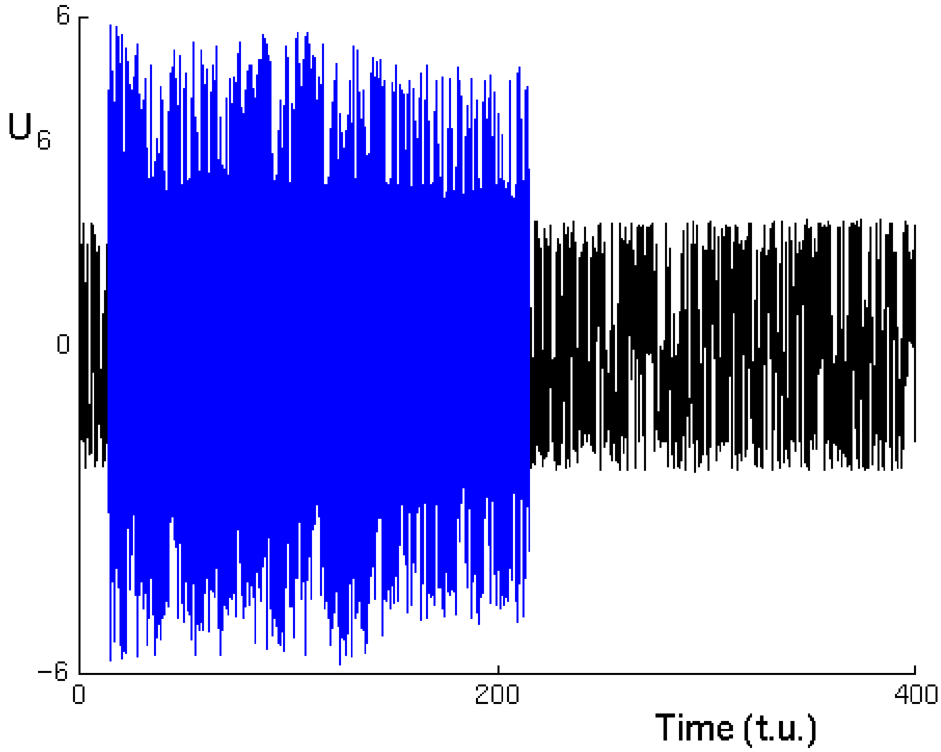
(c)

Fig. 6. (*Continued*)



(a)

Fig. 7. One-dimensional array of Chua's circuits with unidirectional coupling. (a) Space-time plot; first all the circuits are in chaos and, at $t = 30\,t.u.$ the circuit number 1 is perturbed. This perturbation drives the cell out of the chaotic attractor into the periodic behavior. Due to the diffusion, the periodic behavior propagates to the neighboring cell, causing a fast transition wave in the system. As time evolves, the first circuit decays to the chaotic attractor again. This induces a new slow transition wave from periodicity back to chaos again that propagates along the whole system. (b) Temporal evolution of the u-variable for circuit number 9 (periodic episodes are marked in blue). (Same parameters as in Fig. 6 and $D_u = 0.201$ and DIM $= 31$.)



(b)

Fig. 7. (*Continued*)

