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Communication Systems via Chaotic Modulations

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SUMMARY New communication systems via chaotic modulations are experimentally demonstrated. They contain the well-known chaotic circuits as its basic elements - - Chua's circuits and canonical Chua's circuits. The following advantage is found in our laboratory experiments: (a) Transmitted signals have broad spectra. (b) Secure communications are possible in the sense that the better parameter matching is required in order to recover the signal. (c) The circuit structure of our communication system is most simple at this stage. (d) The communication systems are easily built at a small outlay.
key words: *chaos, synchronization, communication systems, Chua's circuit*

1. Introduction

Recently, Pecora and Carroll demonstrated the possibility of synchronizing chaotic systems [1]. In this paper, we propose new communication systems as the possible application of chaotic synchronization [2]. Considering their complicated behavior, chaotic signals are the candidates masking and scrambling the signals [3],[4]. The main idea is to use the chaotic modulation to transmit the informational signals. And the chaotic synchronization mechanism is used to recover the signals [5]–[7].

Next, we show the basic idea to construct the communication systems. First, let us consider the chaotic dynamical system

$$\frac{dx}{dt} = h(x,y), \ \frac{dy}{dt} = k(x,y), \tag{1}$$

where (x,y) is the $n+m$-dimensional space, and h and k are smooth functions of x and y. Then the transmitting system is given by

$$\frac{dx}{dt} = h(x,y) + s(t), \ \frac{dy}{dt} = k(x,y), \tag{2}$$

where $s(t)$ is the informational signal and $x(t)$ is the chaotically modulated transmitted signal. That is, the noise-like masking signal $x(t)$ is transmitted to the channel.

The receiver constructs the following system by using the transmitted signal $x(t)$:

$$\frac{dz}{dt} = k(x,z), \ r(t) = \frac{dx}{dt} - h(x,z). \tag{3}$$

The first equation is the response system driven by the signal $x(t)$. The second equation can retrieve the signal $s(t)$, that is, $r(t)$ is the recovered signal. This is due to the following reason. If all the Lyapunov exponents of the z subsystem are negative, then the y subsystem and the z subsystem will synchronize, that is, $|y(t) - z(t)| \to 0$ as $t \to \infty$ (For details, see Ref. [1]). Thus $s(t)$ is recovered as:

$$r(t) = \frac{dx}{dt} - h(x,z) \to \frac{dx}{dt} - h(x,y) = s(t) \quad \text{for } t \to \infty. \tag{4}$$

We examine the above idea by using Chua's circuits and canonical Chua's circuits.

2. Chua's Circuits and Canonical Chua's Circuits

Chua's circuit is a remarkably simple and robust circuit, which consists of a linear inductor L, a linear resistor R, two linear capacitors C_1 and C_2, and a nonlinear resistor N_R (see Fig. 1 and also Ref. [8]). The state equations are given by

$$\begin{aligned} C_1\frac{dv_1}{dt} &= \frac{v_2 - v_1}{R_1} - g(v_1), \\ C_2\frac{dv_2}{dt} &= \frac{v_1 - v_2}{R_1} + i_1, \\ L_1\frac{di_1}{dt} &= -v_2 - r_1 i_1, \end{aligned} \tag{5}$$

where $g(\cdot)$ is a piecewise-linear function defined by

$$g(v_1) = G_b v_1 + 0.5(G_a - G_b)(|v_1 + B_p| - |v_1 - B_p|), \tag{6}$$

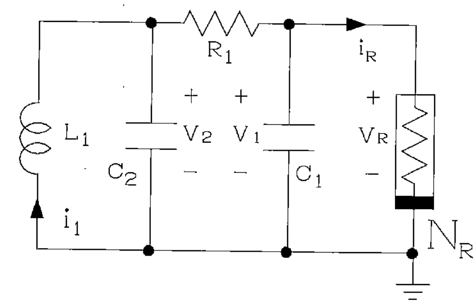

Fig. 1 Chua's circuit.

