The Use of Identified Models in the Control of a Chaotic Circuit

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Abstract— **A number of control techniques require a model of the system to be controlled. There are many ways of estimating models from data obtained form the system. One way is to fit nonlinear polynomials with carefully chosen structure to the data available. The models obtained in such a way have a number of nice properties such a analytical tractability. This paper investigates the usefulness of such models in control problems in which a model is required. The numerical examples discussed in this paper use both monovariable and multivariable discrete models of Chua's double scroll. The results of the paper suggest that the multivariable model is especially well suited for control problems.**

Keywords— **Chaotic systems, control of chaos, Chua's circuit, nonlinear circuits.**

I. INTRODUCTION

The synchronization and control of chaos has recently attracted much attention [1]. Many control and synchronization methods require a model of the system to be controlled. This is especially true for the so-called "control engineering approach" [2–3]. The difficulty with this assumption is that in many real situations a detailed model of the system is not readily available. In order to apply such methods to real systems of which models are not known, it becomes necessary to learn the dynamics of such systems from reasonable amounts of (possibly) noise-corrupted data. A number of techniques have been developed to learn and predict the dynamics of strange attractors [4–5]. Most of such methods, however, yield models which are either piecewise linear or nonlinear with complicated structure and although the resulting models may perform well in prediction and dynamical reconstruction problems, their applicability to control and synchronization of chaos is unclear.

An alternative way of modeling nonlinear systems is fitting NARMAX (Nonlinear AutoRegressive Moving Average model with eXogenous inputs) models to a set of limited data [6]. This type of models has proved very useful in modeling and reconstructing the dynamics of chaotic systems [7,8].

This paper investigates the use of NARMA models in the control of chaos. In particular the Chua circuit is considered in conjunction with both monovariable and multivariable models of the double scroll attractor [7,9]. The main objective of the paper is therefore to investigate how useful such models are in the control of Chua's circuit. This step is believed to be crucial in practical control problems because it uses models which have been identified form data with no *a priori* knowledge of the system.

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II. PRELIMINARIES

Chua's circuit is certainly one of the most studied nonlinear circuits and a great number of papers ensure that the dynamics of this circuit are also well documented [10]. The normalized equations of Chua's circuit are

$$\begin{cases} \dot{x} = \alpha(y - h(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y \ , \end{cases} \tag{1}$$

where

$$h(x) = \begin{cases} m_1 x + (m_0 - m_1) & x \geq 1 \\ m_0 x & |x| \leq 1 \\ m_1 x - (m_0 - m_1) & x \leq -1 \ . \end{cases} \tag{2}$$

In what follows $m_0 = -1/7$ and $m_1 = 2/7$. Varying the parameters α and β the circuit displays several regular and chaotic regimes. The well known double scroll Chua's attractor, for instance, is obtained for $\alpha = 9$ and $\beta = 100/7$.

III. NARMA MODEL ESTIMATION

A general NARMA model can be represented as [6]

$$y(t) = F^{\ell}[\, y(t-1), \ldots, y(t-n_y), e(t), \ldots, e(t-n_e)\,] \ , \tag{3}$$

where n_y and n_e are the maximum lags considered for the process and noise terms, respectively. $y(t)$ is a time series and $e(t)$ accounts for uncertainties, possible noise, unmodeled dynamics, etc. and $F^{\ell}[\cdot]$ is some nonlinear function of $y(t)$ and $e(t)$ with degree of non linearity $\ell \in \mathbb{Z}^+$. For the models used in this paper, the map $F^{\ell}[\cdot]$ is a polynomial of degree ℓ. The parameter vector can be estimated using standard least-squares algorithms [11]. Moreover, least squares minimization is performed using orthogonal techniques in order to effectively overcome two major difficulties in nonlinear model identification, namely i) numerical ill-conditioning and ii) structure selection [12].

