# A NOVEL STATISTICAL APPROACH FOR CHAOS DETECTION IN CHUA'S CIRCUIT

T. J. Maayah, M. A. Khasawneh, and L. M. Khadra

*Department of Electrical Engineering*

*P.O. Box 3030, JUST*

*Irbid 221 10 Jordan, Fax: +962 (2) 295 123*

## ABSTRACT

**A statistical approach for chaos identification in time series is described and applied to numerical data generated form Chua's circuit. This method compares the short-term predictability for a given time series to an ensemble of random data which has the same Fourier spectrum as the original time series. The forecasting error is computed as a statistic for performing statistical hypothesis testing. The forcasting technique is modified by introducing a *Moving predictor*. The results show that this will give more accurate predictions, hence, better capability of distinguishing chaos from random noise in time series.**

## I. INTRODUCTION

Distinguishing deterministic chaos from random process in time series is not straightforward. Both, chaos and random data, can have similar broadband spectra [1]. In our work in this paper we modify and digitally implement a statistical approach, originally proposed in [2], capable of distinguishing chaos from random process in a natural time series. The method relies on the fact that chaos, unlike random data, suggests possibilities for short-term prediction. This new technique compares the predictability of the given data to an ensemble of random control data (surrogate data) which has the same average power spectral density as the original time series. A nonparametric statistic is explored that permits a hypothesis testing approach. The prediction algorithm is modified by using a *moving predictor* rather than a fixed one such that more accurate predictions could be made as the results demonstrate.

The algorithm is applied to numerical data generated by a well-known chaotic system, the Henon-map, and to experimental data which arise in Chua's circuit [3].

## II. THE ALGORITHM

The statistical hypothesis testing used in this technique involves two elements: a null hypothesis that the original time series is generated from a random process, and a discriminating statistic which is computed as the median absolute error (MAE) of the prediction errors.

The first step to calculate the statistic is to do *time-delay embedding* [4], that is, to embed the time series in a state space with a state vector, $\mathbf{X}(n)$, having coordinates:

$$\begin{aligned} x_1(n) &= x(n) \\ x_2(n) &= x(n-1) \\ &\vdots \\ x_d(n) &= x(n-(d-1)) \end{aligned} \tag{1}$$

with $d$ chosen such that $d \geq D$, where $D$ is an assumed attractor dimension, and $n$ takes on the values $1, 2, 3 \ldots N$; $N$ being the number of points in the time series.

The next step is to split the data points in the time series into a fitting set $N_f$ and a testing set $N_t$ with $N_t = N_f = N/2$ [2]. For each point in the testing set we try to predict $x(n+1)$ by searching the fitting set $N_f$ for $k$ nearest neighboring states, $x_1(m) \ldots x_k(m)$, $m \leq n$, such that the *Euclidean Norm* between $\mathbf{X}(n)$ and each of the $k$ neighboring states is minimum. Next we fit a linear polynomial to $k$ pairs $\{(x_1(m), x_1(m+1)) \ldots (x_k(m), x_k(m+1))\}$

$$x(n+1) = \mathbf{a}^{\mathbf{T}} \cdot \mathbf{X}(n) + b \qquad (2)$$

where $\mathbf{a}$ and $b$ are the fitting coefficients, and $\mathbf{X}$(n) is the present state. The prediction error equals the difference between the actual value, $x(n+1)$, and the predicted one.

Repeating the above procedure for all the points in the testing set $N_t$ we end up with a file containing $N/2$ prediction errors, from which the median absolute error *(MAE)* of the $N/2$ errors is computed.

### *A. Generating the surrogate data*

The surrogate data set is constructed based on the null hypothesis that the given time series comes from a Gaussian random process. A nonlinear transformation, called the histogram transform [2], is included to detect chaos even if the raw data is non-gaussian.

To perform the transform, $N$ Gaussian random numbers are generated and their time order is shuffled such that it has the same shape as the original data set.

Now the surrogate time series is generated so that is has the same Fourier spectrum as the transformed time series. The FFT of the transformed time series is computed as

$$X(k) = \sum_{n=0}^{N-1} x(n).e^{i2\pi nk/N} \qquad (3)$$

The phase is randomized by multiplying the complex amplitude at each frequency by $e^{i\phi}$, where $\phi$ is a uniform random variable $\in [0, 2\pi]$. For the inverse FFT to be real, the phases must be symmetrical. The inverse FFT is then calculated, which will result in a Gaussian surrogate time series, and finally the resultant time series is histogram transformed back to have the same amplitude distribution as the original time series.

### *B. Testing the hypothesis*

In the language of hypothesis testing, the significance is calculated according to [2]

$$\chi = \frac{|Q_D - \mu_S|}{\sigma_S} \qquad (4)$$

where $Q_D$ is the statistic *(MAE)* computed for the original time series, $\mu_S$ and $\sigma_S$ are the mean and standard deviation of the distribution of $Q_{S_i}$, respectively; $Q_{S_i}$ being the statistic for the $i$th surrogate data. The level of significance or the p-value is given by [2]

$$p = erfc(\chi/\sqrt{(2)}) \qquad (5)$$

If this value is less than 0.01 then the null hypothesis is rejected and chaos is detected.