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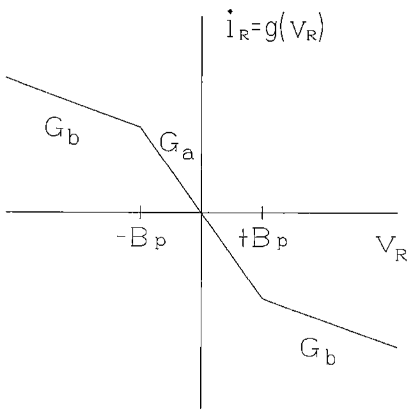

Fig. 2 The v-i characteristic of N_R in the Chua's circuit.

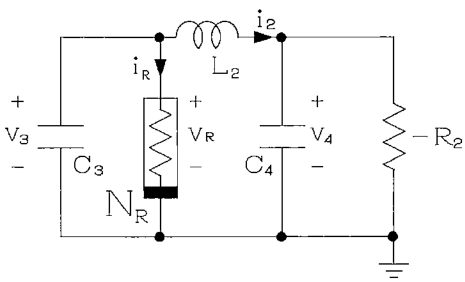

Fig. 3 Canonical Chua's circuit.

and shown in Fig. 2. The term $r_1 i_1$ is added to the ideal Chua's equations in order to account for the small inductor resistance in the physical circuit. The parameters we use are

$$C_1 = 10.3\,\text{nF},\ C_2 = 97.4\,\text{nF},$$
$$L_1 = 20.7\,\text{mH},\ R_1 = 1.41\,\text{k}\Omega,$$
$$B_p = 1.85\,\text{V},\ G_a = -0.87\,\text{mS},$$
$$G_b = -0.52\,\text{mS},\ \text{and}\ r_1 = 64.2\Omega.$$

With these values, (5) has a double scroll attractor.

Similarly, the state equations for the canonical Chua's circuit shown in Fig. 3 is given by

$$\begin{aligned} C_3\frac{dv_3}{dt} &= -f(v_3) - i_2, \\ C_4\frac{dv_4}{dt} &= \frac{v_4}{R_2} + i_2, \\ L_2\frac{di_2}{dt} &= v_4 - v_3 - r_2 i_2, \end{aligned} \tag{7}$$

where $f(\cdot)$ is a piecewise-linear function defined by

$$\begin{aligned} f(v_3) = {} & G'_b v_3 \\ & +0.5(G'_a - G'_b)(|\,v_3 + B'_p\,| - |\,v_3 - B'_p\,|), \end{aligned} \tag{8}$$

and r_2 is the inductor resistance (See Fig. 4 and also Ref. [9]). To generate a chaotic attractor, we used

$$C_3 = 14.9\,\text{nF},\ C_4 = 31.9\,\text{nF},$$

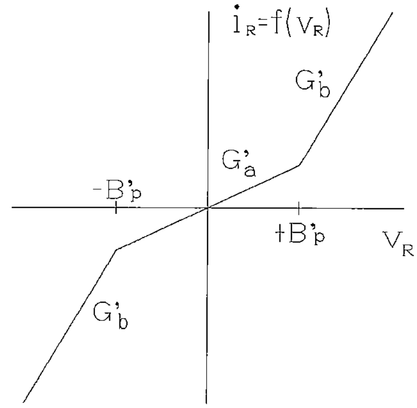

Fig. 4 The v-i characteristic of N_R in the canonical Chua's circuit.

$$L_2 = 20.7\,\text{mH},\ R_2 = 1.00\,\text{k}\Omega,$$
$$B'_p = 0.97\,\text{V},\ G'_a = 0.45\,\text{mS},$$
$$G'_b = 4.00\,\text{mS},\ \text{and}\ r_2 = 52.5\Omega.$$

