A COMPUTER PROGRAM FOR ACCURATE TIME-DOMAIN ANALYSIS OF 1D-ARRAYS OF CHUA'S OSCILLATORS

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ABSTRACT

This paper describes a computer program for the time-domain analysis of one-dimensional arrays of Chua's oscillators. The implemented algorithm exploits the local connectivity, typical of cellular nonlinear networks, and the piecewise linear behavior of the v-i characteristic of the nonlinear elements to obtain an analytical expression of the solution. Examples demonstrate the accuracy and the efficiency, in terms of cpu-time, of the proposed approach with respect to standard simulation tools as SPICE.

1. INTRODUCTION

Cellular Nonlinear Networks (CNNs) [1, 2] are regular arrays of identical cells, each of which generally contains only one nonlinear memoryless element modeled by a piecewise linear (PWL) resistor. They present several applications in the fields of image processing and pattern recognition. Furthermore, they are useful for modeling nonlinear wave phenomena in physics and biology [2].

At present, the only rigorous theoretical results on CNN dynamics concern with either the stability of restricted classes of networks or the dynamic behavior of networks composed of a small number of cells [1]. In a few cases, some approximate techniques, based on spatio-temporal spectral approaches, have been developed to accurately predict several dynamic phenomena [3, 4, 5, 6]: these techniques exploit the fact that the interconnections between the cells of a CNN are local and described by space invariant templates.

As a consequence, most of the CNN applications and dynamic properties have been revealed through extensive numerical simulations. The most basic type of circuit simulation is the computation of all the relevant electric variables as functions of time, starting from given initial conditions. In order to get a global picture of this time evolution with reasonable confidence, the whole state space has to be covered with a fine grid of initial conditions. In other words, a huge amount of time domain solutions have to be calculated for the same circuit. Furthermore, one often is interested in the qualitative aspects of the solutions as the parameters of the circuit are varied, which increases the volume of simulations even more.

These considerations show clearly that the qualitative analysis of circuits calls for efficient simulation tools. To this end, one can exploit the above mentioned local connectivity for time-domain simulations too. In fact, restricting ourselves to the class of PWL CNNs, it gives rise to band Jacobian matrices, from which the eigenvalues and the eigenvectors in each region of linearity can be easily and accurately determined. This allows us to simulate large arrays of CNNs by extending the technique already developed in [7]. It consists of the following two fundamental steps:
(a) in each region of linearity, the solution is formulated in terms of natural modes and calculated analytically, as an explicit function of time, and evaluated for a sequence of time values; each evaluation has basically the same precision and thus the numerical errors do not accumulate as they may in numerical integration methods.
(b) the transition time instants between any pair of adjacent linear regions crossed by the solution are determined; this is the most numerically critical task. In fact, when the circuit exhibits chaotic behavior, the inaccuracies of the crossing points are amplified just as the errors made at each step of a numerical integration algorithm are. However, there are much less boundary crossing than time steps in numerical integration. Furthermore, the accuracy of the crossing time points can reach, in principle, the precision of the computer arithmetic.

Based on the above ideas we have developed a program, called CNNA (Cellular Nonlinear Network Analysis), for the simulation of one-dimensional (1D) CNNs with PWL elements. At the moment, the popular Chua's oscillator is used as cell [8], but other kind of cells will be added in the future. It is based on the algorithm presented in [9] and reviewed in the next Section. The circuit, shown in Fig. 1, is described by assigning the number of cells, the element values of the Chua's oscillator, the values of the coupling and terminating conductances, and the values of initial conditions. The PWL resistor characteristic is specified by its breakpoints. When the transient analysis is terminated, a graphic output allows the researcher to examine each of the node voltages controlling the PWL resistors.

Due to the above considerations, CNNA is expected to use less CPU time and to give more accurate results than those obtainable by general purpose circuit simulation tools, as SPICE, based on numerical integration of nonlinear differential equations. In the last Section, an example is presented to show possibilities and limitations of CNNA as compared to standard circuit simulators.

