Express Letters

Chua's Circuit Can Be Generated by CNN Cells

P. Arena, S. Baglio, L. Fortuna, and G. Manganaro

***Abstract*—A new realization for the unfolded Chua's circuit is introduced. It has been derived from the connection of three simple generalized cellular neural network (CNN) cells. The main theoretical implication of this result is that the CNN cell represents the primitive for realizing high complex dynamics. The circuit implementation of the introduced system and experimental results referring to the double scroll attractor are reported.**

I. Introduction

Chua's circuit is the simplest autonomous third-order nonlinear electronic circuit with a rich variety of dynamical behaviors including chaos, stochastic resonance, $1/f$ noise spectrum and chaos–chaos intermittency [1]–[3]. Moreover, Chua proved that the unfolded Chua's circuit state equation, which is uniquely determined by 7 parameters, is *topologically conjugate* (i.e., equivalent) to a 21-parameter family of continuous odd-symmetric piecewise-linear equations in $\Re^3$ [2]. From this result Chua's circuit is considered the *canonical* circuit for studying chaos. Besides, in recent papers Chua's circuit arrays have been investigated [1], [3].

The CNN cell is the core of the Cellular Neural Network (CNN) [3]–[6] and, in its classical definition [4], is a simple first-order nonlinear circuit. Chua's idea was to use an array of simple, locally interconnected cells to build impressive analog signal processing systems (CNN).

In this paper the unfolded Chua's circuit (also called Chua's oscillator in the literature [1] for simplicity) has been obtained from an appropriate connection of three generalized CNN cells. This result has some important consequences. The main implication is that the CNN cell represents the building block for generating high complex dynamical behaviors.

In the following sections, some background concepts are first summarized, then the new theoretical results are introduced. Finally, the circuit implementation and experimental results are reported.

II. Background

II.1. The Unfolded Chua's Circuit

The unfolded Chua's circuit (shown in Fig. 1(a)) state equations are [2]:

$$\begin{aligned}\frac{dv_1}{d\tau} &= \frac{1}{C_1}[G(v_2 - v_1) - f(v_1)]\\ \frac{dv_2}{d\tau} &= \frac{1}{C_2}[G(v_1 - v_2) + i_3]\\ \frac{di_3}{d\tau} &= -\frac{1}{L}(v_2 + R_0 i_3)\end{aligned} \tag{1}$$

where

$$f(v_1) = G_b v_1 + 0.5\cdot(G_a - G_b)\cdot\left[|v_1 + E| - |v_1 - E|\right] \tag{2}$$

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The authors are with Dipartimento Elettrico, Elettronico e Sistemistico, Università di Catania, 95125, Catania, Italy.

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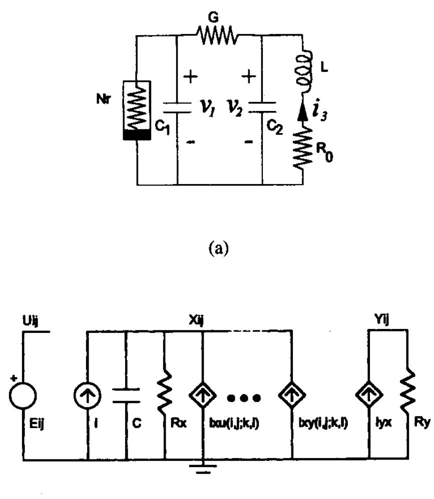

Fig. 1. (a) The unfolded Chua'a circuit. (b) The classical CNN cell.

The nonlinear resistor Nr is the *Chua's diode* and $f(v_1)$ is its characteristic. The difference among the classical Chua's circuit and the unfolded Chua's circuit consists of the resistor R_0, contained in the latter one.

It is often preferred to consider the equivalent dimensionless state equations:

$$\begin{aligned}\dot{x} &= \alpha[y - h(x)]\\ \dot{y} &= x - y + z\\ \dot{z} &= -\beta y - \gamma z\end{aligned} \tag{3}$$

with

$$h(x) = m_1 x + 0.5\cdot(m_0 - m_1)\cdot\left(|x+1| - |x-1|\right) \tag{4}$$

x, y, and z being the state variables and α, β, γ, m_0, m_1 the system parameters. With trivial algebra the relationships among these two representations are found:

$$\begin{aligned}&x = v_1/E;\quad y = v_2/E;\quad z = i_3/(EG);\\ &t = (\tau G)/C_2;\quad m_0 = (G_a/G) + 1;\quad m_1 = (G_b/G) + 1;\\ &\alpha = C_2/C_1;\quad \beta = C_2/(LG^2);\quad \gamma = (C_2 R_0)/(GL);\end{aligned} \tag{5}$$

A zoo of more than 30 strange attractors generated by these equations can be found in [2]. It is nevertheless worth noting that a lot of attractors have been obtained with negative capacitors/inductor.