3. Communication Systems

The basic construction of our communication system for Chua's circuit is shown in Fig. 5. The circuit equations for the transmitting system are given by

$$\begin{aligned} C_1\frac{dv_1}{dt} &= \frac{v_2 - v_1}{R_1} - g(v_1) + \frac{e(t) - v_1}{R_S}, \\ C_2\frac{dv_2}{dt} &= \frac{v_1 - v_2}{R_1} + i_1, \\ L_1\frac{di_1}{dt} &= -v_2 - r_1 i_1. \end{aligned} \tag{9}$$

We use the voltage source $e(t)$ as an informational signal, and $v_1(t)$ as a transmitted signal. That is, $v_1(t)$ is a chaotically modulated transmitted signal.

The circuit equations for the receiving system are given by

$$\begin{aligned} C_1\frac{dv'_1}{dt} &= \frac{v'_2 - v'_1}{R_1} - g(v'_1) - \frac{v'_1}{R_S}, \\ C_2\frac{dv'_2}{dt} &= \frac{v'_1 - v'_2}{R_1} + i'_1, \\ L_1\frac{di'}{dt} &= -v'_2 - r_1 i'_1, \end{aligned} \tag{10}$$

where $v_1(t) = v'_1(t)$ because of the voltage buffer.

Next, we show how the informational signal can be recovered (demodulation process). From the first equation in (9), we obtain

$$e(t) = R_S[C_1\frac{dv_1}{dt} - \frac{v_2 - v_1}{R_1} + g(v_1) + \frac{v_1}{R_S}]. \tag{11}$$

Similarly, the current $j(t)$ in Fig. 5 is given by

$$j(t) = [C_1\frac{dv'_1}{dt} - \frac{v'_2 - v'_1}{R_1} + g(v'_1) + \frac{v'_1}{R_S}]. \tag{12}$$

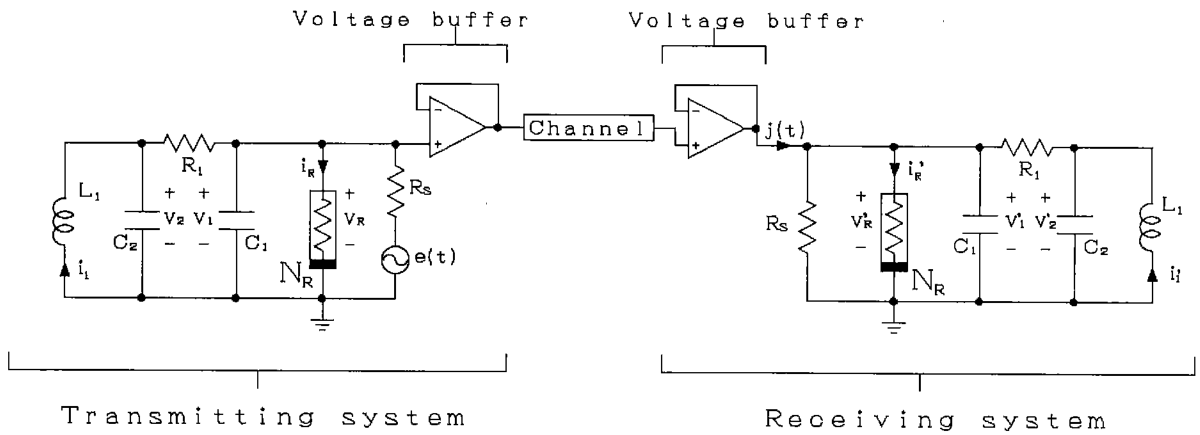

Fig. 5 Communication system which utilizes Chua's circuits.

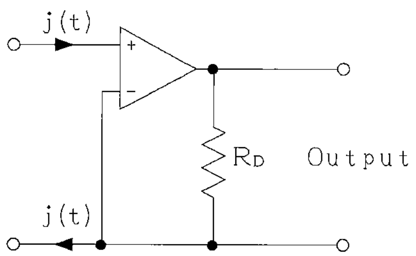

Fig. 6 Current detector.