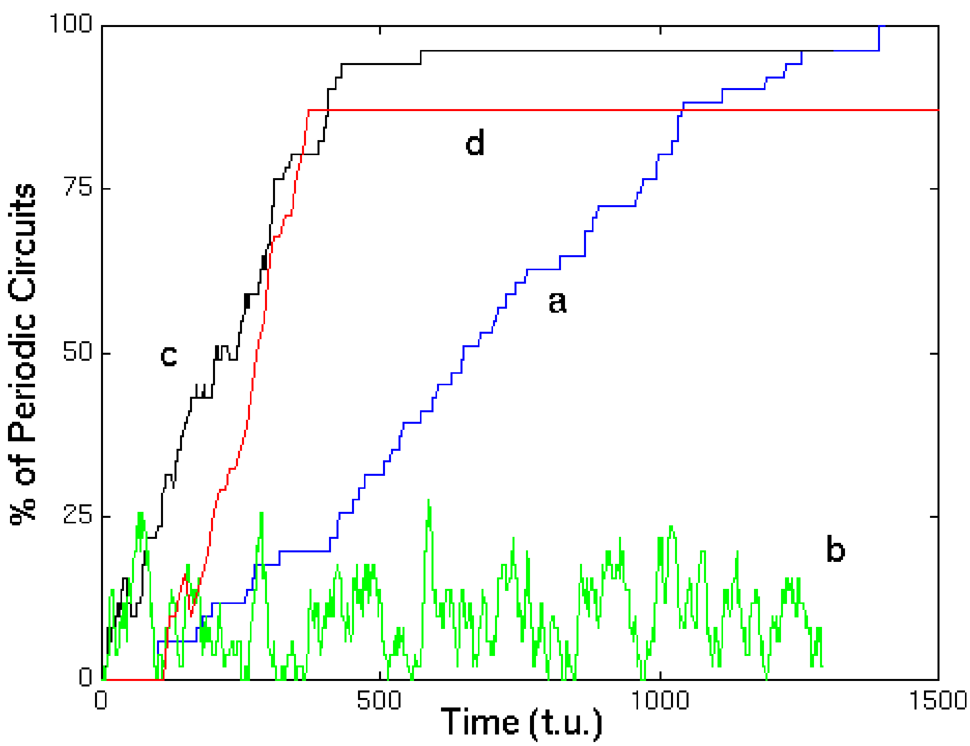


Fig. 8. Percentage of circuits in the periodic regime versus time for the different cases studied: (a) Fig. 2, (b) Fig. 4, (c) Fig. 5 and (d) Fig. 7.

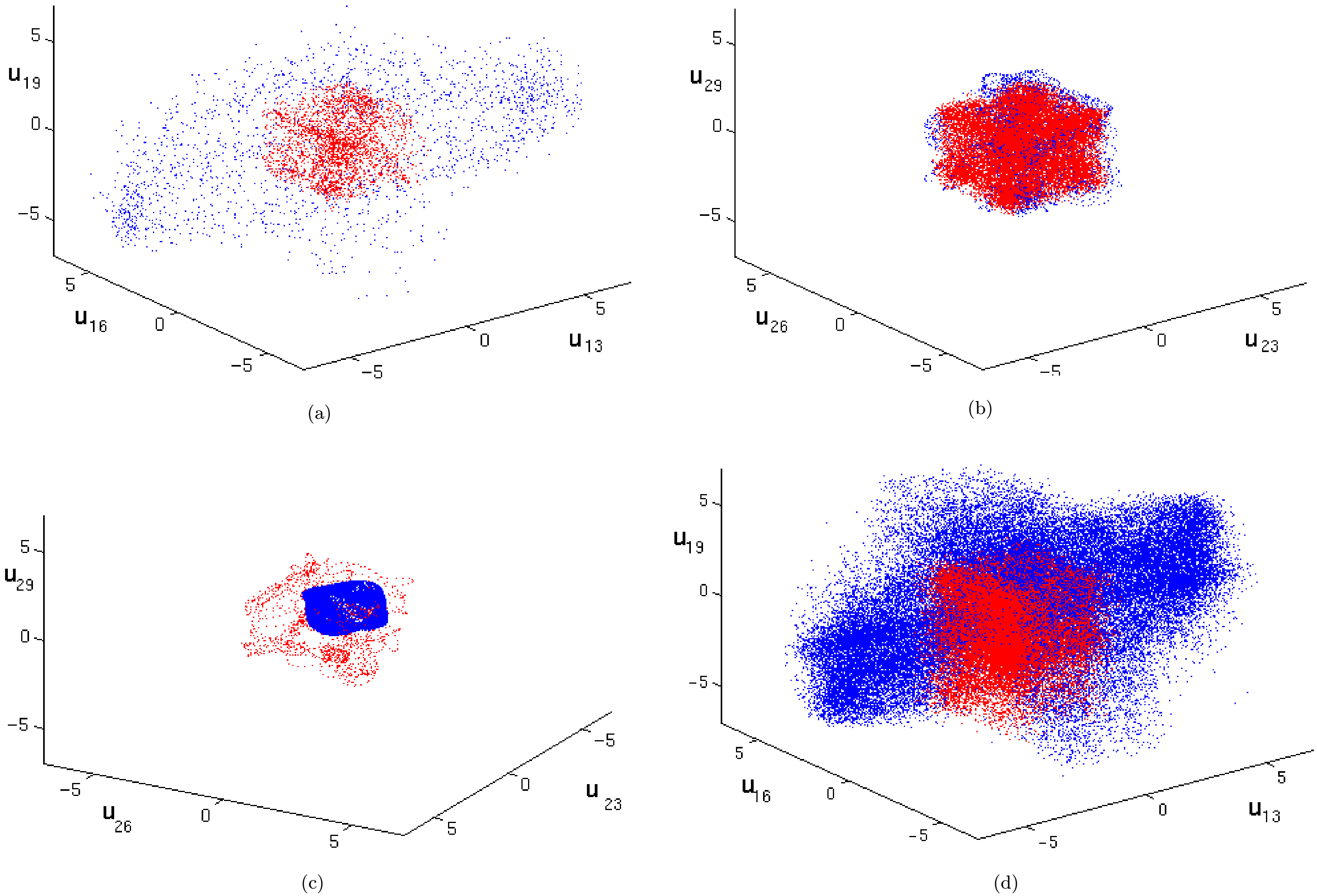


Fig. 9. Values of the u-variable for different circuits in the array are plotted together for the different cases studied: (a) Fig. 2, (b) Fig. 4, (c) Fig. 5 and (d) Fig. 7. (Points marked with blue color are in a periodic state while those in red are chaotic.)

