Controlling Chaotic Motions in Chua's Circuit via Tunnels

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ABSTRACT

A new method is given, which converts a chaotic motion in Chua's circuit to a periodic motion. A tunnel mechanism is used to perform this conversion.

INTRODUCTION

In a chaotic attractor, an infinite number of unstable periodic orbits are typically embedded. Ott, Grebogi, and Yorke [1] introduced the method to stabilize the already existing periodic orbits in a chaotic attractor. Recently, A. Dabrwski et al. [2] modified their method, and applied it to Chua's circuit. Their method requires small time-dependent perturbations in an accessible system parameter, but does not know the dynamics of the systems.

In this paper, a new method is given, which converts a chaotic motion of the system to a periodic motion. A tunnel mechanism is used to control the system. Our method has some disadvantages, that is, we requires some additional circuits and a previous knowledge of the system dynamics. However, the experimental results for Chua's circuit and the forced negative resistance oscillator show the availability of our method.

CONTROLLING CHAOS VIA TUNNELS

Our basic idea of the control mechanism is based on the concept of tunnels related to duck cycles. We shall first give the definition of a duck cycle and a tunnel. They are certain singular solutions of slow-fast systems, which are studied in the theory of relaxation oscillations [3]. The duck cycles were first found for the van der Pol equation: $\epsilon(d^2x/dt^2)+(x^2-1)(dx/dt)+x=a$, and their form resembled that of a flying duck. They are periodic trajectories which at first move along the attracting part of the slow curve and then move to the repelling part and continue along it (See Figure 1). It is well-known that duck cycles generate tunnel solutions around themselves. A tunnel is a bundle of trajectories that at the beginning are at an appreciable distance from one another, and then become infinitely close (a funnel) and deviate considerably from one another (a shower). See Figure 2. At this tunnel section, trajectories are asymptotically stable.

Tunnels are the useful tool that makes the chaotic orbit periodic. The basic idea of our control mechanism is written as follows: First, project the chaotic trajectories onto the two dimensional plane. Then, the system can be regarded as a two dimensional system. Next, find the cross section transversal to a bundle of trajectories in a chaotic attractor. Create the tunnel there. When the trajectory passes through the tunnel, the Lyapunov exponent of the two dimensional system decreases at this section. By adjusting the length of tunnel, we can make the Lyapunov exponents of the target orbit negative "in average". If the chaotic attractor does not turn to be a stable periodic trajectory, then make the tunnel section much longer or find another cross section. By this repeated operations, most of chaotic motions are usually converted to a stable periodic orbit on the two dimensional plane--for the returned trajectories enter the same entry point and leave the same exit point of the tunnel, and their Lyapunov exponent decreases. However, there is the case where we fail to convert. This is due to the following reason: For stable orbits we can expect Lyapunov exponents less than or equal to zero but the converse is not necessarily true. Then we are obliged to abandon the control.

CHUA'S CIRCUIT

Chua's circuit shown in Figure 3 is a simple oscillator which exhibits a variety of bifurcation and chaotic phenomena [4]. The circuit equations are given by

$$C_1(dv_1/dt)=(v_2-v_1)/R-f(v_1),$$
$$C_2(dv_2/dt)=(v_1-v_2)/R+i,$$
$$L(di/dt)=-v_2-ri, \qquad (1)$$

where v_1, v_2, and i are the voltage across C_1, the voltage across C_2, and the current through L, respectively. The resistance r is added to the ideal Chua's circuit in order to account for the small inductance resistance in the physical circuit. The characteristic of the nonlinear resistor is given by

$$f(v_1)=G_b v_1+0.5(G_a-G_b)[|v_1+B_p|-|v_1-B_p|]. \quad (2)$$

For our experiment, we used the following parameters:

$$C_1=10.1nF,\ C_2=101nF,\ L=20.8mH,\ R=1420\Omega,$$
$$r=63.8\Omega,\ G_a=-0.865mS,\ G_b=-0.519mS,\ B_p=1.85V. \quad (3)$$

The experimental circuit possesses the double scroll attractor shown in Figure 4.

