ADAPTIVE SYNCHRONIZATION OF CHAOTIC SYSTEMS BASED ON SPEED GRADIENT METHOD WITH APPLICATION TO COMMUNICATIONS

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Abstract

A problem of synchronizing two nonlinear multidimensional systems with unknown parameters is considered. Two general procedures for adaptive synchronization law design based on speed-gradient method are proposed and investigated. Application of the proposed algorithms for communications is discussed.

1 Synchronization by speed-gradient method

Consider two interconnected systems

$$\dot{x}_i = F_i(x_i, u, t), \tag{1}$$

where $i = 1, 2$, F_i are some vector-functions, $x_i \in R^n$ are state vectors, $u \in R^m$ is interconnection (or coupling) signal.

The problem is to choose synchronization algorithm

$$u = U(x_1, x_2, u, t) \tag{2}$$

or adaptive synchronization algorithm

$$u = U(x_1, x_2, \theta, t) \tag{3}$$
$$\dot{\theta} = \Theta(x_1, x_2, \theta, t) \tag{4}$$

where $\theta \in R^N$ is vector of adjustable parameters ensuring the synchronization goal.

$$x_1(t) - x_2(t) \to 0 \text{ when } t \to \infty. \tag{5}$$

If we interpret coupling signal $u(t)$ as control input, then both synchronization and adaptive synchronization problems can be considered as generalized control problem. Therefore speed-gradient method [2] is applicable which yields the following procedure of synchronization algorithms design.

Step 1. Choose the goal function $Q(x) \geq 0$, $x \in R^n$, such that boundedness of $Q(x)$ implies boundedness of x and synchronization goal (5) can be expressed as

$$Q(x_1(t) - x_2(t)) \to 0 \text{ when } t \to \infty. \tag{6}$$

In many cases the choice is just to take quadratic form $Q(x_1 - x_2) = (x_1 - x_2)^T P(x_1 - x_2)$, where $P = P^T \geq 0$.

Step 2. Calculate speed $\frac{dQ}{dt} = \omega(x_1, x_2, u, t)$ and speed-gradient $\nabla_u \dot{Q} = \nabla_u \omega$ of the goal function.

Step 3. Check conditions of the theorem for finite form of speed-gradient algorithm [2]. If speed-gradient $\nabla_u \omega$ depends only on measurable variables and known parameters then the problem is solved and synchronization algorithm (in finite form) looks as follows [2]:

$$u = -\psi(x, u, t), \tag{7}$$

where $\psi(x, u, t)$ forms sharp angle with the speed-gradient $\nabla_u \omega(x, u, t)$ $(\psi^T \nabla_u \omega \geq 0)$[1].

The typical forms of algorithm (7) are linear $u = -\Gamma \nabla_u \omega(x, u, t)$ and relay ones $u = -\Gamma \text{sign}(\nabla_u \omega(x, u, t))$, where components of vector $\text{sign}(z)$ are signs of the corresponding components of vector z and $\Gamma = \Gamma^T > 0$ is $m \times m$ gain matrix.

Step 4. If speed-gradient algorithm (7) depends on vector of unknown parameters $\xi \in R^N$ then replace ξ by the vector of adjustable parameters $\theta \in R^N$:

$$u = -\Gamma \nabla_u \omega(x_1, x_2, \theta, t) \tag{8}$$

and consider the system (1) together with algorithm (8) as new system. Choose adaptation algorithm (4) in combined or differential ($\psi = 0$) forms [2] by speed-gradient method applied to the new system (1),(8) and the goal (6) in assumption that vector θ is new input

$$\frac{d}{dt}(\theta + \psi(x, \theta, t)) = -\Gamma \nabla_u \omega(x, \theta, t). \tag{9}$$

If conditions of theorem for combined form of SG algorithm are fulfilled [2] and adaptation algorithm depends

[1] One can apply the proposed procedure to the ideal control law u_* which satisfies the attainability condition of the theorem for finite form speed-gradient algorithm [2] instead of algorithm (7).

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only on measurable variables and known parameters then the problem is solved.

The method was investigated on several examples, such as Chua circuits and generators on tunnel diodes and show a good ability to synchronize.

2 Adaptive synchronization of passifiable nonlinear systems

Given equations of the two systems in the form:

$$\begin{cases} \dot{x}_i = f_i(x_1, x_2, t) + B_i u \\ y_i = C x_i \end{cases} \tag{10}$$

where $i = 1, 2$, $x_i \in \mathcal{R}^n$ are state vectors, $y_i \in \mathcal{R}^l$ are measurable outputs, f_i are some functions consisting of linear and nonlinear parts, C is some matrix, B_i ($B_1 \neq B_2$) are gain matrices, $u \in \mathcal{R}^m$ is a control variable.