IV. NARMA MODELS OF THE DOUBLE SCROLL ATTRACTOR

In this paper two models of the double scroll attractor are used. The identification and validation of such models have been discussed in some detail in [7,9]. It seems appropriate, however, to include such models here for completeness. Thus, the monovariable model is

$$
\begin{aligned}
z(k) &= 0.17799\times 10\, z(k-1) + 0.46703 z(k-3) z(k-4)^2 \\
&- 0.87039 z(k-3) + 0.91053\times 10^{-1} z(k-1)^2 z(k-4) \\
&+ 0.63361\, z(k-5) + 0.13195\times 10^{-2} z(k-4)^2 z(k-5) \\
&+ 0.57917 z(k-1) z(k-4) z(k-5) - 0.34940\, z(k-4) \\
&- 0.91067\times 10^{-1} z(k-1)^3 + 0.081258\, z(k-1)^2 z(k-2) \\
&- 0.10603\, z(k-5)^3 - 0.16898\, z(k-1)^2 z(k-5) \\
&+ 0.073569\, z(k-2) z(k-5)^2 - 0.012979 z(k-1) z(k-2)^2 \\
&- 0.39490 z(k-1) z(k-5)^2 - 0.031911 z(k-2) z(k-4)^2 \\
&+ 0.022167 z(k-1)^2 z(k-3) + 0.56608 z(k-3) z(k-5)^2 \\
&- 0.78674 z(k-3) z(k-4) z(k-5) - 0.40725 z(k-1) z(k-4)^2 \\
&+ 0.19246 z(k-1) z(k-3) z(k-5) - 0.15206 z(k-3)^2 z(k-5) \\
&+ \Psi_{\xi}^{\mathrm{T}}(k-1)\hat{\Theta}_{\xi} + \xi(k)\ , \qquad (4)
\end{aligned}
$$

and the multivariable model is

$$
\begin{aligned}
x(k) &= 1.1282 x(k-1) + 0.55867\, y(k-1) - 0.04719 x(k-1)^3 \\
&+ 0.039895 y(k-1) z(k-1)^2 - 0.31229\times 10^{-2} z(k-1)^3 \\
&+ 0.018363 z(k-1) + \Psi_{\xi_x\xi_y\xi_z}^{\mathrm{T}}(k-1)\hat{\Theta}_{\xi_x\xi_y\xi_z} + \xi_x(k) \\
y(k) &= 0.91948\, y(k-1) - 0.10392\times 10^{-3} z(k-1)^3 \\
&+ 0.70843\times 10^{-1} x(k-1) + 0.67800\times 10^{-1} z(k-1) \\
&- 0.0013424 x(k-1)^3 + 0.44206\times 10^{-3} x(k-1)^2 y(k-1) \\
&+ \Psi_{\xi_x\xi_y\xi_z}^{\mathrm{T}}(k-1)\hat{\Theta}_{\xi_x\xi_y\xi_z} + \xi_y(k) \\
z(k) &= 0.96628\, z(k-1) - 0.95854\, y(k-1) - 0.036719 x(k-1) \\
&- 0.55765\times 10^{-1} y(k-1)^3 + 0.10333\times 10^{-2} x(k-1)^3 \\
&+ 0.0020536\, x(k-1) y(k-1) z(k-1) \\
&+ \Psi_{\xi_x\xi_y\xi_z}^{\mathrm{T}}(k-1)\hat{\Theta}_{\xi_x\xi_y\xi_z} + \xi_z(k)\ . \qquad (5)
\end{aligned}
$$

It suffices to say that these models reproduce the main dynamical properties of the double scroll such as the largest Lyapunov exponent, correlation dimension, attractor shape, etc. The sampling times used were $T_s = 0.15$ and $T_s = 0.07$, respectively.

