Problem 1: Derive the period of oscillation of a pendulum with length $\ell$ as pictured. You will need to use the small angle approximation. That is, since $\theta << 1$, $\cos \theta \approx 1$ and $\sin \theta \approx \theta$. Also, assume that acceleration in the $y$-direction is essentially zero.

Problem 2: Assume you have a clock whose timekeeping depends on a pendulum. What would happen to the clock’s timekeeping ability if your “friend” shortened the pendulum to 1/4 of its original length? What if your “friend” lengthened the pendulum by a factor of 2?

Problem 3: Assume that a particle’s motion can be described by the following expression: $x(t) = A \cos(\omega t)$.

a) Write expressions for the particle’s velocity and acceleration as functions of time.

b) Plot position, velocity, and acceleration of this particle.

Problem 4: Consider a spring-mass system resting on a frictionless table with spring constant $k$ and mass $m$ that is undergoing a simple harmonic oscillation.

a) What is the period of oscillation?

b) When is the spring’s potential energy at a maximum?

c) When is the spring’s kinetic energy at a maximum?

d) Describe what’s going on with energy in this system in a sentence or two.
Problem 5: Circuit analysis time! It’s okay if you know nothing about electromagnetics. The purpose of this exercise is to show you the power of mathematical abstraction. The inductor-capacitor or LC circuit is a well-known example of an electrical oscillator.

a) The sum of the voltages around the circuit must equal zero (assuming no external magnetic field couples to the circuit). This results in the following second order differential equation for the charge stored on the capacitor:

\[
L \ddot{q} + \frac{1}{C} q = 0
\]

\( L \) is the inductance, \( C \) is the capacitance, and \( q \) is the amount of charge stored on the capacitor as a function of time. Solve for \( q(t) \), given that \( q(0) = 1 \) Coulomb, and \( \dot{q}(0) = 0 \) Amperes.

b) Compare the differential equation we derived in class for the spring-mass system with the differential equation describing the LC circuit. Assume that position \( x \) is analogous to charge \( q \). Which circuit element is analogous to mass? Which is analogous to springs?

c) Inductors store energy in the form of magnetic fields and capacitors store energy in the form of electric fields. Can you describe the transfer of energy in this system in a sentence or two?

d) Extra credit! If I added a resistor \( R \), an element that dissipates energy when charge flows through it, how would \( q(t) \) change (describe qualitatively)? What could you add to our familiar spring/mass system that would cause a similar change to \( x(t) \)?