Derivative Control of the Steady State in Chua's Circuit Driven in the Chaotic Region

Gregg A. Johnston and Earle R. Hunt

Abstract—**We experimentally demonstrate that Chua's circuit, operating in the double-scroll chaotic regime, may be brought to either of the two unstable, stationary state fixed points by means of derivative control.**

THE CONTROL of dynamical chaos has been demonstrated in a number of vastly different physical systems including electronic circuits [1], lasers [2], rabbit hearts [3], and chemical systems [4]. The elimination of unstable behavior in such systems is generally advantageous, as the system under study is transformed from a state of unpredictability into one of the absolute regularity. Most work in the field of controlling chaos has concentrated on the stabilization of periodic orbits embedded in the chaotic attractor which can be achieved by means of feedback methods following from the Ott-Grebogi-Yorke (OGY) technique [5]. However, in a number of chaotic systems there exists at least one solution, which is nonoscillatory. In many such cases, this steady state is the most practical operation mode, and the onset of chaotic or periodic oscillations is a hinderance to the system. A practical example is a laser, as in [6], which has a steady state output intensity for a limited range of pump excitation power followed by the onset of regimes of oscillations and chaos as input power is increased. The applicability of such a system is limited by the range of its steady state operation, hence the extension of this range is the goal of recent research effects.

The occasional proportional feedback (OPF) method [1], a fast analog modification of the OGY technique, has been successful in stabilizing periodic orbits in a number of systems including Chua's circuit [7]. It was found that, in addition to unstable periodic orbits, the steady states in Chua's circuit may be stabilized as well with periodically applied corrections [8]. These steady states can be tracked through the chaotic regime by constant adjustment of control parameters as the system is moved through its otherwise fully chaotic range. Thus the OPF method was demonstrated as a means of dramatically increasing the regime of non-oscillatory operation.

In a recent article Bielawski *et al.* [9] suggest that many systems which have an unstable steady state destabilizing by a Hopf bifurcation, as does Chua's circuit, may not be readily controlled with feedback proportional to the signal alone. They suggest and demonstrate the use of continuous feedback which

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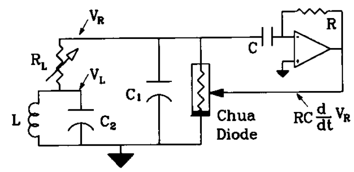

Fig. 1. Simple diagram of Chua's circuit, with chaotic variable V_R, and the differentiating circuit. The feedback signal, which is proportional to the time derivative of V_R, continuously modulates the negative resistance value of the Chua diode.

is proportional to the derivative of the chaotic signal. They successfully use the method in controlling the steady state of a optical-fiber laser where the light intensity is the chaotic variable.

To further demonstrate the effectiveness of this method, we apply the same derivative method to Chua's circuit. The system parameter which is varied in the circuit is the negative resistance of the nonlinear Chua diode. We use a simple operational amplifier differentiator circuit to generate the continuous feedback signal which is proportional to the time derivative of the voltage across the negative resistance, V_R, as shown in Fig. 1. The control parameter R_N (negative resistance) may be varied using the circuit shown in ref. [7] and can be effectively expressed as

$$R_N(t) = R_{\mathrm{No}} + \beta(dV_R/dt)$$

where R_{No} determines the regime of the uncontrolled oscillations. The second term on the right can be considered the damping or drag, which can˜suppress oscillations with appropriate choice of β. This constant is determined by the reaction of the variable resistance to a given voltage and the electronic components of the differentiator.

In the given example, R_{No} is set so that the system oscillates chaotically in the double-scroll strange attractor regime with no feedback applied. In Fig. 2 prior to $t = 0$, the chaotic signal measured across the Chua diode oscillates chaotically about two unstable foci which are the unstable steady state fixed points. At $t = 0$ a switch connecting the derivative feedback with the variable negative resistance is closed and the system quickly settles on the desired unstable fixed point. Reversing the polarity of the feedback has the result of stabilizing the other unstable focus which lies in the negative V_R region. The time required to bring the oscillations down to the noise level is determined by the magnitude of the constant β. If β is reduced below some critical value, which depends on

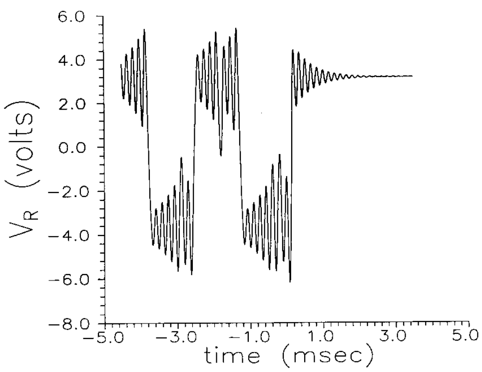

Fig. 2. The chaotic voltage V_R is shown before and after control is initiated at $t = 0$ as captured on a digital storage oscilloscope. It is interesting to note that the damping is effectively negative about the lower focus and kicks the system towards the other fixed point about which the damping is positive.

the system parameters, the fixed point is not reached and the system settles into some oscillatory behavior. In addition, β need not be adjusted in order to track the steady state through the range of oscillations Chua's circuit once the state is stabilized.

It should be emphasized that the system is in the chaotic regime when the derivative feedback is applied. This is unlike the control of the steady state using the OPF method, in which case the system is tracked into that region from the stable region.

The success of the two seemingly different procedures suggests a possible connection. One clue is the peculiar result of the OPF method that the frequency at which the feedback pulses (the synchronizing frequency) are applied must be at least roughly 10% higher (or lower) than the frequency of the stable limit cycle. This result motivated the following analysis of the OPF method.

With a sinusoidal input signal it can be shown by Fourier analysis that the correction signal generated by the OPF circuit has a component that is proportional to the derivative of the input. Specifically, this component is proportional to $\mathrm{Sin}^2(\omega T_s) * \mathrm{Sin}^2((1/2)f\omega T_S)$ where ω is the frequency of the input, T_s is the reciprocal of the synchronizing frequency, and f is the width of the correction pulse as a fraction of T_s. If we attribute control of the steady state to this component, we find excellent agreement with experimental results.

We consider here a few special cases. Experimentally, we find a total lack of control when the frequency of our correction pulses is set very near to the frequency of the oscillations, i.e., $\omega T_s = 2\pi$. Clearly from the results of our analysis, the derivative component approaches zero in this case. A second case to consider is one which results in easy control of the steady state, for example $\omega T_s = 4\pi/3$. The derivative component can be maximized with adjustment of the pulse width f. We find experimentally that $f = 3/4$ gives robust control and note that this value also gives the maximum value for the derivative component.

From this, we conclude that the steady state control reported in [6], [8] can be attributed to the derivative component that arises in the OPF method. The fact that a derivative component arises allows one to consider the feedback as a damping term as in the case of normal derivative control.

We have shown the derivative control of the steady state in Chua's circuit to be and effective way of suppressing oscillations in a chaotic regime. The emphasized difference between this method and the OPF method is that control can be initiated in any regime with the derivative method, while the OPF method involves tracking the fixed point from stable regime to unstable regime. Last, we attribute the control of the steady state by the OPF method to the component of the feedback signal which is proportional to the derivative of the input. We confirm that this component is large when steady state control is realized in experiment and quite small when control can not be initiated. In a practical application, OPF control could be used in a system with substantial noise levels such that taking the derivative could introduce even more noise. Hence both derivative and OPF control methods offer an effective way to stabilize the steady state fixed points in a chaotic attractor and could be interchanged in situations where one proves to be more advantageous.

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