To obtain the synchronization algorithm and achieve the goal (5) one should follow the procedure:

Step 1. Write down the error equation and function Φ in the form:

$$\begin{aligned} \dot{e} &= Ae + \Phi(x_1, x_2, t) + Bu, \\ \Phi &= \sum_{k=1}^{m} B_k \left[\xi_k^T z_k(x_1, x_2, t) + v_k(x_1, x_2, t)\right], \end{aligned}$$

where $e(t) = x_1(t) - x_2(t)$ is an error vector, A is linear part of the error equation, B_k are the columns of matrix $B = B_1 - B_2$, $\xi_k \in \mathcal{R}^N$ are vectors of unknown parameters and the values of vector-functions $z_k(\cdot) \in \mathcal{R}^N$ and scalar functions $v_k(\cdot)$ are measurable.

Step 2. Write control algorithm in the form

$$u_k = \theta_{0k}^T (y_1 - y_2) + \theta_{1k}^T z_i(x_1, x_2, t) - v_k(x_1, x_2, t), \tag{11}$$

where $\theta_{0k} \in \mathcal{R}^l$, $\theta_{1k} \in \mathcal{R}^N$ are vectors of adjustable parameters.

Step 3. Choose the adaptation algorithm in general, differential ($\psi_{jk} = 0$), or finite ($\Gamma_{jk} = 0$) forms (see Sec.1):

$$\theta_{jk}(t) = -\psi_{jk}(w_{jk}(t)) - \Gamma_{jk} \int_0^t w_{jk}(s) ds \tag{12}$$

where $j = 0, 1$; $w_{0k} = g_k^T (y_1 - y_2)(y_1 - y_2)$; $w_{1k} = g_k^T (y_1 - y_2) z_k$, $\Gamma_{jk} = \Gamma_{jk}^T \geq 0$ are gain matrices, $g_k \in \mathcal{R}^l$ are columns of some matrix G and $\psi_{jk}(w)^T w \geq 0$ for all w (for example $\psi = w$ or $\psi = \text{sign}(w)$).

Step 4. Obtain matrix G such that the system with transfer function $W = G^T C(\lambda I - A)^{-1} B$ is hyper-minimum phase[2] [2].

Step 5. If function $z_k(x_1, x_2, t)$ is bounded in any region $\{(e, t) : \|e\| \leq r,\ t \geq 0\}$ then the goal $\lim_{t \to \infty} e(t) = 0$ is achieved, else the goal $e(\cdot) \in L_2(0, t_*)$ where t_* is maximal time of existence of solution of (10), (11), (12) is achieved.

Step 6. Choose matrices $\Gamma_{jk} = \Gamma_{jk}^T \geq 0$.

One can apply the proposed procedure to design synchronization law for systems with different structure as it was shown in [1].

3 Secure communications

In recent years much attention has been attracted to the methods of secure communications utilizing chaos [3, 4]. Various methods for transmitting signals via chaotic synchronization were proposed: chaotic signal masking [5, 3], parameters modulation [8], chaotic binary communications [5, 6], etc.

The ability of the proposed algorithms to identify the system parameters can be utilized for communications using parameters modulation technique and its special case - chaotic binary communications.

Transmitter and receiver should have identical structure and identical values of parameters. The basic idea is to modulate one of the transmitters coefficients with information-bearing waveform and transmit the chaotic signal. At the receiver side the coefficient modulation will be reproduced by its estimate and the information can be detected.

For example, take a pair of Chua circuits:

$$\dot{w}_i = A_i w_i + D_i f_i(w_i) + B_i u, \tag{13}$$

$$f_i(w_i) = M_{0i} x_i + 0.5(M_{1i} - M_{0i})(|x_i + 1| - |x_i - 1|),$$

where $i = 1, 2$, $A_i = \begin{pmatrix} 0 & p_i & 0 \\ 1 & -1 & 1 \\ 0 & -q_i & 0 \end{pmatrix}$ and $D_i = (-p_i\ 0\ 0)^T$ are matrices of parameters, $w_i = (x_i\ y_i\ s_i)^T$ is a state vector, $B_1 = \begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \end{pmatrix}^T$, $B_2 = 0$ are gain matrices, u is a control signal. Suppose that p_1, q_1 are unknown. To achieve synchronization aim:

$$\lim_{t \to \infty} e(t) = 0 \tag{14}$$

where $e(t) = w_1(t) - w_2(t)$ is an error vector, one can apply the proposed procedure and obtain the following synchronization algorithm:

$$u_1 = \theta_{01}^T (v_1 - v_2) + \theta_{11}(y_1 - f_1(x_1)) + p_2(y_2 - f_2(x_2)),$$