II.2. The CNN Cell Model

Chua and Yang introduced the Cellular Neural Network (CNN) in 1988 [4]. They defined the CNN cell as the nonlinear first order circuit shown in Fig. 1(b); u_{ij}, y_{ij} and x_{ij} being the input, the output and the state variable of the cell, respectively. The output is related to the state by the nonlinear equation:

$$y_{ij} = 0.5\cdot\left[|x_{ij} + 1| - |x_{ij} - 1|\right] \tag{6}$$

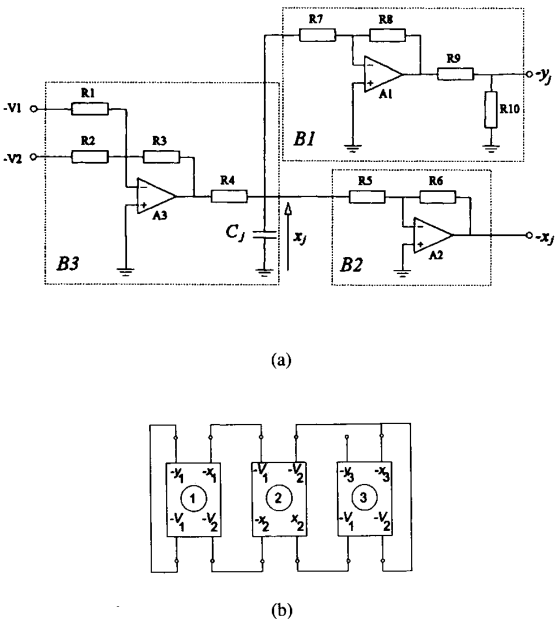

Fig. 2. (a) Generalized cell circuit. (b) Cell connection scheme.

The simplest CNN is defined as a two-dimensional array of $M \times N$ cells. Each cell mutually interacts with its nearest neighbors by means of the voltage-controlled current sources $I_{xy}(i,j;k,l) = A(i,j;k,l)y_{kl}, I_{xu}(i,j;k,l) = B(i,j;k,l)u_{kl}$, where the constant coefficients $A(i,j;k,l)$ and $B(i,j;k,l)$ are known as the *cloning templates*. If the templates are equal for each cell then they are called *space-invariant*; otherwise they are *space-variant*. The CNN is described by the state equations of all cells:

$$\begin{aligned} C\dot{x}_{ij}(t) = -\frac{1}{R_x}x_{ij}(t) + \sum_{c(k,l)\in N_r(i,j)} A(i,j;k,l)y_{kl}(t) \\ + \sum_{c(k,l)\in N_r(i,j)} B(i,j;k,l)u_{kl} + I; \\ 1 \le i \le M, \quad 1 \le j \le N \end{aligned} \tag{7}$$

where

$$\begin{aligned} N_r(i,j) = \{C(k,l) | \max(|k-i|, |l-j|) \le r, \\ 1 \le k \le M; \quad 1 \le l \le N\} \end{aligned} \tag{8}$$

is the *r-neighborhood* and

$$x_{ij}(0) = x_{ij0}; \quad C > 0; \quad R_x > 0. \tag{9}$$

However, as in the case of Chua's circuit it is often preferable to study the dimensionless state equations. After the publication of [4], many generalizations of the previous model have been introduced [3]–[6]. Recently, Chua and Roska formulated a more general definition for CNN's [5] as follows:

The CNN is an n-dimensional array of mainly identical dynamical systems, called cells, which satisfies two properties: a) most interactions are local within a finite radius r, and b) all state variables are continuous valued signals.

III. New Theoretical Results

III.1. The Generalized CNN Cell Model

A new generalized CNN cell model is introduced. It is described by the following dimensionless nonlinear state equation:

$$\dot{x}_j = -x_j + a_j y_j + G_o + G_s + i_j \tag{10}$$

j being the cell index, x_j the state variable, y_j the cell output given as:

$$y_j = 0.5 \cdot (|x_j + 1| - |x_j - 1|) \tag{11}$$

a_j a constant parameter and i_j a threshold value. In (10) the terms G_o and G_s are linear combinations of the outputs and state variables, respectively, of the connected cells being considered. This generalized model fits perfectly the mentioned Chua-Roska definition. Moreover, it differs from the original Chua-Yang definition just by the presence of G_s.