V. Derivation of Transfer Functions

One of the main advantages of using NARMA polynomials with few terms is that the resulting models become amenable to analytical handling. For instance, the Jacobian of such models can be readily evaluated at, say, the fixed points. Thus evaluating the Jacobian of model (5) at the nontrivial fixed points (note that these points can be easily and accurately obtained from the identified models [7]) and considering the z component as the output and the control applied to this component as the input, a discrete transfer function was obtained. There are many ways of obtaining a continuous-time model from the discrete counterpart [13]. One approach yielded the following model

$$
G_z(s) = \frac{14.1969 s^2 + 79.6019 s - 77.3048}{s^3 + 5.0778 s^2 + 7.2894 s + 50.9878}\ , \qquad (6)
$$

which has poles at $s = -5.455$ and $s = 0.1886 \pm j3.0515$. These singularities are reasonably close to the eigenvalues of the Jacobian of the original system evaluated at the nontrivial fixed points, $\{-3.9421,\ 0.1854 \pm j3.0470\}$. The fact that the complex eigenvalues are better approximated by the identified model can be explained considering that it is easier to estimate information from the system which lies on the unstable manifold of the fixed-point because it is on such a manifold that the system is mostly excited. In other words, because very little motion takes place on the stable manifold, the stable eigenvalue is 'less observable' than the unstable eigenvalues. Fortunately, this does not seem to be restrictive in control problems in which the aim will be usually to stabilize the fixed points of the system and in such cases the unstable part of the system seems to be the most important anyway.

A similar transfer function for the model in equation (4) was also obtained. In this case, however, the singularities were not retrieved accurately. Two reasons are suggested: i) the monovariable model was estimated from the measurements of just one component and this seems to blur some information, and ii) in order to estimate a model which reproduces the main dynamical features of the attractor, the embedding dimension n_y had to be increased. Consequently, the Jacobian of the model in equation (4) has a pair of spurious eigenvalues in addition to the three 'original' ones. In order to avoid this problem a model was identified from the same set of data but with $n_y = 3$. Such a model was not as accurate as (4) as far as attractor reconstruction is concerned, but did reproduce the original eigenvalues quite well.

VI. Controller Design

When it comes to controlling a system, the number of options is enormous. An overview of methods is not intended here (see [1] for a survey) nor is it claimed that the control techniques used are the best suited in this case. The objective of this section is to verify if the models (4) and (5) convey analytical information which can be usefully employed in designing a controller. Moreover, it is assumed that the utmost aim of the controller is to suppress chaos, that is, to force the circuit to a regular orbit without altering any of its parameters. In particular, two methods were investigated, namely i) proportional control [14] and ii) approximate model matching control [15].

6.1 Proportional control

The effectiveness of the control $u = K(\bar{z} - z)$ applied to the z component in equation (1) was investigated in [14]. In that paper $\bar{z}$ was taken to be an unstable limit cycle embedded in the double scroll. Thus forcing the system to follow such a reference orbit resulted in the suppression of chaos. Here the aim will be to try to stabilize the nontrivial fixed points by means of a controller. It therefore seems meaningful to compare the root loci of the real system and that of equations (4) and (5) as the gain of the controller is varied [2]. In the case of the original system the root locus was obtained plotting the eigengvalues of the Jacobian evaluated at the fixed point which should be stabilized. The root loci of the original system and of the model (5) under the proportional feedback $u = K(\bar{z} - z)$ are shown in Fig. 1a and Fig. 1b, respectively.

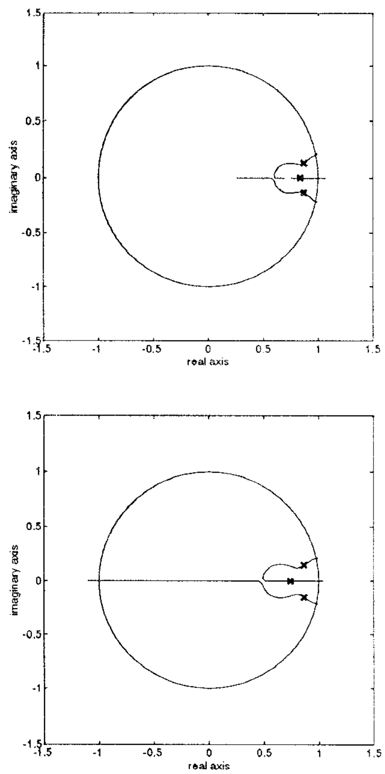

Fig.1 Root loci of (a) Chua's equations and (b) model (5) under proportional feedback.

