ON THE CONTROL OF AN ARRAY OF CHUA'S SYSTEMS

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Abstract

The control of an array of Chua systems is discussed. For the particular array coupling chosen, it is shown that the control becomes more difficult as the number of Chua lumps increases when controlling from the boundary of the array.

Introduction

In the last twenty years, much progress has been made toward understanding the behavior of continuous-time chaotic systems. This is particularly true in the last five years, where both the control of chaotic systems and the utilization chaotic systems for specific purposes has been demonstrated [1]. With these successes, the last two years has seen increased interest in arrays of interacting chaotic systems (although much work has been done previously by Kaneko [2]). Here the research has been considering synchronization of the array [3,4], traveling wave fronts [5,6,7], pattern formation [8], and the generation of hyperchaos [9]. Clearly one of the underlying themes of these studies has been an attempt to gain insight into more complicated systems such as turbulent flows, reactive media, combustion, and plasma dynamics. This paper contains an initial attempt at understanding the control of arrays of chaotic systems. In this study, control will primarily mean to drive the chaotic system to a fixed steady state point.

To gain more insight, it is useful to choose a chaotic system that is somewhat well understood. Chua's circuit is one such system which has also been used in many of the studies on chaotic arrays, and will be used here. At this point it is necessary to choose the way the array of Chua circuits interact with each other for this study. Many choices have been made in the past including, diffusive coupling through resistors in both one and two dimensions [5,8], transmission line-like coupling [7], and one directional convective coupling through any one of the states [9] Many other choices are available. For this study, one directional convective coupling through the x-states of the Chua system with cubic nonlinearity [10] is chosen, as it demonstrates many of the problems involved in controlling an array of chaotic systems. Thus the equations for the j-th Chua circuit, or j-th lump, of the array are

$$\dot{x}_j = \alpha\left(y_j + \frac{1}{7}\left(x_j - 2x_j^3\right)\right) - c\left(x_j - x_{j-1}\right)$$
$$\dot{y}_j = x_j - y_j + z_j$$
$$\dot{z}_j = -\frac{100}{7}y_j + u_j$$

Here, the u variable is a local control input which acts on a particular lump directly as in [10], and the input to the lump from the previous lump enters through the x equation via x(j-1). There will be no zeroth lump and x_0 will be a boundary input to the array. Clearly the flow of information along the array is in one direction only, in that the lower lump numbered outputs can effect the higher lump numbers but not conversely. For this paper, $\alpha = 9$ and $c = 0.25$ exclusively. With $u(j) = 0$, the steady state, or reference, value of x_0 is chosen to be $\sqrt{0.5}$, which then forces one of the steady state values of the x(j) to also be $\sqrt{0.5}$ for all j. Then corresponding steady state values of $y(j) = 0$ and $z(j) = -\sqrt{0.5}$ for all j. It should be noted that there are other steady state values for each lump which could be stabilized in what follows, but these will not be considered here as they become very complicated. Note that the two other steady states for x(1) are -0.1869 and-0.5052, and that there are a total of 3^j steady state points for j-lumps. It is observed that these points become very close together and may add to some of the sensitivity problem to be discussed later. For the given parameters, the chosen steady state point described above is unstable. Each Chua circuit is basically chaotic (given the proper IC's), but it is also being driven by a chaotic Chua circuit from the left. Thus the result is probably a hyperchaotic system similar to [9], but this has not been verified. A phase portrait is given in Figure 1.

The paper is organized as follows. Discussed first is the use of local measurements and local control. Then control only at the left boundary is used throughout the remainder of the paper. The paper progresses by presenting the design of controllers for

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increasing numbers of lumps along with the problem encountered. Five lumps are the maximum considered in this study. Note that only the regulator problem is solved here, and tracking is not considered.

Local Measurment and Local Control

When local control through the u-variable can be used, it is fairly easy to drive the system to a steady state fixed point through the use of any of the classical feedback strategies used in [10]. Here it is necessary to measure a local output from a lump, say x(j), and feed it back through a compensator into the local input to the lump, u(j). If there are n-lumps, then there must be n-compensators, one for each lump. It is possible to control the system in this way, as the input from the previous lump appears as a disturbance to the present lump, see Figure 2. This is a common control configuration, and the disturbances are easily rejected with proper design of the sensitivity function in the Nyquist plane [11].

As this control approach requires as many sensors and actuators as there are lumps, it is fairly expensive. In the remainder of this study, it is assumed that only one actuator will be used, and that it will act from the boundary. Thus all of the u(j) inputs will be set to zero.