2. REVIEW OF THE ALGORITHM

We consider 1D arrays with $(K-1)$ identical series branches and K identical shunt branches as shown in Fig. 1. Each series branch is a simple linear resistor of conductance G_s, whereas each shunt branch is the Chua's oscillator shown in Fig. 2 [8] and represents a cell of the array. The Chua's oscillator consists of the parallel connection of a third-order dynamic linear one-port (with admittance $Y(s)$) and a PWL two-terminal resistor (Fig. 2). The characteristic of the PWL resistor considered in this paper is shown in Fig. 3 [8] and is represented by

$$\hat{i}(v_k(t)) = G_{h(k)}v_k(t) + I_{h(k)} \qquad (1)$$

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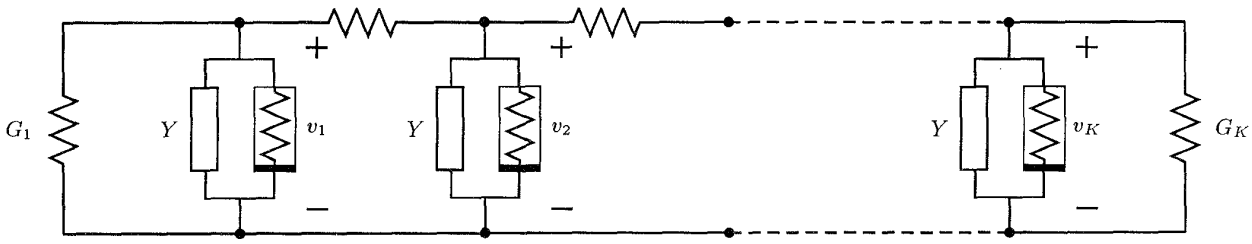

Figure 1: *1D array of Chua's oscillators*

where $h(k)$ denotes the segment of $\hat{\imath}(\cdot)$ on which the k-th cell operates, and $G_{h(k)}$ and $I_{h(k)}$ are the parameters of the Norton model related to this segment. Finally, the 1D array is terminated by two linear resistors of conductances G_1 and G_K.

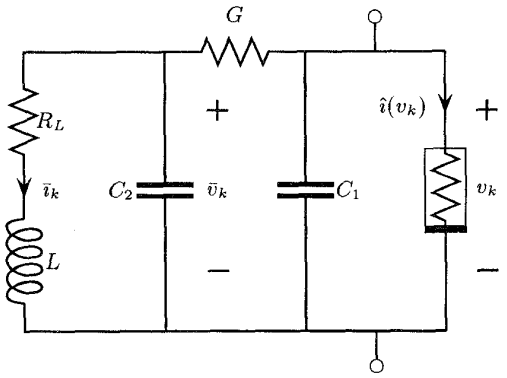

Figure 2: *Chua's oscillator*

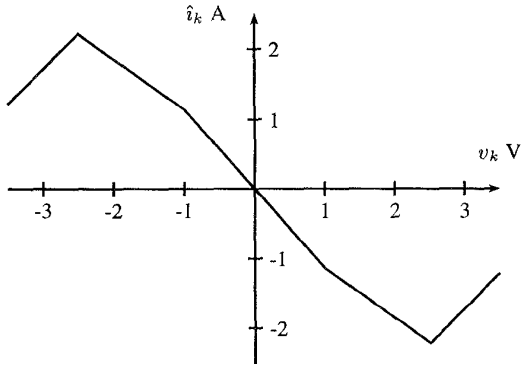

Figure 3: *Characteristic of the PWL resistor of Fig. 2*

The time-domain behavior of the whole circuit of Fig. 1 is described by $3K$ state variables. However, the circuit is best studied by resorting to the voltages $v_1(t), v_2(t), \ldots, v_K(t)$ across the PWL resistors. In fact, for such variables and in each of the n_s^K possible linear regions of the circuit (n_s being the number of segments of the characteristic of Fig. 3) the following set of K linear 3rd-order differential equations holds [9]

$$P(\mathcal{D})[v(t)] = \boldsymbol{A}Q(\mathcal{D})[v(t)] - q_0\hat{\boldsymbol{I}} \tag{2}$$

