China 1991 International Conference on Circuits and Systems, June 1991, Shenzhen, China

PERIOD DOUBLING AND PERIOD PLUS ONE LAW — CHAOTIC PHENOMENA IN SIMILAR CHUA'S CIRCUIT

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Abstract

This paper describes some interesting phenomena discovered in a Similar Chua's Circuit. First we look briefly at the circuit structure and state equation, and then determines the possible chaotic behavior from the property of the eigenvalues according to Shilnikov theorem. Some valuable results are obtained by both experiments and spectrum analysis, i.e., period doubling leads to chaos from the equilibrium and then it obeys the law of period-chaos-period plus 1. Finally, a bifurcation formula is given and a possible numerical range leading to chaos from bifurcation is offered when parameter G is changed.

Introduction

The Chua's Circuit is shown in Fig. 1. Period-chaos-period plus 1 law has been discovered in [1]. Now more perfect chaotic phenomena was observed in Similar Chua's Circuit, shown in Fig. 2, i.e., it obeys alternative period doubling and period plus one law. It is found both in experiment and spectrum analysis. Furthermore, this law can be not only obtained by changing one of G, C_1, C_2 and L, but also by changing the op's voltage which is the power of the nonlinear resistor. The characteristic of it is shown in Fig. 3. Finally, we have found a Hopf bifurcation expression. And taking G-bifurcation as an example, a possible numerical range leading chaos is given.

The State Equation and Chaos

The state equation in Fig. 2 can be written as:

$$
\begin{aligned}
C_1\, dv_{C1}/dt &= -i_L - g(v_{c1}) \\
C_2\, dv_{C2}/dt &= -Gv_{c2} + i_L \qquad (1) \\
L\, di_L/dt &= v_{c1} - v_{c2}
\end{aligned}
$$

Therefore, via the rescaling.

$x \triangleq v_{c1}/B_p$, $y \triangleq v_{c2}/B_p$, $z \triangleq i_L/B_pG$, $\tau \triangleq tG/C_2$, $a \triangleq m_1/G$, $b \triangleq m_o/G$, $\alpha \triangleq C_2/C_1$ and $\beta \triangleq C_2/LG^2$, Equation (1) is transformed into the following simpler dimensionless form:

$$
\begin{aligned}
dx/dt &= -\alpha\,[z + f(x)] \\
dy/dt &= -y + z \qquad (2) \\
dz/dt &= \beta x - \beta y
\end{aligned}
$$

where $f(x) \triangleq g(x;\, 1,\, b,\, a)$

$$
= \begin{cases} bx + a - b, & x \geq 1 \\ ax, & |x| \leq 1 \\ bx - a + b, & x \leq -1 \end{cases}
$$

provided $a, b \neq -1$ we can obtain 3 equilibrium in 3-dimensional phase space:

$P^+ = (k, k, k)$, $P^o = (o, o, o)$, $P^- = (-k, -k, -k)$

where $k = (b-a)/(b+1)$.

It is not difficult to prove that equation (2) is symmetric to the origin, as under the transformation

$(x, y, z) \longrightarrow (-x, -y, -z)$

the vector field is not variable. Therefore, the trajectory in the phase space of the circuit is symmetric to the origin.

Let $1/C_1 = 48$, $1/C_2 = 60$, $1/L = 16$, $G = 0.7$, $m_o = -0.5$, $m_1 = -0.8$, $B_p = 2.25$. After calculation, the eigenvalues of the system at P^o, P^+ and P^- are

$$\nu_o \approx 0.32, \quad \alpha_o \pm j\omega_o \approx -0.17 \pm j\,0.4$$

$$\nu_p \approx -0.5, \quad \alpha_p \pm j\omega_p \approx 0.035 \pm j\,0.53$$

Since $\nu_o > 0$, one dimension manifold associated with P^o is unstable, known as W_u^o; and since $\alpha_o < 0$, two dimension manifold associated with P^o is stable, known as W_s^o. For $|\alpha_o / \gamma_o| < 1$, it has countable horseshoe by purtubating the neighbor of $P^o(o, o, o)$, and this is a mathematical mechanism for generating chaos [2].

The Experiment and Spectrum Analysis

Traditionally, we can fix three of C_1, C_2, L and G and change the remaining one, then observe the bifurcation and chaotic motion. Here it is more convenient to adjust R, C_1 and C_2 than L. However, when four parameters are all fixed, more finer structures will be shown by changing the op's voltage, i.e., the characteristic of the nonlinear resistor. Let V_+(15v) and V_-(-15v) are the voltages of the operational amplifiers which are the components of the nonlinear resistor. When V_+ is set in 15v, V_- is adjusted from ov, the experiment result is shown in Table 1.

Table 1. Phase diagram when V_+ is 15v, and V_- is variable.