Control of One Lump From the Boundary

To control the one lump system from the boundary, a steady state analysis must be performed. The system equations are

$$\dot{x} = \alpha\left(y + \frac{1}{7}(x - 2x^3)\right) - c\,(x - x_0)$$
$$\dot{y} = x - y + z$$
$$\dot{z} = -\frac{100}{7}\,y$$

The steady state values chosen for the variables, with $x_0 = \sqrt{0.5}$ are , $x_{ss} = \sqrt{0.5} = -z_{ss}$ and $y_{ss} = 0$. The linearized small perturbation equations at this steady state are then given by $\delta\dot{\xi} = A\,\delta\xi + B\,\delta u$ with output $\delta\eta = C\,\delta\xi$ where $\delta\xi$ is a 3-vector of the linearized states, δu is the linearized scalar control input, and $\delta\eta$ is a small perturbation output. Notice then that A, B, and C are all matrices of the appropriate dimensions. For this system

$$A = \begin{bmatrix} \frac{-2\alpha}{7} - c & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & \frac{-100}{7} & 0 \end{bmatrix},\quad B = \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix},$$

and assuming that the x-variable is the system output, C = [1 0 0]. This is then a single-input-single-output-system whose transfer function can be found via $H(s)=C[sI-A]^{-1}B$, where s is the Laplace variable. It is

$$\frac{\delta x(s)}{\delta x_0(s)} = H(s) = \frac{0.2500\,s^2 + 0.2500\,s + 3.5714}{s^3 + 3.8214\,s^2 + 8.1071\,s + 40.3061}$$

with zeros at $s = -0.5000 \pm j3.7464$ and poles at $s = -4.1854,\ 0.1820 \pm j3.0979$. The linearized system then has two poles in the right half s-plane and the steady state point is clearly unstable. A root locus analysis indicates that this system can easily be stabilized by using proportional control, $\delta x_0 = -k\,\delta x$ and is stable for any proportional gain $k > 17.81$; $k=100$ is a good choice. To implement this linear control law, the perturbed variables are fed back. Thus, when implemented into the original equations, this controller yields

$$\dot{x} = \alpha\left(y + \frac{1}{7}(x - 2x^3)\right) - c\left(x - \left[\sqrt{1/2} - k\left(x - \sqrt{1/2}\right)\right]\right)$$
$$\dot{y} = x - y + z$$
$$\dot{z} = -\frac{100}{7}\,y$$

The utility of this controller was verified via simulation and it was somewhat robust with respect to perturbations from the steady state point.

Control of 2 Lumps From the Boundary

Here the control of two coupled lumps from x_0 is considered. The system equations are then

$$\dot{x}_1 = \alpha\left(y_1 + \frac{1}{7}(x_1 - 2x_1^3)\right) - c\,(x_1 - x_0)$$
$$\dot{y}_1 = x_1 - y_1 + z_1$$
$$\dot{z}_1 = -\frac{100}{7}\,y_1$$
$$\dot{x}_2 = \alpha\left(y_2 + \frac{1}{7}(x_2 - 2x_2^3)\right) - c\,(x_2 - x_1)$$
$$\dot{y}_2 = x_2 - y_2 + z_2$$
$$\dot{z}_2 = -\frac{100}{7}\,y_2\,.$$

Again a small perturbation analysis must be performed. As stated in the first section, however, the chosen steady state is shared by each lump. In fact, as a state space linearization would show, because the coupling is only one directional, the small perturbation transfer function from δx_0 to δx_j is simply the transfer function from δx_0 to δx_1 raised to the j-th power. We thus now have the small perturbation transfer function for any number of lumps, namely

$$\frac{\delta x_j(s)}{\delta x_0(s)} = [H(s)]^j = \left[\frac{0.2500\ s^2 + 0.2500\ s + 3.5714}{s^3 + 3.8214\ s^2 + 8.1071\ s + 40.3061}\right]^j$$

and we can concentrate on the control design. The poles and zeros were given in the previous section, and there are now j of each of them. The pole excess is then j. When there are two lumps, the pole excess is two and one would expect to be able to stabilize this system using some large gain proportional control. Root locus analysis indicates that this system will be stable for proportional gain k>1230, while k=2000 is reasonably robust to perturbations.

Control of 3 Lumps From the Boundary

In this situation, the linearized transfer function about the chosen steady state has a pole excess of three, with six poles in the right-half s-plane. This is indeed a difficult control problem. Initially, a second order lead compensator is used. It is

$$\frac{\delta x_0(s)}{\delta x_3(s)} = -k\,\frac{s^2 + 4s + 65}{s^2 + 40s + 400}$$

which is stable in the linearized model for approximately 0.8e+06<k<1.9e+06. It was implemented on the nonlinear model with k=1.1e+06 and very stable responses were obtained for small (<0.001) initial perturbations, however, both k=1.0e+06 and k=1.2e+06 gave small limit cycling responses about steady state. Due to this sensitivity, an optimal state feedback control law was designed which used the feedback gain vector k=[9.3587, 29.8136, 3.9736, 27.6571, -61.2762, 80.1885, -172.8133, -752.8481, -113.5225]^T. This state feedback law uses measured displacements from steady state for all of the states in every lump. It thus acts whenever any of the states start to vary from their equilibria. It should also be noticed that the gains are larger for the states of the third lump. This seems reasonable since they are the farthest away from the control signal, which is effectively diminished by a factor of (c=0.25) for every lump. It should also be noted that this type of control is very expensive, as it requires sensors for all the states.