Establishing the difference $p(t) = v_2(t) - v_2'(t)$ and $q(t) = i(t) - i'(t)$, we get

$$C_2 \frac{dp}{dt} = -\frac{p}{R_1} + q,$$
$$L_1 \frac{dq}{dt} = -p - rq. \tag{13}$$

Since the origin of (13) is globally asymptotically stable, we conclude that $| p |=| v_2 - v_2' | \to 0$ and $| q |=| i - i' | \to 0$ as $t \to \infty$. That is, the (v_2, i)-subsystem and the (v_2', i')-subsystem will synchronize. (We get the same results by calculating the Lyapunov exponents of the (v_2', i')-subsystem; see Ref. [1].) Thus, $j(t)$ is given by:

$$j(t) = [C_1 \frac{dv_1'}{dt} - \frac{v_2' - v_1'}{R_1} + g(v_1') + \frac{v_1'}{R_S}] \to \frac{e(t)}{R_S}. \tag{14}$$

It implies that the current $j(t)$ varies in proportion to the informational signal $e(t)$. That is, the informational signal can be recovered by using the current detectors in Fig. 6. (The circuit parameters in our experiments are $R_S = 33.9\,\mathrm{k\Omega}$ and $R_D = 2.00\,\mathrm{k\Omega}$.)

The similar communication system is also given in Fig. 7, which utilizes canonical Chua's circuits. Its details are omitted here, since the basic idea of this communication system is as same as that of Chua's circuit.

4. Experimental Results

We built the two types of communication systems, that is, the communication system via Chua's circuits and the one via canonical Chua's circuits. Both systems are tested by using various kinds of signals, for example, human voices, music, and so on. Figures 8–11 and Figs. 12–15 show the experimental results for the Chua's circuit and those for the canonical Chua's circuit, respectively. From these results, we conclude the followings:

(a) The communication systems are easily built at a small outlay.

(b) The waveforms of the transmitted chaotic signal $x(t)$ can mask the input signal $s(t)$ if the amplitude of $s(t)$ is small enough (see Fig. 8 and Fig. 12).

(c) The transmitted signals have broad spectra, and also can mask the spectra of the input signals (see Fig. 9 and Fig. 13) .

(d) The informational signals are recovered with sufficient quality from the transmitted signals (see Fig. 10 and Fig. 14).

(e) These systems have the high sensitivity of the parameter mismatch. (see Fig. 11 and Fig. 15). To eliminate the masking signal, very accurate knowledge of the parameter of the system is required to synchronize the chaotic signal. That is, the parameters of the systems serve as the "encryption key". In this sense, our communication systems are secure.

5. Transmission Methods

At this stage, three different methods have proposed for transmitting an informational signal $s(t)$ via chaotic signal $u(t)$. We discuss them briefly. The first, chaotic masking consists of adding $s(t)$ to the chaotic signal $u(t)$ [3], [4]. This method is very simple, but contains the following disadvantages:

(a) The power level of the informational signal must be significantly lower than that of the chaotic signal in