V_- (v)	0	1.1	3.5	4		4.5	4.8		5.3	5.6
phase dia.	point	1p	2p	4p	ch	2p	4p	ch	3p	6p

V_- (v)		6.2		7.1		8		8.8		9.1	
phase dia.	ch	4p	ch	5p	ch	6p	ch	7p	ch	8p	ch

V_- (v)	9.9		10.2		10.9		11.2		12	
phase dia.	9p	ch	10p	ch	11p	ch	12p	ch	13p	ch

where np as period n, ch as chaos, point as equilibrium.

The two dimensional projections corresponding to the equilibrium, period 1 - 10 and the chaotic attractor are shown in Fig. 4 (a - l). In order to confirm the experimental results, we observe the corresponding spectrum, as shown in Fig. 5 (a - m). It is obvious that the results obtained by both the experiments and the spectrum analysis are almost the same. That is to say, it obeys period doubling-chaos-period plus one law.

Hopf Bifurcation Range (G-bifurcation)

We have two goals in studying chaotic circuits. One is how to generate chaos so as to use it in some cases, the other is how to avoid chaotic failure. Chaos results from continuous bifurcation, so the first thing is to find the bifurcation range.

In the circuit, we only consider 3 inner regions. At these regions, let $g(x) = \mu x$ [3], where μ is the slope.

Then, from equation (1), the characteristic equation may be described as

$$W(s) = \det (sI - A')$$

where

$$A' = \begin{bmatrix} -a\mu & 0 & -a \\ 0 & -bG & b \\ 1 & -1 & 0 \end{bmatrix}$$

so $$W(s) = s^3 + (bG + a\mu)s^2 + (al + bl + abG\mu)s + ablG + abl\mu$$

According to Routh-Hurwitz criterion, the stability conditions of the linearized system is to satisfy the following unequalities:

$$bG + a\mu > 0 \qquad (3)$$

$$(a + b)l + abG\mu > 0 \qquad (4)$$

$$G + \mu > 0 \qquad (5)$$

$$(a+b)l + abG\mu - \frac{ablG + abl\mu}{bG + a\mu} > 0 \qquad (6)$$

Equations (3), (4), (5) and (6) are 4 straight lines or curves.

We know that the unstable situation results from continuous bifurcation. So we must consider the comprehensive bifurcation expression containing 5 parameters.

Let $W(s) = s^3 + As^2 + Bs + C$

and let it has 3 roots, i.e., λ_0, $\lambda_1 = \sigma + j\omega$, $\lambda_2 = \sigma - j\omega$. When $\sigma = 0$, λ_1 and λ_2 are a pair of pure virtual roots. Then we have

$$\lambda_0 + \lambda_1 + \lambda_2 = \lambda_0 .$$

Therefore, the Hopf bifurcation condition is

$$C = AB$$

Then the comprehensive bifurcation expression is

$$a^2bG\mu^2 + (ab^2G^2 + a^2l)\mu + b^2lG = 0$$

where 5 parameters a, b, l, G and μ are all variable. As any one of them changes, Hopf bifurcation point may be obtained.

Now taking G as an example Let a = 24, b = 25, l = 9, $m_o = -0.5$, $m_1 = -0.8$.

First, after calculation, Hopf bifurcation value at O and $P^{\pm}$ are 0.796 and 0.810.