III.2. Main Result

The dynamic model of three fully connected generalized CNN cells in accordance with the state equation, (10), is:

$$\begin{aligned} \dot{x}_1 &= -x_1 + a_1 y_1 + a_{12} y_2 + a_{13} y_3 + \sum_{k=1}^{3} s_{1k} x_k + i_1 \\ \dot{x}_2 &= -x_2 + a_{21} y_1 + a_2 y_2 + a_{23} y_3 + \sum_{k=1}^{3} s_{2k} x_k + i_2 \\ \dot{x}_3 &= -x_3 + a_{31} y_1 + a_{32} y_2 + a_3 y_3 + \sum_{k=1}^{3} s_{3k} x_k + i_3 \end{aligned} \tag{12}$$

where x_1, x_2, and x_3 are the state variables, y_1, y_2, and y_3 the corresponding outputs. If:

$$\begin{aligned} &a_{12} = a_{13} = a_2 = a_{23} = a_{32} = a_3 = a_{21} = a_{31} = 0; \\ &s_{13} = s_{31} = s_{22} = 0; \quad i_1 = i_2 = i_3 = 0; \end{aligned} \tag{13}$$

Equation (12) becomes:

$$\begin{aligned} \dot{x}_1 &= -x_1 + a_1 y_1 + s_{11} x_1 + s_{12} x_2 \\ \dot{x}_2 &= -x_2 + s_{21} x_1 + s_{23} x_3 \\ \dot{x}_3 &= -x_3 + s_{32} x_2 + s_{33} x_3 \end{aligned} \tag{14}$$

Looking at (3) it can be seen how the unfolded Chua's circuit equation is a particular case of (14). In fact, assuming:

$$\begin{aligned} &a_1 = \alpha(m_1 - m_0); \quad s_{33} = 1 - \gamma; \quad s_{21} = s_{23} = 1; \\ &s_{11} = 1 - \alpha \cdot m_1; \quad s_{12} = \alpha; \quad s_{32} = -\beta, \end{aligned} \tag{15}$$

the unfolded Chua's circuit (1) is obtained with x_1, x_2, and x_3, respectively, equal to x, y, and z.

IV. Generalized CNN Cell Design and Implementation

A circuit implementation, inspired by [4] and [6], for the new generalized model is presented. It is shown in Fig. 2(a). It is essentially constituted by three blocks:

1) $B1$ forms the output nonlinearity. It exploits the natural output saturation of the amplifier $A1$; so R_7 and R_8 are chosen such that $A1$ output saturates when $|x_j| > 1$. The subsequent voltage divider (R_9 and R_{10}) is designed to scale the output voltage

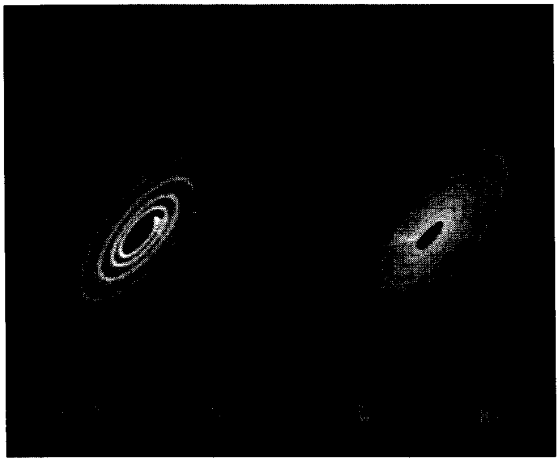

Fig. 3. Observed phase portrait in x_1-x_2 plane

$-y_j$ in the range $[-1, 1]$. Hence the following design equations hold:

$$R_8/R_7 = V_{\text{sat}A}/V_{\text{sat}x} \tag{16a}$$

$$R_7/R_8 = R_{10}/(R_9 + R_{10}) \tag{16b}$$

where $V_{\text{sat}A}$ is the output saturation voltage of $A1$, while $V_{\text{sat}x}$ is its corresponding input voltage (i.e., in our case $V_{\text{sat}x} = 1$). The input and output impedances of $B1$ are R_7 and the parallel of R_9 and R_{10}, respectively.