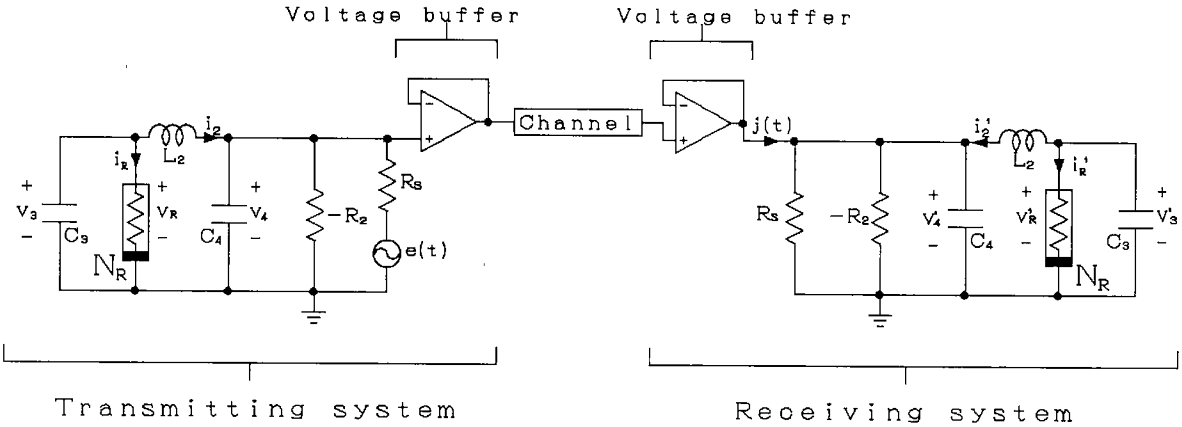

Fig. 7 Communication system which utilizes canonical Chua's circuits.

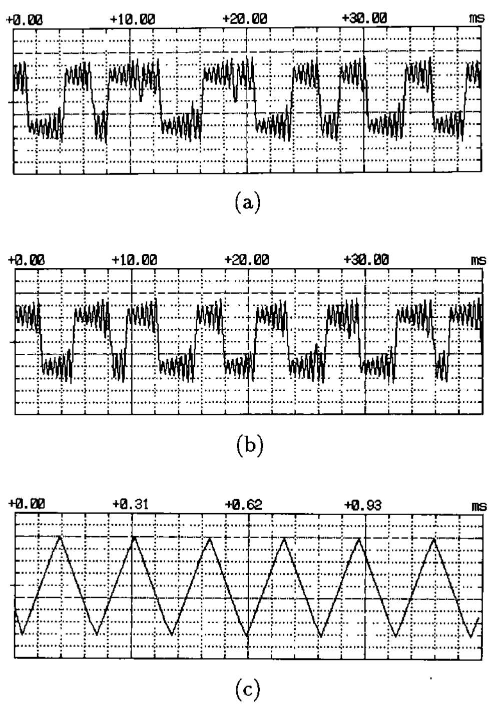

Fig. 8 Time waveforms of the transmitted chaotic signals and the input signal. (a) Transmitted chaotic signal with no input. Horizontal scale: 2.0 ms/div. Vertical scale: 1.0 V/div. (b) Transmitted chaotic signal with an input. Horizontal scale: 2.0 ms/div. Vertical scale: 1.0 V/div. (c) Input signal of frequency 5.0 kHz and amplitude 0.4 V. Horizontal scale: 0.062 ms/div. Vertical scale: 0.1 V/div.

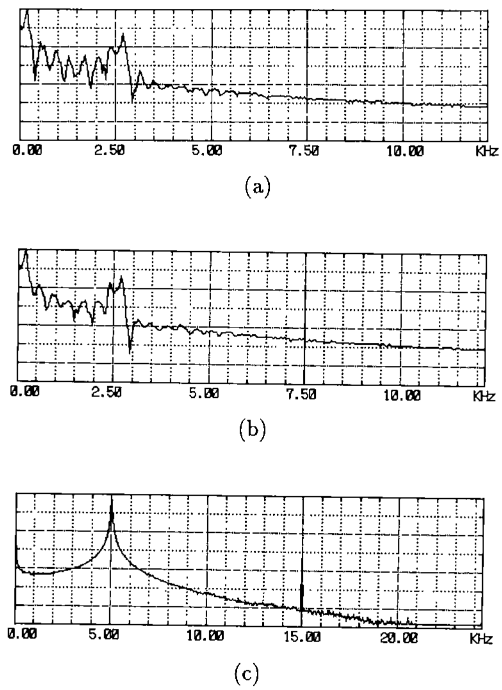

Fig. 9 Spectra of the transmitted chaotic signals and the input signal. (a) Spectrum of the transmitted chaotic signal with no input. Horizontal scale: 0.5 kHz/div. Vertical scale: 10 dB/div. (b) Spectrum of the transmitted chaotic signal with an input. Horizontal scale: 0.5 kHz/div. Vertical scale: 10 dB/div. (c) Spectrum of the input signal. Frequency of the input signal is 5.0 kHz. Amplitude of the input signal is 0.4 V. Horizontal scale: 1.0 kHz/div. Vertical scale: 10 dB/div.

order for the demodulation to be possible.
(b) The demodulation is disrupted when the informational signal approaches to the natural frequency of the LC tank in Chua's circuit [4].