The root locus of the original equation was mapped unto the z-plane to facilitate comparison. The eigenvalues in this case were mapped using $\lambda_d = e^{\lambda_c T_s}$, where $T_s = 0.07$. It seems appropriate to point out that in order to compare the gains meaningfully, these should also be mapped, by multiplying the continuous-time gains by T_s in order to obtain the discrete-time counterparts.

As figures 1a and 1b make it plainly clear, the NARMA model (5) (via the linearized transfer function $G_z(s)$) can be used to obtain a root locus which is quite similar to the original one. The critical gains of these loci are: *complex pair entering the circle*: $K \approx 0.03 \times 0.07 = 0.02$ (original) and $K \approx 0.02$ (model (5)). *Real pole crossing* $z = 1$: $K \approx 5.70 \times 0.07 = 0.40$ (original) and $K \approx 0.66$ (model (5)). Finally, only the locus of the model (5) crosses $z = -1$ for $K \approx 2.0$. Thus the stability range inferred from Fig. 1b is $0.02 \geq K \geq 0.66$ while the range of the original system seems to be around $0.02 \geq K \geq 0.40$. Although the upper bound on the gain could be rather misleading, in many applications this would not be too grave a problem because the poles are usually placed in a region which is not too close to $z = 1$.

In order to illustrate the design, the gain value $K = 0.18$ was (rather arbitrarily) chosen. The location of the roots for this gain value have been indicated in Figs. 1a e 1b with '**x**'. It should be noted that because the location of such roots in both diagrams is quite similar it seems appropriate to use the identified model in the design. Figure 2, shows two simulations using the original system with the control law $2.5(\bar{z} - z)$ where the value $K \approx 2.5 \approx 0.18/0.07$ is the 'continuous counterpart' of the value chosen from the discrete system with $T_s = 0.07$. In Fig. 2, two different reference orbits were used, namely i) the fixed point $\bar{z} = -1.5767$ around which the Jacobian of model (5) was linearized, and ii) the periodic signal $\bar{z}(t) = \cos(1.77t)$.

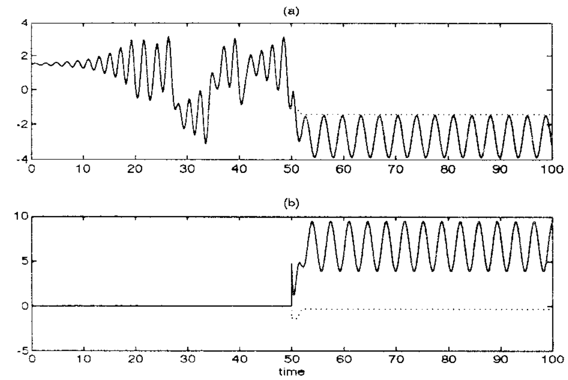

Fig.2 Proportional feedback control. (a) z-component (b) control effort. References $(\cdots)$ $\bar{z} = -1.5767$ (—) $\bar{z}(t) = \cos(1.77t)$.

The root locus obtained using the linearized transfer function of model (4) did not prove helpful. As mentioned before, there are two spurious poles in such a model and these distort the root locus of the controlled system. Moreover, even when third-order models were used to avoid the spurious poles, the resulting root locus did not yield useful information. At this stage we can only conjecture that the difficulty with the monovariable model is that some information is lost due to the inherent difficulty of retrieving dynamical information from a set of measurements in $\mathbb{R}$ of dynamics which exist in $\mathbb{R}^3$.

6.2 Approximate model matching

The objective of this section is to apply linear controller design techniques using $G_z(s)$ and to verify if the resulting controller in fact suppresses chaos or not. In order to do so an approximate model matching algorithm was used [15]. Of course, many other options exist.