2) $B2$ is an inverting amplifier with unity gain ($R_5 = R_6$). Its input impedance is R_5 while the output impedance is 0.
3) $B3$ is the core of the cell. If the parallel of the input impedances of $B1$ and $B2$ is very high, compared with the output impedance of block $B3$ (that is $R_4/(1+j\omega R_4 C)$) then blocks $B1$ and $B2$ do not load the capacitor C_j. This is clearly true if $R_7 R_5/(R_7 + R_5) \gg R_4$. In this case, the generalized CNN cell state equation is:

$$C_j \dot{x}_j = -\frac{x_j}{R_4} + \frac{R_3}{R_1 R_4} V_1 + \frac{R_3}{R_2 R_4} V_2 \tag{17}$$

The impedances seen from the two inputs $-V_1$ and $-V_2$ are R_1 and R_2, respectively.

Proposition 1: Let us consider three generalized cells, and represent with x_1, x_2, x_3 each state variable, and with y_1, y_2, y_3 the corresponding outputs. If it is assumed that $V_1 = y_1$ and $V_2 = x_2$ for the first cell, $V_1 = x_1$ and $V_2 = x_3$ for the second one, and $V_1 = -x_2$ and $V_2 = x_3$ for the third one, as shown in Fig. 2(b), then the three connected cell equations are equivalent to system (14). Therefore, they generate Chua's circuit equation.

In fact (with the assumptions in hypothesis) it follows that (14) are the dimensionless versions of (17). By comparison between (14) and (17), the design of $B3$ is straightforward.

V. Experimental Results

A double scroll attractor is observed in Chua's circuit dynamic if $\alpha = 9$, $\beta = 14.286$, $\gamma = 0$, $m_0 = -1/7$ and $m_1 = 2/7$. From expressions (15), the parameter values for the three CNN cells are $a_1 = 3.857$, $s_{33} = s_{21} = s_{23} = 1$, $s_{11} = -1.5714$, $s_{12} = 9$, $s_{32} = -14.286$. In accordance with the previous design considerations, suitable values for the cell components are:

- *cell 1*:
  $R_1 = 13.2$ KΩ; $R_2 = 5.7$ KΩ; $R_3 = 20$ KΩ;
  $R_4 = 390\,\Omega$; $R_5 = 100$ KΩ; $R_6 = 100$ KΩ;
  $R_7 = 74.8$ KΩ; $R_8 = 970$ KΩ; $R_9 = 27$ KΩ;
  $R_{10} = 2.22$ KΩ; $C_1 = 51$ nF;
- *cell 2*:
  $R_1 = R_2 = R_3 = R_5 = R_6 = 100$ KΩ;
  $R_4 = 1$ KΩ; $C_2 = 51$ nF;
- *cell 3*:
  $R_1 = 7.8$ KΩ; $R_2 = R_3 = R_5 = R_6 = 100$ KΩ;
  $R4 = 1$ KΩ; $C_3 = 51$ nF;
- *power supply*: $V_{CC} = +15$ V; $V_{EE} = -15$ V.

Remark: In the previous paragraph the complete scheme of the generalized cell has been introduced. However, from the above considerations (in particular, see (14)), it can be noted that the second and third cell outputs $-y_2$ and $-y_3$ are not actually exploited in order to realize the unfolded Chua's circuit. Therefore, for these cells the nonlinear block is unnecessary. It can be replaced with a noninverting buffer in order to provide the variable x_2 value for the third cell. In Fig. 3, the phase portrait on the $x_1 - x_2$ plane of the experimentally observed attractor for the circuit generated by the three cells is reported. It shows the classical double-scroll attractor on the x-y plane.

VI. Conclusion

In this paper it has been proved that the unfolded Chua's circuit can be realized by three generalized CNN cells. In this way the cornerstone concept inside the CNN cell is emphasized, making it the primitive circuit for generating Chua's circuit; hence, a Chua's circuit array can be realized by a three-layer CNN. Another peculiarity of the proposed realization with respect to the classical one consists of using natural amplifier nonlinearities instead of the nonlinear Chua's resistor. Besides, with traditional realizations, some sets of the unfolded Chua's circuit parameters can be achieved only using negative capacitors/inductor, while this is not necessary with the scheme proposed in which just appropriate choices for positive resistors and, possibly, sign inversion of some cell signals are required.

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