In the second, chaotic switching [10]–[13], a binary signal drives a switch between two parameters values of the transmitting system. On the receiver side, two systems, one for each parameter value, try to synchronize to the transmitted signal. In this case, the system has the following properties:
(a) The informational signal is a digital signal.
(b) The circuit structure of the receiving system is complicated.

Finally, chaotic modulation proposed in this paper uses directly the synchronization method of Ref. [2]. Our system has the following properties:
(a) The circuit structure of the communication system is most simple.

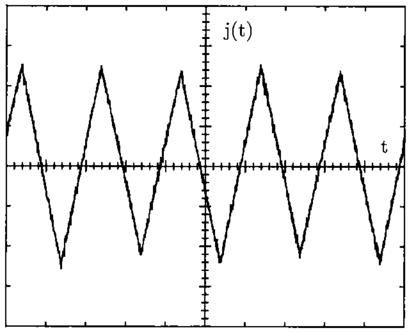

Fig. 10 Time waveform of the recovered signal. Horizontal scale: 0.1 ms/div. Vertical scale: 10 mV/div.

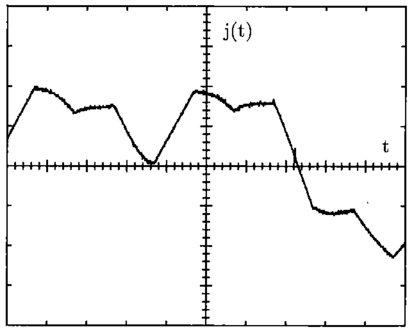

Fig. 11 Time waveform of the recovered signal with 3% resistor mismatch. Horizontal scale: 0.1 ms/div. Vertical scale: 50 mV/div.

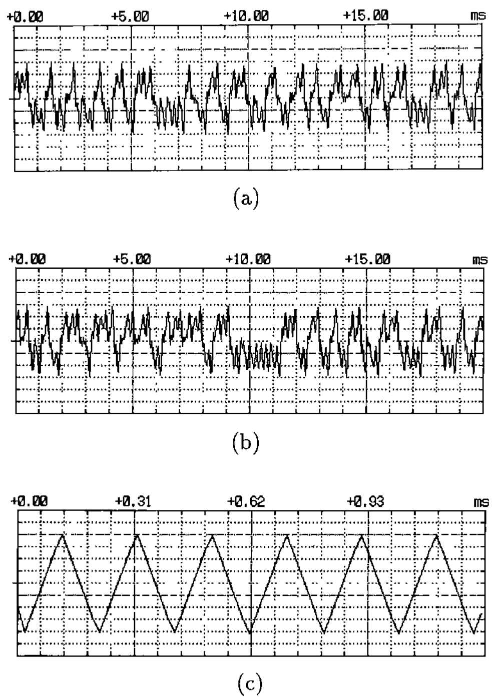

Fig. 12 Time waveforms of the transmitted chaotic signals and the input signal. (a) Transmitted chaotic signal with no input. Horizontal scale: 1.0 ms/div. Vertical scale: 0.5 V/div. (b) Transmitted chaotic signal with an input. Horizontal scale: 1.0 ms/div. Vertical scale: 0.5 V/div. (c) Input signal of frequency 5.0 kHz and amplitude 0.4 V. Horizontal scale: 0.062 ms/div. Vertical scale: 0.1 V/div.