In the above equations, $v(t) = [v_1(t)\ v_2(t)\ \ldots\ v_K(t)]^{\mathrm{T}}$, $\hat{\boldsymbol{I}} = [I_{h(1)}\ I_{h(2)}\ \ldots\ I_{h(K)}]^{\mathrm{T}}$, $\mathcal{D}$ represents the time derivative operator, P and Q are the numerator and denominator polynomials of $Y(s)$, respectively, which are specified by the following coefficients

$$\begin{aligned} p_0 &= G \\ p_1 &= C_1 + GRC_2 + C_1GR_L \\ p_2 &= C_1GL + C_1C_2R_L + GLC_2 \\ p_3 &= C_1C_2L \end{aligned} \tag{3}$$

$$\begin{aligned} q_0 &= GR_L + 1 \\ q_1 &= C_2R_L + GL \\ q_2 &= C_2L \end{aligned}$$

and $\boldsymbol{A}$ is the tridiagonal symmetric matrix defined by

$$\begin{aligned} [\boldsymbol{A}]_{11} &= -(G_s + G_{h(1)} + G_1) \\ [\boldsymbol{A}]_{KK} &= -(G_s + G_{h(K)} + G_K) \\ [\boldsymbol{A}]_{kk} &= -(2G_s + G_{h(k)}),\ \ k = 2, 3, \ldots, K-1 \\ [\boldsymbol{A}]_{(k-1)k} &= [\boldsymbol{A}]_{k(k+1)} = G_s \end{aligned} \tag{4}$$

The closed-form solution of (2) in every linear region can be written as

$$v(t) = \sum_{k=1}^{K} \boldsymbol{w}_k(C_{k1}e^{\lambda_{k1}t} + C_{k2}e^{\lambda_{k2}t} + C_{k3}e^{\lambda_{k3}t}) + v_0 \tag{5}$$

In the above equation, v_0 is the (virtual or actual) equilibrium point of the linear region (i.e the solution of the algebraic equation obtained from (2) by setting $\mathcal{D} = 0$), λ_{km}, $m = 1, 2, 3$, are the solution of the K third-order algebraic equations

$$P(\lambda_{km}) - \lambda_k Q(\lambda_{km}) = 0,\ \ k = 1, 2, \ldots, K \tag{6}$$

where λ_k is the k-th eigenvalue of matrix $\boldsymbol{A}$, and $\boldsymbol{w}_k$ is the corresponding eigenvector. Solution (5) holds because the λ_{km} are distinct [9]. In summary, the piecewise solution algorithm amounts to the computation of the $3K$ natural frequencies λ_{km} ($k = 1, 2, \ldots, K$; $m = 1, 2, 3$) via the evaluation of the eigenvalues of the tridiagonal $K \times K$ symmetric matrix $\boldsymbol{A}$ and the solution of the K 3rd-order algebraic equations (6). Coefficients C_{km}, ($k = 1, 2, \ldots, K$; $m = 1, 2, 3$), instead, are obtained by applying the continuity properties of $v(t)$ every time the circuits enters a new linear region [9].

As an example, we consider a chain of $K = 20$ Chua's circuits with $G_1 = G_K = 0$. Each cell is described by the normalized parameter values $G = 1$, $C_2 = 1$, $C_1 = 0.125$, $L = 0.0667$. Moreover we assume that the nonlinear characteristic of Chua's diode is of the type shown in Fig. 3 and given by:

$$\hat{i}(v) = v + \tfrac{3}{14}(|v+2.5| - |v-2.5|) - \tfrac{3}{14}(|v+1| - |v-1|) \quad (7)$$

It is easily shown that, with this choice of parameters, in absence of coupling (i.e. $G_s = 0$) , each cell may exhibit only two limit cycles: they mainly lies in the upper and in the lower part of the characteristic (7) and will be denoted with $+1$ and -1 respectively. Therefore for $G_s = 0$ the entire network presents 2^K attractors, that will be denoted by a sequence of ± 1.

As reported in [10, 11], by increasing the coupling conductance G_s the following phenomena occur: first, all the attractors represented by a string of the type $\ldots + 1, -1, +1 \ldots$ and $\ldots - 1, +1, -1, \ldots$ disappear; then the whole system becomes chaotic. Such phenomena have been observed through the program CNNA.

