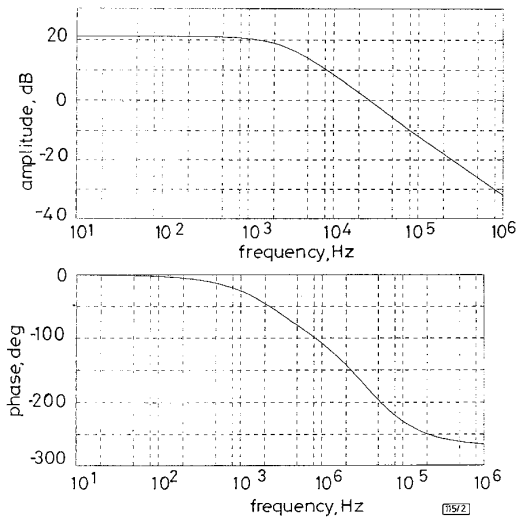


**Results:** Fig. 2 presents the frequency response of a half-wave ZVS boost converter. The values of the resonant elements are  $L_r = 47\mu\text{H}$  and  $C_r = 13.3\text{nF}$  and the load resistance is  $R = 30\Omega$ . The control system was designed using Unitrode's UC3864 resonant mode power supply controller.



**Fig. 2** Frequency response of half-wave ZVS boost converter

**Conclusions:** The method described in the Letter shows how to determine the mathematical expression of the transfer function of a quasi-resonant boost converter. It can be applied for each quasi-resonant DC/DC converter operating with continuous current. Knowing the control-to-output transfer function of the power circuit, the design of the feedback loop and compensation circuit is simpler, and better dynamic performances can be achieved. The algorithm can be implemented easily in computer simulation programs.

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A. Szabo, M. Kansara, P.G. Holmes and E.S. Ward (Department of Electrical Engineering, The Nottingham Trent University, Burton Street, Nottingham, NG1 4BU, United Kingdom)

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## Wien bridge based RC chaos generator

Ö. Morgül

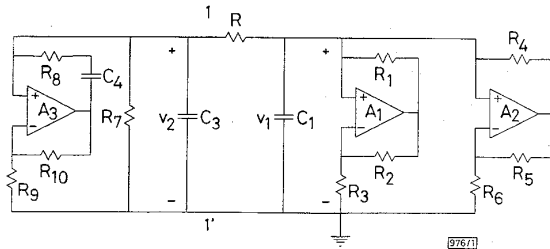
*Indexing terms:* Chaos, Oscillators

A new circuit, which is formed by coupling a Chua diode with a Wien bridge oscillator in parallel, is presented. This circuit contains only resistors, capacitors and operational amplifiers. By choosing element values appropriately, this circuit is shown experimentally to exhibit various forms of chaotic behaviour.

**Introduction:** The study of chaotic behaviour in nonlinear dynamic systems has attracted great interest because of many possible applications in various fields of science. Since simple nonlinear circuits may exhibit such behaviour, the analysis and design of elec-

tronic circuits which generate chaotic signals has received a great deal of attention in recent years [1]. Most of the chaotic signal generators proposed in the literature contain inductors (see, e.g. [2]), which is inconvenient for various reasons. For example, inductors are less standard as compared to other circuit elements and have to be prepared separately in most applications. They are not as ideal as other circuit elements, and in terms of spatial dimensions they are bigger in size than the other circuit elements, unless the inductance is rather small. The circuits which contain capacitors instead of inductors are more convenient in this sense; however, most of such chaotic circuits proposed in the literature are rather complex (see [3] and the references therein).

In this Letter we propose a simple circuit which contains a Wien bridge and a three-segment piecewise linear resistor known as the Chua diode. We show experimentally that this circuit may generate various forms of chaotic signals.



**Fig. 1** Wien bridge based chaos generator

**Proposed chaos generator:** We consider the circuit shown in Fig. 1. In Fig. 1 the operational amplifiers and the associated resistances ( $R_1, \dots, R_6$ ) are used to realise the three-segment piecewise linear resistance called the Chua diode [2]. The circuit to the left of 1-1' is a Wien bridge circuit. The resistor  $R$  is a potentiometer and can be used to tune the circuit to observe chaotic behaviour. This circuit is first proposed in [4], and for this reason, we used the same element labelling used in [4]. However, the mode of operation of the Wien bridge in this Letter is different than the one proposed in [4], which is described below.