TUNNELS IN CHUA'S CIRCUIT

In the first step of our control, we must project the trajectory on the two dimensional space. Let us consider the circuit in Figure 5. Three elements, that is, a diode D, a capacitor C_3, and a resistor R_3 are added to the Chua's circuit. These additional elements do not have a great effect to Chua's circuit, since C_3 is sufficiently small. (C_3= 8pF, $R_3=20\sim200k\Omega$: Note that a diode D and a capacitor C_3 are included in a 4066B IC chip) Figure 6 shows the trajectories projected on the two dimensional (v_2,v_3)-plane.

In the next step, we find the cross section on the (v_2,v_3)-plane. Glancing over Figure 6, we can easily find it. (Note that the attractor shape is changeable by adjusting R_3 if necessary.)

In the third step, we create the tunnel. The mechanism of tunnels (funnels and shower) is realized by the circuit in Figure 7. Its circuit behavior is explained as follows: The switch S closes at the moment the trajectory intersects the cross section $M=\{(v_2,v_3)\,|\,v_2=e_2, v_3\geq e_1\}$ (e_i are the control parameters). Then the capacitor C_3 will be charged instantaneously. That is, the voltage v_3 across the C_3 will hold the value e_1. Thus, the trajectories become infinitely close to the line $v_3=e_1$. (It forms a funnel.) After that, the switch is open again, and the trajectory deviate. (It forms a shower.) The length of the tunnel is decided by the switching time T of 4098B chip. (The period T is given by $T=0.5R_xC_x$.) Furthermore, the position of the tunnel is changeable according to the value e_2.

In the fouth step, we adjust the length and the position of a tunnel so that the chaotic motions are converted to a periodic orbit. The converted trajectory is shown in Figure 8.

Consequently, our control procedure can be described as follows: If a chaotic circuit is given, then find two independent terminals, and connect the control circuit with them. Next, find the cross section on the (v_2,v_3)-plane, and create a tunnel there (v_2 indicates the terminal voltage connected to the LF356). Adjust the length and the position of a tunnel such that the chaotic motions are converted to a periodic orbit. Then, various kinds of periodic orbits may appear according to the position and length of tunnels. Note that our aim is not the classification of the converted orbits--we will discuss it elsewhere.

EXPERIMENTAL RESULTS FOR CHUA'S CIRCUIT

Applying our method to Chua's circuit, we succeeded to convert the chaotic attractor to various types of periodic orbits:

$$\{ P_{ij}\ |\ i=(1,2,...,6) \text{ and } j=(1, 2, ..., 10) \}. \quad (4)$$

The symbol P_{ij} indicates the periodic trajectory which circles the upper plane with i times and the lower j times on the (v_1,v_2)-plane. Some of them are shown in Figure 9.

APPLICATION TO THE FORCED NEGATIVE RESISTANCE OSCILLATOR

We also applied our method to the negative resistance oscillator in Figure 10 [5]. The circuit equations are given by

$$Ldi/dt=-Ri-v-E\cos 2\pi ft,$$
$$Cdv/dt=-f(v)+i, \quad (5)$$

where $f(v)=k_2v+0.5(k_1-k_2)[|v+B_p|-|v-B_p|]$. The equation (6) has a chaotic attractor under the following parameters:

$$L=79.7mH,\ C=21.3nF,\ R=1.41k\Omega,\ E=0.66V,$$
$$f=2.40kHz,\ k_1=-1.8mS,\ k_2=-0.25mS,\ B_p=1.6V. \quad (6)$$

This chaotic attractor was converted to various kinds of periodic orbits by the same control circuit (see Figure 11). These results show the availability of our method.

CONCLUSION

The method to convert a chaotic motion to a periodic motion was proposed. We are now applying our method various kinds of circuits. We will report it elsewhere.

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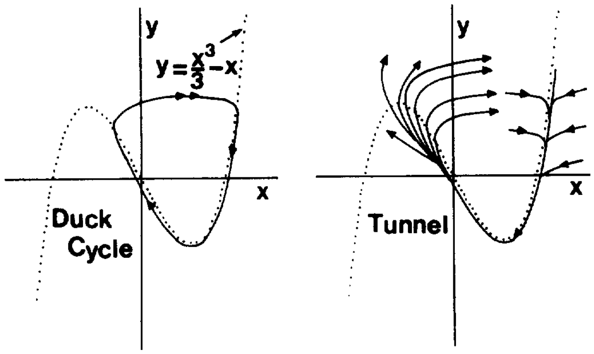

Figure 1 Duck cycles.