In order to evaluate accuracy and speed of CNNA we have compared it with SPICE. As a case study we have considered two coupling conductances: $G_s = 0.00625$ (which give rise to a periodic attractor) and $G_s = 0.025$ (which generates a chaotic attractor). In both cases, for $v_k(0)$, $\bar{i}_k(0)$ and $\bar{v}_k(0)$ (see Fig. 2) we have chosen the following initial conditions:

$$\begin{cases} v_k(0) = 1.1 \text{ V},\ \bar{i}_k(0) = 1.1 \text{ A},\ \bar{v}_k(0) = 0.1 \text{ V} \\ \quad \text{for} \quad k = 1, 4, 5, 7, 8, 11, 14, 15, 18, 19, 20 \\ v_k(0) = -1.1 \text{ V},\ i_k(0) = -1.1 \text{ A},\ \bar{v}_k(0) = -0.1 \text{ V} \\ \quad \text{for} \quad k = 2, 3, 6, 9, 10, 12, 13, 16, 17 \end{cases} \quad (8)$$

The comparison between CNNA and SPICE outputs is summarized in Table 1 (for $G_s = 0.00625$) and 2 (for $G_s = 0.025$), where the voltage across the capacitor C_1 of the 15-th cell ($v_{15}(t)$) is reported for different time instants. The first column of both the Tables reports the time instants in which the voltage $v_{15}(t)$ is evaluated; the second column reports the SPICE results with automatic time-step size; the other columns refer to SPICE with different time-step sizes; finally the last column reports the CNNA results. The last row contains the cpu-time (in seconds) needed for the simulation. The waveforms $v_{15}(t)$ for $G_s = 0.00625$ and for $G_s = 0.025$ computed by CNNA are shown in Figs. 4 and 5, respectively.

The following considerations hold about the simulation values reported in Table 1 and Table 2:

(a) for the periodic attractor case (Table 1), the accuracy of SPICE becomes comparable to that of CNNA for a time-step equal to 0.0003 s, that requires a cpu-time of 8702.21 s. (i.e approximately 60 times the cpu-time required by CNNA).

(b) for the chaotic attractor case (Table 2), the accuracy of SPICE (after 100 s) is never comparable to that of CNNA even for very small time-steps. However it is worth noticing that by decreasing the SPICE time-step the convergence to the CNNA results improves: this confirms that CNNA results are very accurate also for the simulation of chaotic waveforms.

Table 1: Comparison between values of $v_{15}(t)$ (expressed in V) computed by SPICE and by CNNA for $G_s = 0.00625$ (periodic attractor). The last row shows the cpu-time in seconds

t	*SPICE*				*CNNA*
s	*auto*	*0.01* s	*0.001* s	*0.0003* s	
0	1.1	1.1	1.1	1.1	1.1
10	1.30063	1.54049	1.54117	1.54118	1.541181
20	1.71239	1.46454	1.46779	1.46782	1.467820
30	1.33912	.593554	.591177	.591157	.5911553
40	1.74813	1.76414	1.75936	1.75932	1.759316
50	.630826	1.49496	1.50346	1.50353	1.503537
60	1.96359	.556479	.552603	.552570	.5525663
70	.607552	1.74500	1.73642	1.73634	1.736328
80	1.96551	1.45413	1.46789	1.46802	1.468037
90	.721497	.587609	.580656	.580589	.5805820
100	1.86736	1.82621	1.81579	1.81569	1.815683
Cpu	10	226	2078	8702	145

Table 2: Comparison between values of $v_{15}(t)$ (expressed in V) computed by SPICE and by CNNA for $G_s = 0.025$ (chaotic attractor). The last row shows the cpu-time in seconds

t	*SPICE*				*CNNA*
s	*auto*	*0.01* s	*0.001* s	*0.0003* s	
0		1.1	1.1	1.1	1.1
10	1.11975	1.58039	1.58067	1.58067	1.580669
20	1.10622	1.24046	1.24333	1.24335	1.243353
30	1.52389	.499709	.498043	.498027	.4980259
40	.940608	1.46026	1.44971	1.44966	1.449657
50	.960284	1.33948	1.31740	1.34702	1.347336
60	1.63789	.670158	.589809	.619336	.6197242
70	.538656	.868835	.939869	.964861	.9649737
80	1.36089	1.44907	1.45168	1.44442	1.444262
90	1.27642	1.28212	1.63072	1.59282	1.592374
100	.886548	.827604	1.08769	1.29559	1.298076
Cpu	11	218	1939	6706	164

4. CONCLUSIONS

We have developed a program, named CNNA, for the analysis of cellular nonlinear networks composed by identical cells, each containing only one PWL resistor . It is more accurate than general-purpose simulation tools based on numerical integration, because the solution is computed exactly in each linear region, according to the algorithm developed in [7]. Moreover, by exploiting the local connectivity of the structure, which results in a band Jacobian matrix, it allows the analysis of large networks that cannot be dealt with by [7]. Finally, it uses the identity of the cells to accelerate the calculation of eigenvalues.