$c_3 = 3.8$ and DIM $= 31$. These new parameters produce a nonlinearity as plotted in Fig. 6(a), that corresponds with the dynamic in the (u, v)-plane shown in Fig. 6(b). The temporal evolution of the u-variable is plotted in Fig. 6(c). The system now has only one chaotic attractor but, for a sufficiently large perturbation, the system does not return immediately to the chaotic regime but it rather stays temporary in a quasiperiodic orbit before decaying into the chaotic attractor again. This is just a transient effect at the cell level, but when a one-dimensional array of such cells is considered, cooperative phenomena are observed. This is shown in Fig. 7(a) ($D_u = 0.201$), first all the circuits are in chaos and, at $t = 30$ t.u. the circuit number 1 is perturbed by a pulse of $u_1 = -5$. This perturbation drives the cell out of the chaotic attractor into periodic behavior. Due to the diffusion, the periodic behavior propagates to the neighboring cell, causing a fast transition wave in the system. As time evolves, the first circuit decays into the chaotic attractor again. This induces a new slow transition wave from periodicity back to chaos again that propagates along the whole system. The temporal evolution of the circuit number 9 is plotted in Fig. 7(b), first the system lays in a chaotic attractor and suddenly, when the first fast transition wave arrives, it is driven out of the chaotic regime and into the periodic one. After a time, the second slow transition wave reaches the cell and induces its decay to the chaotic attractor again. Note that a small perturbation induces a dramatic change in the whole system dynamics; because periodicity is associated in some systems (like the brain neurons) with malfunctioning, this means that the perturbation of one single cell (caused by some intrinsic or external causes) can cause a temporary malfunctioning of the whole system.

All results presented in this section are summarized in Fig. 8. The percentage of circuits in a periodic regime is plotted as a function of time for the different cases studied. Two qualitatively different behaviors observed; on the one hand, a rapid increase of the number of periodic cells (cases a, c and d in the figure), for which it is possible to define a transition velocity (from 0% to 100%); on the other hand, the system can remain in some intermediate state, with strong oscillations (case b in the figure). Figure 9 also compares the different behaviors studied. Here the values of the u-variable for different circuits in the array are plotted together; the geometrical figures of the different attractors occupy different parts of the space (blue color is used to mark the periodic behavior while red marks the chaotic one).

4. Conclusion

By using a CNN, whose basic unit cell is a generalized Chua's circuit, we produced chaos-periodicity transitions similar to some transitions observed in the brain during epileptic seizures. For the different couplings and cases considered in the paper, small perturbations to the system are able to trigger some of the cells out of the chaotic regime and into a periodic one. Depending on the different couplings (diffusive or unidirectional) or values for the parameters used, it is observed that the periodic behavior propagates to the neighboring cells, appearing as a transition wave. The velocity of this transition wave is nonconstant as it strongly depends on the local values of the circuit variables and small differences may induce the transition in one of the cells or not. For the unidirectional case (closer to the type of coupling in neurons) another transition wave is observed driving the circuits back to the chaotic behavior from the periodic one. In some other cases, clusters of circuits in periodicity appear and disappear as time evolves, surrounded by chaotic circuits.

All this points out the possibility of qualitatively modeling complex behaviors (such as some observed in the brain) by using artificial neural networks. The extensive study of these systems can be employed not only to reproduce the dynamics of some existing systems but also to obtain exhaustive information about this type of dynamics because for the proposed CNNs the link between the behavior at the cell level and the macroscopic level is understandable. As an extension of the described behaviors, it is possible then to think about designing systems where different types of structures may coexist, which enables the study of their interaction on a real homogeneous system, opening the possibilities for future work.

Acknowledgments

Alberto P. Muñuzuri acknowledges Dr. V. Pérez-Muñuzuri for helpful discussions. The research was undertaken at the University of California at Berkeley, partially within the framework of a Concerted Action Project MIPS of the Belgian

Flemish Community and the IUAP P4-02 programme, initiated by the Belgian government. Calculations were performed at the *Centro de Supercomputación de Galicia* (*CESGA*), Spain.

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