On the other hand, from (3) - (6), we have

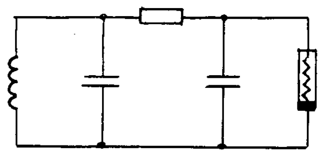

Fig. 1 Chua's Circuit

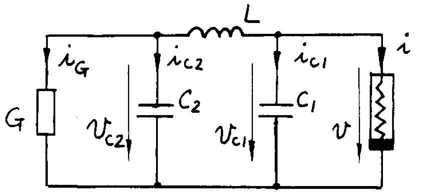

Fig. 2 Similar Chua's Circuit

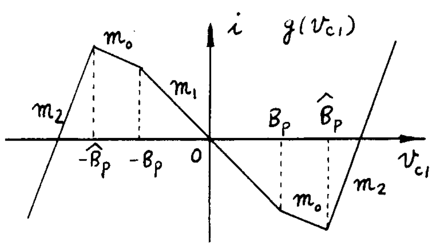

Fig. 3 v - i characteristic

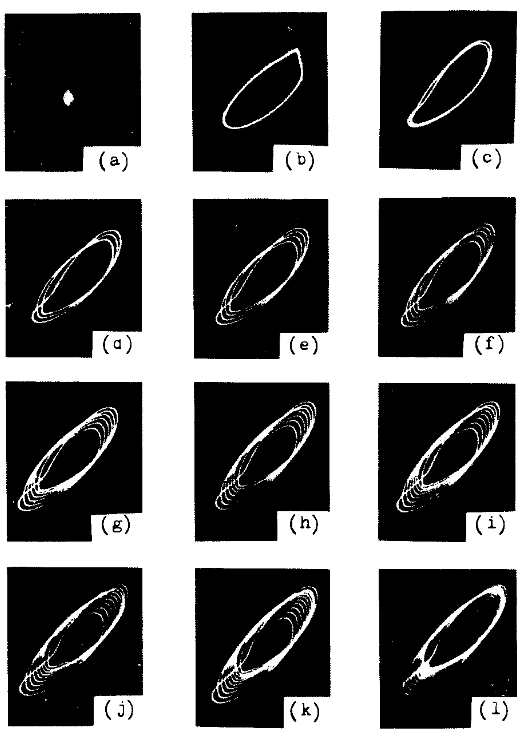

Fig. 4 Phase diagram

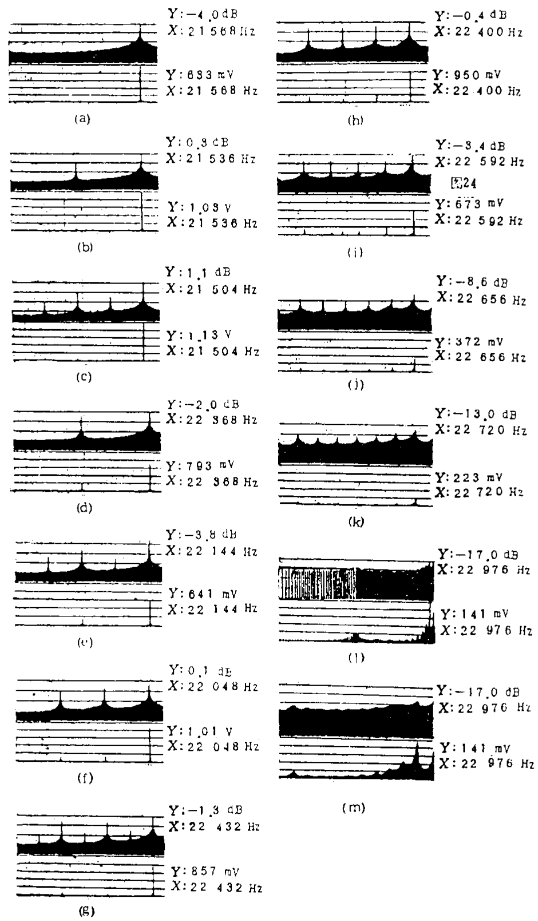

Fig. 5 Spectrum of v_{C2}

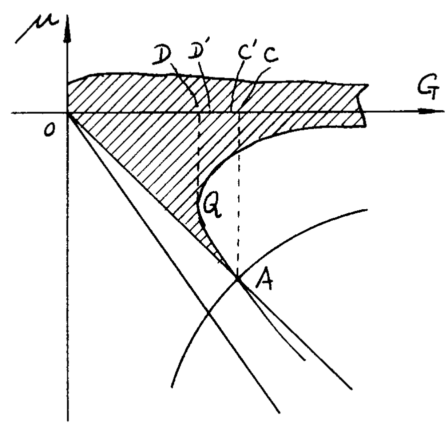

Fig. 6 Stable region at G-bifurcation

$$\mu > -bG / a \tag{7}$$

$$\mu > -(a+b)\, l / abG \tag{8}$$

$$\mu > -G \tag{9}$$

$$a^2 bG\mu^2 + (ab^2G^2 + a^2 l)\mu + b^2 lG > 0 \tag{10}$$

Let $a < b$, the curves corresponding to (7)-(10) are shown in Fig. 6.

It can be proved that (8), (9) and (10) interest at the same point A ($\sqrt{l(a+b)/ab}$, $-\sqrt{l(a+b)/ab}$). And from (10), the coordinates of point Q is $[\sqrt{l}\,(\sqrt{b}+\sqrt{b-a}) / b$, $-(b^2G^2 + al) / 2abG]$. After computation, the abscissa of A and Q are 0.857 and 0.720($1/\Omega$).

It follows that the Hopf bifurcation point of C is exactly between point C and D. And perhaps chaos is within C' and D'.

Discussion

We have obtained very interesting and nice results by changing op's voltage, i.e., the characteristic of the nonlinear resistor. And experimental method and spectrum analysis reveal the same law, period-chaos-period plus 1. In fact the spectral method is very effective in studying the bifurcation and chaotic phenomena, and it will become fashionable again.

Reference

[1] Anshan Huang, "A study of the chaotic phenomena in Chua's Circuit", presented at the IEEE ISCAS, Finland 1988.

[2] Smale S., Differentiable dynamic systems, Bull, Amer. Math. Soc., 1967, 73. 747-817.

[3] Maciej J. Ogorzalek, Chaotic regions from double scroll. IEEE Trans., 1987, CAS-34(2): 201-203.