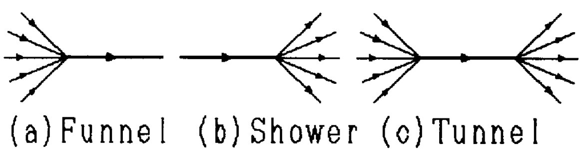

Figure 2 A funnel, a shower, and a tunnel.

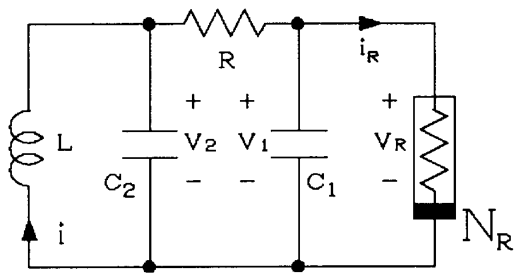

Figure 3 Chua's circuit.

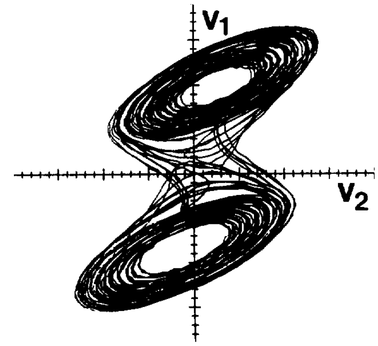

Figure 4 Double scroll attractor in Chua's circuit.

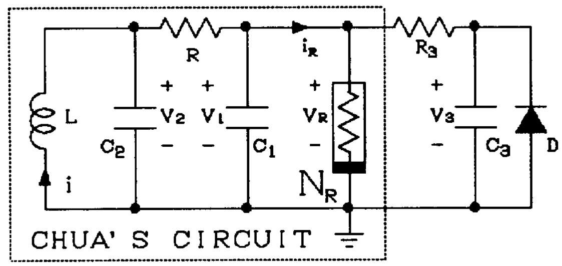

Figure 5 Chua's circuit with additional elements.

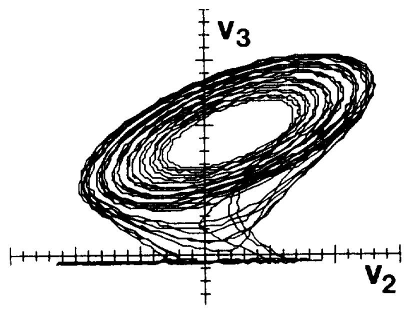

Figure 6 Trajectories on the (v_2,v_3)-plane.