Assuming that the operational amplifier A3 is ideal and operates in the linear region, the impedance  $Z(s)$  of the Wien bridge circuit in Fig. 1 (i.e. to the left of 1-1') can be found as

$$Z(s) = \frac{R_7 R_9 (s C_4 R_8 + 1)}{s^2 C_3 C_4 R_7 R_8 R_9 + s (R_8 R_9 C_4 + R_7 R_9 C_3 - R_7 R_{10} C_4) + R_9} \quad (1)$$

We note that all resistors and capacitors in this circuit are passive (i.e. resistances and capacitances are positive). A simple calculation shows that the impedance  $Z(s)$  given in eqn. 1 is a positive real function (i.e. stable and  $\text{Re}\{Z(j\omega)\} \geq 0 \forall \omega \in \mathbf{R}$ ) if and only if the following condition holds:

$$R_8 R_9 \geq R_7 R_{10} \quad (2)$$

If eqn. 2 holds, then the impedance  $Z(s)$  given above can also be realised by using only passive  $R$ ,  $L$  and  $C$  elements. In particular, if the equality  $R_8 R_9 = R_7 R_{10}$  holds, then the resulting impedance  $Z(s)$  can also be realised by using a series combination of a resistor  $R_0$  and an inductor  $L$  with a parallel combination of a capacitor  $C$ , with  $R_0 = R_7$ ,  $L = C_4 R_7 R_8$  and  $C = C_3$ . In this case, the circuit given in Fig. 1 is equivalent to the Chua oscillator [2, 4]. Since in this case the impedance  $Z(s)$  is positive real, which is related to passivity, we call this mode of operation of the Wien bridge the 'passive mode'. It is known that the Chua oscillator generates chaotic signals [2], and it was shown in [4] that similar behaviour can also be observed in the proposed circuit when the Wien bridge operates in the passive mode.

In the Chua oscillator, the resistor  $R_0$  represents the internal resistance of the inductor. In the ideal case the condition  $R_0 = 0$  should hold, in which case the parallel/series combination of  $R_0$ ,  $L$  and  $C$  becomes a harmonic oscillator. We note that this case is ideal and cannot be realised in practice. However, if the following condition is satisfied,

$$R_8 R_9 C_4 + R_7 R_9 C_3 - R_7 R_{10} C_4 = 0 \quad (3)$$

then sustained oscillations can easily be obtained in the Wien bridge. For this reason we say that the Wien bridge operates in

'oscillatory mode' when eqn. 3 is satisfied. Since in this case the denominator of eqn. 1 is in the form  $s^2 + \omega_0^2$ , the oscillation frequency  $\omega_0$  (rad/s) is given by

$$\omega_0 = \frac{1}{C_3 C_4 R_7 R_8} \quad (4)$$

We note that in the oscillatory mode, the impedance  $Z(s)$  given by eqn. 1 is not positive real, and hence cannot be realised by using only passive elements. In the next Section we show experimentally that the proposed circuit can exhibit various forms of chaotic behaviour when the Wien bridge operates in the oscillatory mode.

**Experimental results:** For the Chua diode we choose the following parameters, which are taken from [2]:  $R_1 = R_2 = 220\Omega$ ,  $R_3 = 2.2\text{k}\Omega$ ,  $R_4 = R_5 = 22\text{k}\Omega$ ,  $R_6 = 3.3\text{k}\Omega$ , and A1 and A2 are operational amplifiers (AD712 or equivalent). To set the Wien bridge in oscillatory mode, for simplicity we choose  $R_7 = R_8$  and  $C_3 = C_4$ . In this case, eqn. 3 reduces to  $R_{10} = 2R_9$ .

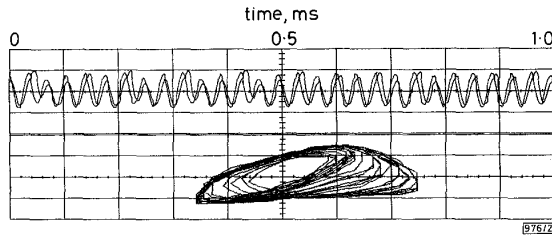


Fig. 2  $v_2-v_1$  characteristics for  $C_1 = 33\text{pF}$ ,  $R = 1890\Omega$

Sensitivity:  
Channel 1: 400mV/div  
Channel 2: 100mV/div  
Function 1: 60mV/div

We obtained chaotic behaviour for a large number of parameter values. However, we only report some of them here. We used a  $2\text{k}\Omega$  potentiometer for  $R$  and a  $1\text{k}\Omega$  potentiometer for  $R_{10}$ . The role of  $R_{10}$  is to tune the Wien bridge oscillator to oscillatory mode (i.e. to satisfy eqn. 3). In all these experiments, we first disconnect  $R$ , and change  $R_{10}$  till oscillations are observed for  $v_2$ . Note that these oscillations should be as close to a pure sinusoidal waveform as possible. Then, we connect and tune  $R$  to observe chaotic behaviour. We also note that the chaotic behaviour is very sensitive to  $R_{10}$ . Nevertheless, tuning to chaos can be achieved rather easily. We choose  $R_7 = R_8 = R_9 = 100\Omega$ ,  $R_{10} = 200\Omega$  and  $C_3 = C_4 = 47\text{nF}$ . As explained above,  $R_{10}$  is a potentiometer and is used to tune the Wien bridge oscillator to the oscillatory mode.  $R_{10} = 200\Omega$  is the theoretical value to obtain pure sinusoidal oscillations (see above); actual measurement is very close to this value ( $R_{10} = 207\Omega$ ). The oscillation frequency is  $33.87\text{kHz}$  in this case (see eqn. 4). Then we used  $R$  to obtain chaotic behaviour for various values of  $C_1$  and  $R$ . For the parameter values stated above, we observed chaotic behaviour for various values of  $C_1$  and  $R$  in the range  $1\text{pF} \leq C_1 \leq 470\text{pF}$ ,  $1345\Omega \leq R \leq 1890\Omega$ . For the case  $C_1 = 33\text{pF}$ , the waveforms of  $v_1$  and  $v_2$ , and  $v_2 - v_1$  characteristics are given in Figs. 2 and 3 for  $R = 1890\Omega$  and  $R = 1360\Omega$ , respectively.