Control of 5 Lumps From the Boundary

A five lump problem is now considered, skipping over four lumps, which was also controlled using the following strategy. It was initially planned to consider an arbitrary number of lumps, but the increasing sensitivity to initial conditions indicates that the five lump problem is demonstrative of the difficulty in controlling any higher number of lumps. Fuethermoere, the control Gramians were found to become more singular as more lumps were added. This implies that as more lumps added the control becomes more difficult.

Also, based on the experience with the three lump problem, state feedback is used exclusively from the beginning. Note that the problem now has a pole excess of five and has ten poles in the right half plane. As in the three lump case, an optimal linear regulator design is used to stabilize the five lump system. This controller worked well, but the initial state perturbations must be less than 0.0001, and the controller drequired very large gains. These gains are not listed as the controller was not very sensative to the choice of Q and R in the linear requlator design.

As discussed earlier, state feedback is a very expensive control approach as it requires sensors on every state, in this case 15 of them. A much cheaper approach is to use observer feedback. Two observer feedback designs were considered. One using the error signals from all of the x-variables (5 in this case), and the other using the error signal from the last state, x(5), which is more similar to the input-output controller from before. The observer was implemented using the exact nonlinear system equations with the following observer gains feeding the error signal back to all the states:

all x-measurements

L_{15x5} =

4.4220	0.0783	0.0009	0.0000	-0.0000
1.6396	0.0158	-0.0001	-0.0000	-0.0000
-3.6970	-0.1273	-0.0024	-0.0000	-0.0000
0.0783	4.4246	0.0783	0.0009	0.0000
-0.0126	1.6399	0.0159	-0.0001	-0.0000
0.0478	-3.6942	-0.1273	-0.0024	-0.0000
0.0009	0.0783	4.4246	0.0783	0.0009
-0.0000	-0.0126	1.6399	0.0159	-0.0001
-0.0005	0.0478	-3.6942	-0.1273	-0.0024
0.0000	0.0009	0.0783	4.4246	0.0783
0.0000	-0.0000	-0.0126	1.6399	0.0159
-0.0000	-0.0005	0.0478	-3.6942	-0.1273
-0.0000	0.0000	0.0009	0.0783	4.4254
-0.0000	0.0000	-0.0000	-0.0126	1.6401
0.0000	-0.0000	-0.0005	0.0478	-3.6954

x(5)-measurement

L_{15x1} = 1.0e+04 *

0.00161687384192
0.00402886754185
0.00018101427868
0.01445351093760
0.02755833966399
0.02053391616153
-0.09854688167118
-0.58797411896576
0.04445504697133
-0.55507704374031

-0.57688432093263
-0.92309258856449
0.89584503104179
5.47499076757945
0.12783857953928

Interestingly enough, these observers were very robust, and allowed perfect tracking of the true system states, even in the uncontrolled system, and for large perturbations (>0.1). In this sense, they forced synchronization between the two 15-th order potentially hyperchaotic systems. When used with the controller, the initial state errors were the same size as the initial perturbations of the states, but of the opposite sign.

Conclusion

In this paper it has been demonstrated that an array of Chua systems can be stabilized and forced to regulate to one of the system fixed points. For this particular system, the controllers were very sensitive to initial perturbations from the desired unstable steady state. This problem became more difficult as more lumps were added. Observer feedback was also implemented by using the nonlinear system equations in the observer. It was found that this observer configuration allowed state tracking even with no controller in the loop, thereby perhaps providing a general means of synchronizing chaotic systems.

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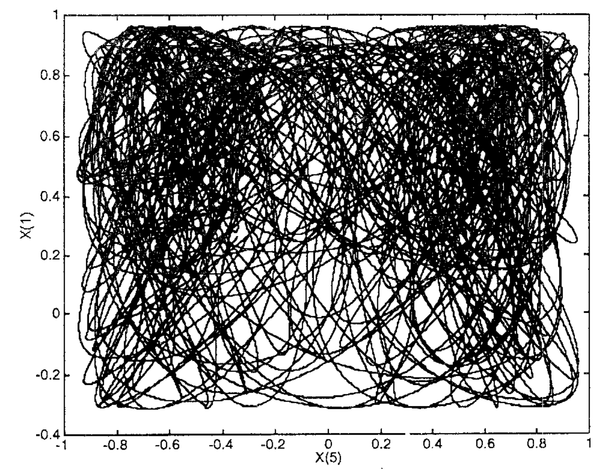

Figure 1. Uncontrolled 5-lump array, x(5) vs x(1).

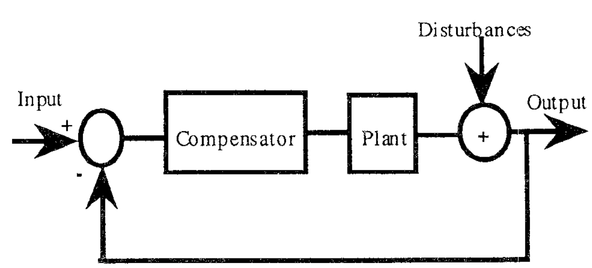

Figure 2. Typical feedback configuration.