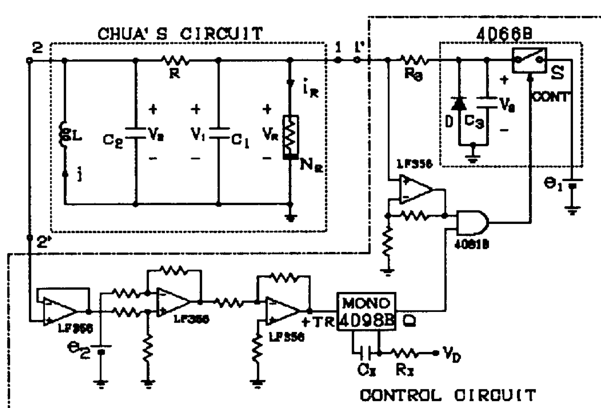

Figure 7 Control circuit. →

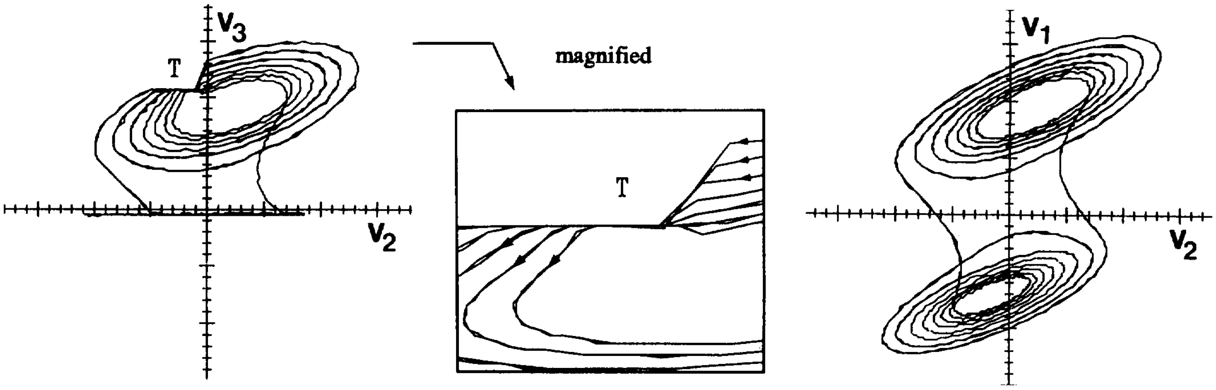

Figure 8 Trajectories of the tunnel on the (v_2,v_3)-plane and those on the (v_1,v_2)-plane.

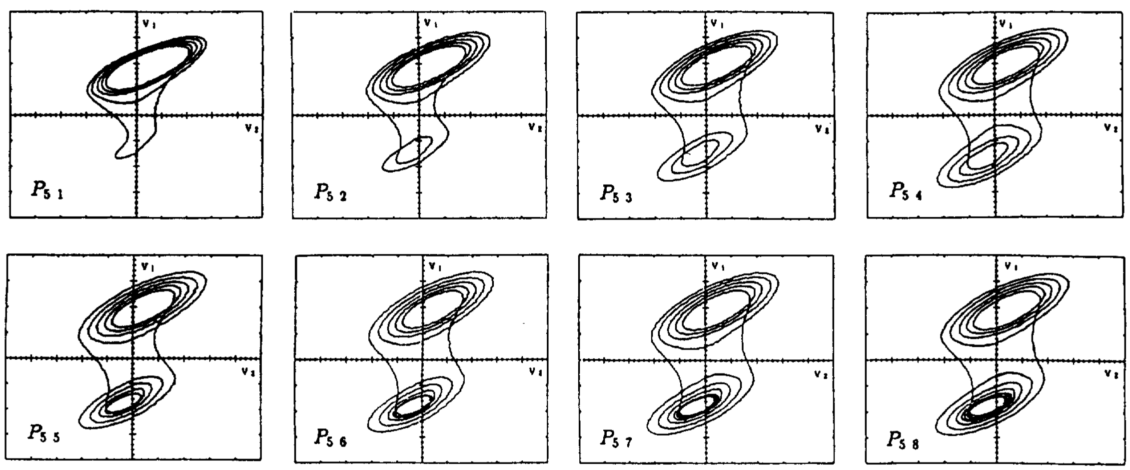

Figure 9 Converted Periodic orbits.

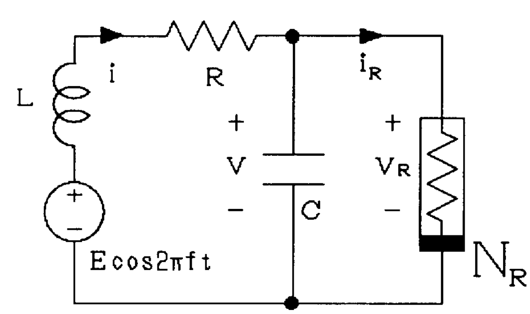

Figure 10 Forced negative resistance oscillator.

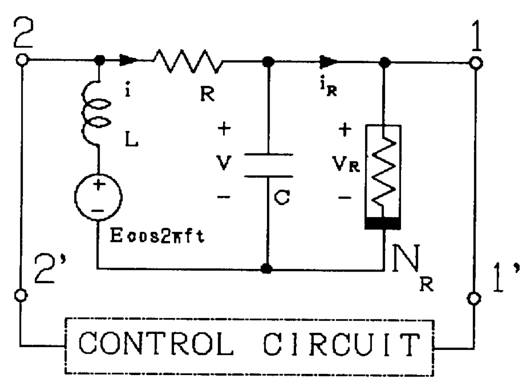

Figure 11 Control circuit.