Although the design method used is straightforward, it entails the choice of some 'variables' such as the structure of the controller and the reference model used in the design. A detailed description of the design cannot be undertaken here and will be the subject of a future paper. Thus the objective is *not* to describe the procedure but to assess the utility of the linearized model $G_z(s)$ in the design.

The design was carried out in two different ways. In the first example it was not required that the closed-loop should have unitary steady-state gain (K_{DC}), and in the second example $K_{DC} = 1$ was imposed. Thus the following controllers were obtained

$$C_1(s) = \frac{0.6733s + 1.3120}{1.3571s^2 + 3.0033s + 2.1970}\,, \qquad (7)$$

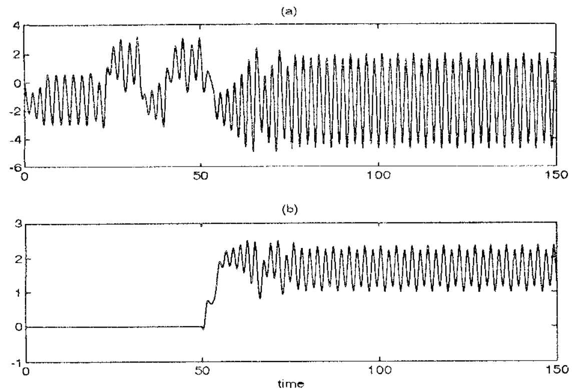

Fig. 3. Using $C_1(s)$, (a) z component (b) control effort.

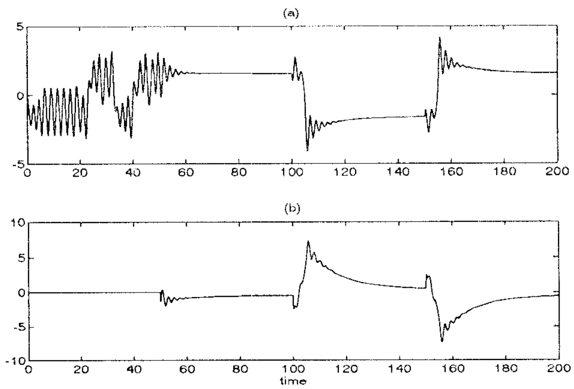

Fig. 4. Using $C_2(s)$, (a) z component (b) control effort.

$$C_2(s) = \frac{-0.7510s^2 + 0.6621s - 6.1324}{s^2 + 15.3540s} \quad (8)$$

Connecting $C_1(s)$ to the system at time $t = 50$ yielded the results shown in Fig. 3. Clearly, the system is stabilized and after transients have died out the system displays a periodic motion. Similarly, $C_2(s)$ was connected to the system at time $t = 50$, and the reference was switched from 1.5767 to -1.5767 at $t = 100$ and back to 1.5767 at $t = 150$. It should be noted that these reference values correspond to the nontrivial fixed points of equation (5) around which this model was linearized to yield $G_z(s)$. The results of this simulation are shown in Fig. 4 which reveals that the system does reach the reference value in steady state. This is probably due to the integrator which appears as a consequence of imposing that the closed-loop should have unitary steady-state gain.

On the other hand, the monovariable model (4) did not prove helpful in controller designs of this type. No stabilizing controller was found. We believe that the reasons for this are as conjectured in Sec. 6.1.

VII. Conclusions

This paper has investigated the use of identified models of Chua's double scroll in the design of control laws to suppress chaos. Monovariable and multivariable models were considered. The latter model proved helpful in a number of situations because such a model actually conveys important dynamical information regarding the original system such as the accurate location and stability information of the fixed points. On the other hand, the monovariable models did not yield satisfactory results when used to stabilize the fixed points of the double scroll. Some reasons for this have been conjectured. However, identified monovariable models can be quite effective in many other methods of synchronization and control of chaos. Two examples of such methods are model-following synchronization problems [16] and when an unstable periodic trajectory is required to synchronize the system [14,17]. In both cases, the monovariable model can be used to provide both the stable and unstable trajectories. In some cases these trajectories can also be obtained directly from the system but at a higher cost [17].

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