[2] System $\dot{x} = Ax + Bu$, $y = Cx$ where $u \in \mathcal{R}^m$, $y \in \mathcal{R}^l$ is called *hyper-minimum-phase* if it is *minimum-phase* (i.e. the polynomial $\varphi(\lambda) = \det(\lambda I - A) \det W(\lambda)$, where $W(\lambda) = C(\lambda I - A)^{-1} B$ is stable) and the matrix $CB = \lim_{\lambda \to \infty} \lambda W(\lambda)$ is symmetrical and positive definite.

$$u_2 = \theta_{02}^T(v_1 - v_2) + \theta_{12} y_1 - q_1 y_2,$$

where $v_i = (x_i \; y_i)^T$; $\theta_{01}^T = (\theta_{01}^{(1)} \; \theta_{01}^{(2)})$, $\theta_{02}^T = (\theta_{02}^{(1)} \; \theta_{02}^{(2)})$, θ_{11}, θ_{12} are tunable parameters,

$$\begin{cases} \dot{\theta}_{01} = -\Gamma_{01} g_1^T (v_1 - v_2)(v_1 - v_2), \\ \dot{\theta}_{02} = -\Gamma_{02} g_2^T (v_1 - v_2)(v_1 - v_2), \\ \dot{\theta}_{11} = -\Gamma_{11} g_1^T (v_1 - v_2)(y_1 - f_1(x_1)), \\ \dot{\theta}_{12} = -\Gamma_{12} g_2^T (v_1 - v_2) y_1, \end{cases} \tag{15}$$

where $\Gamma_{jk} = \Gamma_{jk}^T > 0$ $(k, j = 1, 2)$, g_k are columns of matrix $G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$ and $b_1 > 0$, $b_2 > 0$, $g_{11}g_{22} - g_{12}g_{21} > 0$.

For information transmission one should modulate the transmitter (system 1) coefficients $p_1 = p_{1*} + m(t)$ or $q_1 = q_{1*} + m(t)$ by information signal $m(t)$ (two information signals can be transmitted simultaneously by two coefficients) and transmit the chaotic signals x_1, y_1. At the receiver end information signal will reproduced be corresponding estimates of this parameters θ_{11}, θ_{12}.

It is necessary that modulated parameters remain in the chaotic region of bifurcation diagram during information transmission. It can be achieved by proper choice of initial value of modulated parameters p_{1*}, q_{1*} and amplitude of the information signal $m(t)$.

Note that parameters θ_{11}, θ_{12} estimates the values p_1/b_1 and q_1/b_2. Therefore one can increase the receivers sensitivity by decreasing the value of parameters b_i.

The method permits transmission of discreet and continuous signals if speed of their change is quite slow (it is connected with a transient process of parameters tuning). If the number of modulated parameters (and tunable variables respectively) decrease then transient process of estimation also decrease and speed of information signal change can be faster.

In example with Chua circuits we demonstrate the ability of information transmission utilizing the proposed procedures. One can apply proposed procedures to another systems and obtain algorithms of information transmission by means of one chaotic signal.

Note that independence of synchronization process from the values of transmitter parameters and ability of the proposed methods to identify the systems parameters can be applied also to decoding the chaotically coded signal, in particular - chaotic binary communications [5, 6]. However this issue needs further investigation.

References

[1] Markov A.Yu. Fradkov A.L. Adaptive synchronization of coupled chaotic systems. Conf. on Chaos and Fractals in Chemical Engineering. Rome. 1996.

[2] Fradkov A. L. Adaptive control of complex systems. Moscow, Nauka. (in Russian). 1990.

[3] Kocarev L, Halle K.S, Eckert K, Chua L.O. Experimental demonstration of secure communication via chaotic synchronization. Int. J. Bif. Chaos, vol.2, No.3, pp.709-713, 1992.

[4] Cuomo K.M, Oppenheim A.V, Isabelle S.H. Spread spectrum modulation and signal masking using synchronized chaotic systems. MIT Res. Lab. Electron. TR 570, Feb. 1992.

[5] Cuomo K.M, Oppenheim A.V, Strogatz S.H. Synchronization of Lorenz-based chaotic circuits with application to communications. IEEE Trans. Circ. Syst. - II: Analog and Digital signal processing, vol.40, No.10, pp.626-633, 1993.

[6] Dedieu H, Kennedy M.P, Hasler M. Chaos shift keying: modulation and demodulation of chaotic carrier using self-synchronized Chua's circuits. IEEE Trans. Circ. Syst. - II: Analog and Digital signal processing, vol.40, No.10, pp.634-642, 1993.

[7] Blekhman I., Fradkov A., Nijmeijer H. and Pogromsky A., (1996). Self-synchronization and controlled synchronization of dynamical systems (submitted to ECC'97).

[8] Parlitz U, Kocarev L. Multichannel Communication Using Autosynchronization. Int. J. Bif. Chaos. Vol 6, No.3. pp. 581-588. 1996.