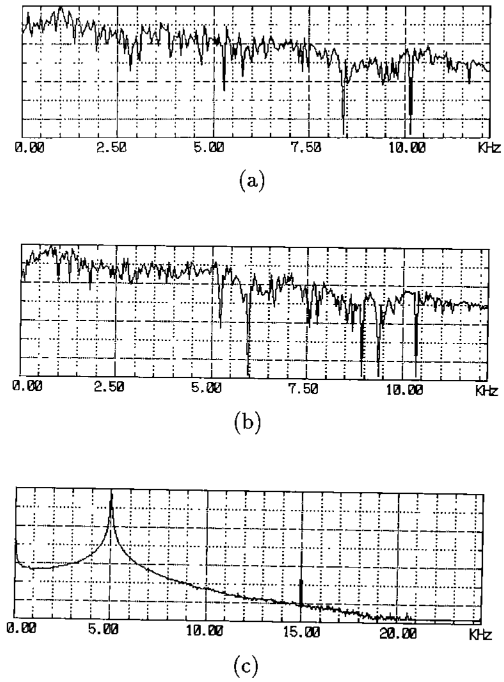

Fig. 13 Spectra of the transmitted chaotic signals and the input signal. (a) Spectrum of the transmitted chaotic signal with no input. Horizontal scale: 0.5 kHz/div. Vertical scale: 10 dB/div. (b) Spectrum of the transmitted chaotic signal with an input. Horizontal scale: 0.5 kHz/div. Vertical scale: 10 dB/div. (c) Spectrum of the input signal. Frequency of the input signal is 5.0 kHz. Amplitude of the input signal is 0.4 V. Horizontal scale: 1.0 kHz/div. Vertical scale: 10 dB/div.

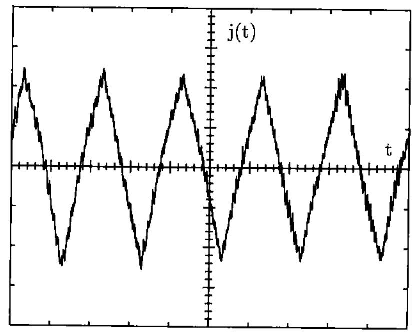

Fig. 14 Time waveform of the recovered signal. Horizontal scale: 0.1 ms/div. Vertical scale: 10 mV/div.

(b) The informational signal $s(t)$ is usually analog, but digitally modulated signals can also be used.
(c) This method is easily applied to some other chaotic circuits.

In our transmitting system a signal injected into the chaotic circuit. This addition modifies the behavior of the circuit, and some care should be taken in order that the addition does not destroy the chaotic behavior and spectra. However, this method does not have the above mentioned disadvantage. Furthermore, this care makes the cording function used in Ref. [5] unnecessary. (The results in Ref. [5] were written in early stage of this

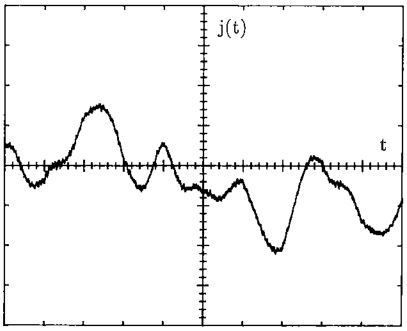

Fig. 15 Time waveform of the recovered signal with 3% resistor mismatch. Horizontal scale: 0.1 ms/div. Vertical scale: 50 mV/div.

work.)

6. Conclusion

We proposed the new communication systems via chaotic modulations and chaos synchronizations. Furthermore, three different transmission methods are discussed. The discrete version of our communication system is given in Refs. [14]–[16]. These systems have some advantages and disadvantages. The performance of the communication systems which are given in this paper and Refs. [14]–[16] will be discussed elsewhere.

Lastly, we refer the work of Ushio [17], in which a new communication system via neural networks is given. It may bring a new scheme in this field.

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