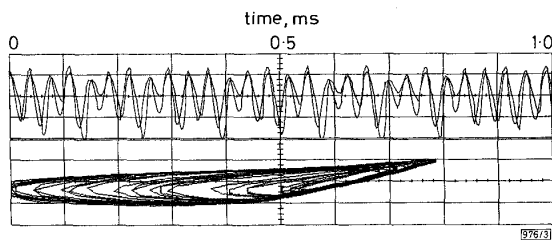


Fig. 3  $v_2-v_1$  characteristics for  $C_1 = 33\text{pF}$ ,  $R = 1360\Omega$

Sensitivity:  
Channel 1: 40mV/div  
Channel 2: 10mV/div  
Function 1: 14mV/div

Since we observed chaotic behaviour even for  $C_1 = 1\text{pF}$ , which is the smallest standard capacitance available in our laboratory, we omitted  $C_1$  in some experiments to see if we can still observe chaotic behaviour. Indeed, even if  $C_1$  is not used, we observed

chaotic behaviour in our experiments. For this case (i.e.  $C_1$  is not used), Figs. 4 and 5 show the waveforms  $v_1$  and  $v_2$ , and  $v_2 - v_1$  characteristics for  $R = 1840\Omega$  and  $R = 1680\Omega$ , respectively. Since a second-order autonomous system with continuous nonlinearity cannot exhibit chaotic behaviour, the observed chaotic behaviour when  $C_1$  is not used, requires explanation. A possible explanation could be the existence of parasitic capacitances. Another explanation could be a possible hysteresis type nonlinearity occurring in some part of the circuit.

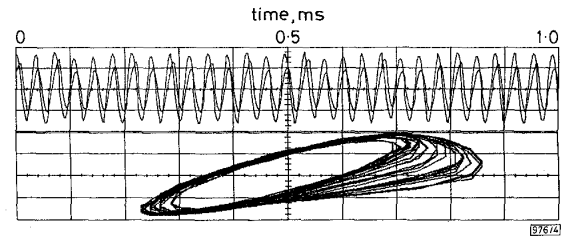


Fig. 4  $v_2-v_1$  characteristics for  $R = 1840\Omega$  ( $C_1$  not used)

Sensitivity:  
Channel 1: 200mV/div  
Channel 2: 40mV/div  
Function 1: 36mV/div

In all the Figures we first observed the  $v_2 - v_1$  characteristics in an analogue oscilloscope in the  $X - Y$  mode. Then the same Figure is obtained in a digitising oscilloscope (HP 54502A). After storing the screen in the memory of the oscilloscope, the screen is printed on a plotter (HP 7475A) by using an HB-IB bus. The signals above are  $v_1$  and  $v_2$  against time, and below are  $v_2 - v_1$  graphics. Since the important information is the  $v_2 - v_1$  graphics, we do not label the signals  $v_1$  and  $v_2$ .

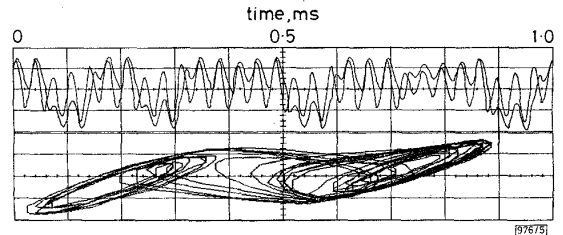


Fig. 5  $v_2-v_1$  characteristics for  $R = 1680\Omega$  ( $C_1$  not used)

Sensitivity:  
Channel 1: 200mV/div  
Channel 2: 40mV/div  
Function 1: 36mV/div

**Conclusion:** In this Letter we have presented a new chaotic circuit. This circuit contains a Wien bridge coupled with a three segment piecewise linear resistor, known as the Chua diode. Since the proposed circuit does not contain an inductor, it may be more suitable for integrated circuit applications. We have shown experimentally that this circuit exhibits many types of chaotic behaviour. Since this circuit has more parameters which could be used for chaos tuning, it may be useful for various practical applications.

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Ö. Morgül (Bilkent University, Department of Electrical and Electronics Engineering, 06533, Bilkent, Ankara, Turkey)

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