## III. Simulations

We applied the algorithm to the Henon map, which is a choatic system with fractal dimension 1.3. Figs. 1 and 2 show the attractor and the time domain trajectory of the Henon map, respectively. With zero added noise, the level of significance is 0, that is, the probability of rejecting a true null hypothesis (that the input time series is random) equals 0, therefore chaos is detected as expected. Fig. 3 shows the p-value, for both the moving & fixed predictors, versus $\sigma$ which is the level of added noise according to the following relation:

$$X_{new} = (1-\sigma) * X_{old} + \sigma * r \qquad (6)$$

where $r$ is a random variable uniformly distributed on $[-1, 1]$. Note that with the fixed predictor, the threshold at 0.01 was reached soon after the noise level of 0.4; using the new modification, however, allows the same threshold to be reached at 0.7 of noise level. This result helps to illustrate the improvement the new algorithm possesses over the fixed predictor technique. We also applied the algorithm to the canonical Chua's circuit [3]. Figs. 4 and 5 show the state space and time domain trajectories. Using the new method we were able to detect chaos from the time series with a p-value of $5.334 \times 10^{-5}$. This, again, demonstrates the enhanced capability of the proposed technique over the *fixed predictor* approach.

## IV. Conclusions

In this paper, the problem of determining whether the erratic fluctuations observed in a given time series are, in fact, generated from a random process or from the underlying chaotic dynamics. The method used relies on the nonlinear prediction as a discriminating property and is capable of distinguishing between random colored noise and deterministic chaos [5]. As the simulations demonstrated, the introduction of the *moving predictor* to the prediction algorithm improves the robustness of the overall approach for chaos detection against the noise level.

## References


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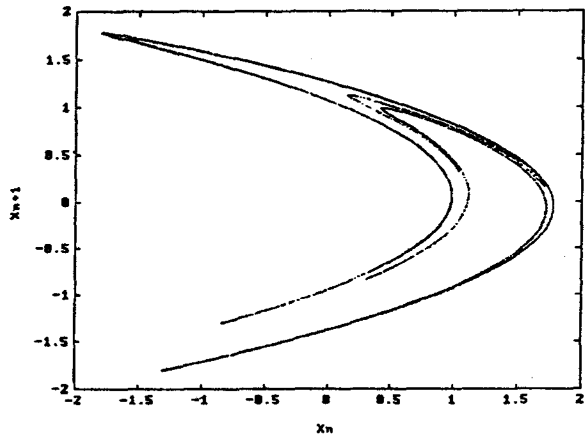


Fig.1: The Henon attractor.

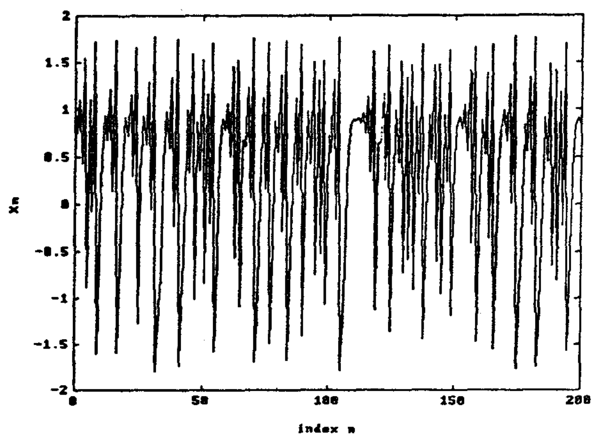


Fig.2 Time trajectory of the Henon map.

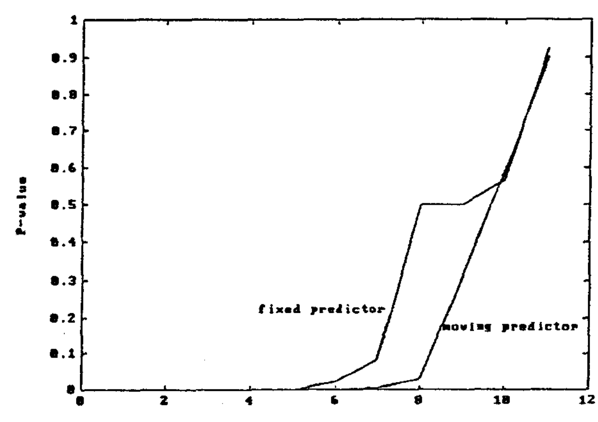


Fig.3: p-value vs. noise level for both methods.

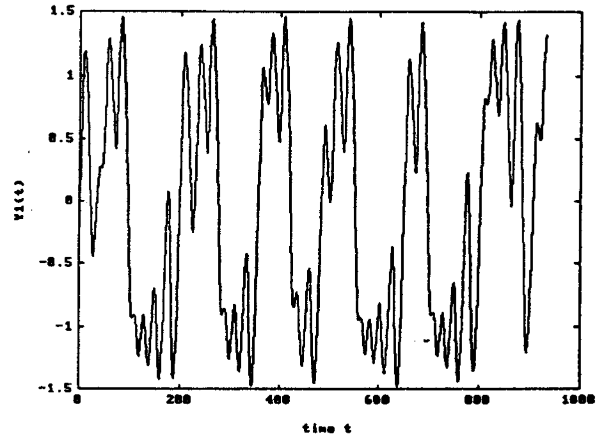


Fig.4: Time trajectory for Chua's circuit.

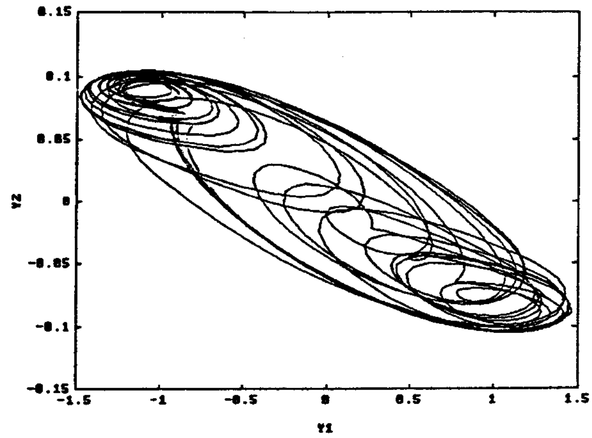


Fig.5: State-space trajectory for Chua's circuit.