Extensive comparisons have proved that CNNA is more accurate and fast than SPICE, for the simulation of both periodic and chaotic attractors. Presently, CNNA concentrates on the particular case of 1D arrays of Chua's oscillators. However, it can be easily extended to other kind of PWL cells and to include non reciprocal coupling. Such extensions are under development along with the

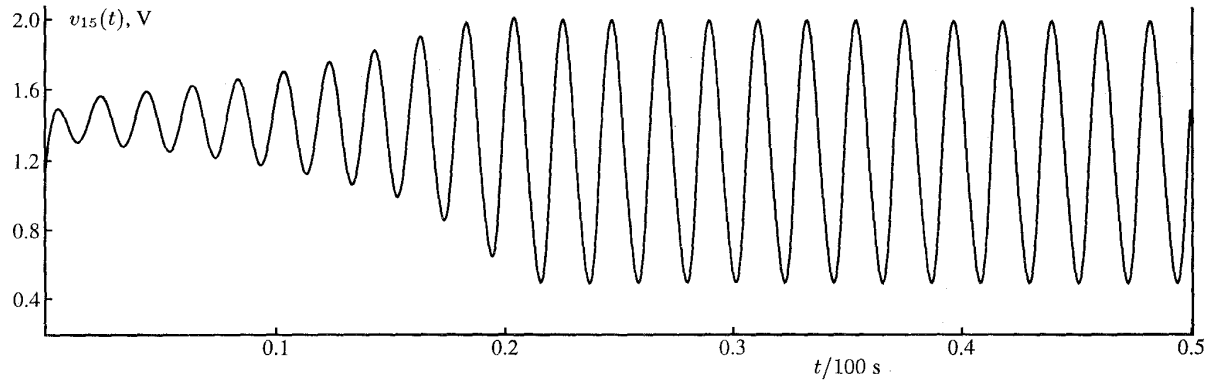

Figure 4: *Time waveform of the voltage across the capacitor C_1 of the 15-th cell, for $G_s = 0.00625$.*

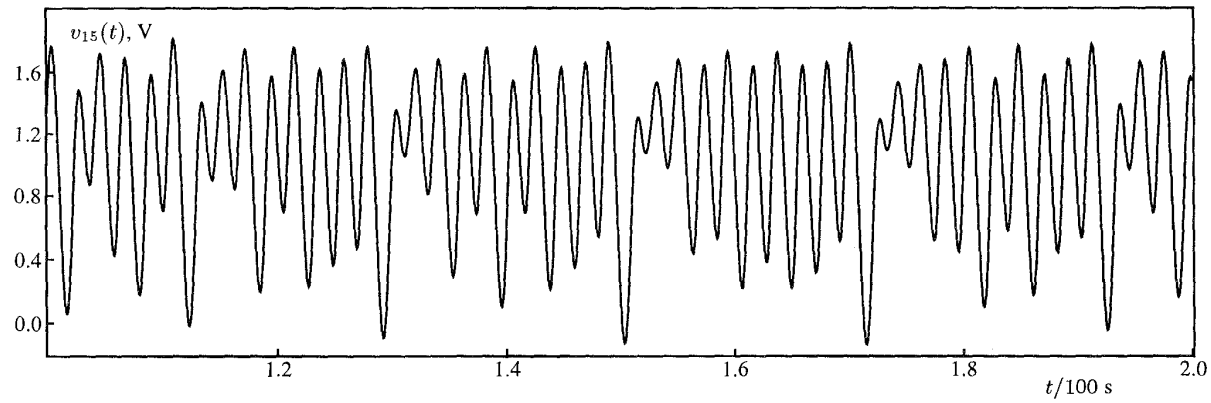

Figure 5: *Time waveform of the voltage across the capacitor C_1 of the 15-th cell, for $G_s = 0.0250$.*

study of bifurcation processes.

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