Problem 1: Consider a roller coaster in which cars start from rest at a height $h_0$, and roll down into a valley whose shape is circular with radius $R$, and then up a mountain whose top is also circular with radius $R$, as shown in the figure. Assume the contact between the car and the roller coaster is frictionless. The gravitational constant is $g$. Assume that the wheels of the car run inside a track which follows the path shown in the figure above, so the car is constrained to follow the track.

a) Find an expression the speed of the cars at the bottom of the valley.

b) If the net force on the passengers is equal to $8mg$ at the bottom of the valley, find an expression for the radius $R$ of the arc of a circle that fits the bottom of the valley.

c) The top of the next mountain is an arc of a circle of the same radius $R$. If the normal force between the car and the track is zero at the top of the mountain, what is the height $h_{top}$ of the mountain?

Problem 2: A ball of negligible radius is tied to a string of radius $R$. An engineer whirls the string and stone in a vertical circle. Assume that any non-conservative forces have negligible effect. Show that if the string is to remain taut at the top of the circle, the speed at the bottom of the circle must be at least $\sqrt{5gR}$. 
Problem 3: A object of mass $m$ is pushed against a spring at the bottom of a plane that is inclined at an angle $\theta$ with respect to the horizontal and held in place with a catch. The spring compresses a distance $x_0$ and has spring constant $k$. The catch is released and the object slides up the inclined plane. At $x = 0$ the object detaches from the spring and continues to slide up the inclined plane. Assume that the incline plane has a coefficient of kinetic friction $\mu_k$. How far up the inclined plane does the object move from the point where the object detaches from the spring?

Problem 4: Two children are playing a game, which they try to hit a small box using a spring-loaded marble gun, which is fixed rigidly to a table and projects a marble of mass $m$ horizontally from the edge of the table. The edge of the table is a height $h$ above the top of the box. The spring has a spring constant $k$ and the edge of the box is some unknown horizontal distance $\ell$ away from the table. The first child compresses the spring a distance $x$ and finds that the marble falls short of its target by a horizontal distance $y$. How far should the second child compress the spring in order to land in the box? Let $g$ denote the gravitational acceleration. Express your answer in terms of $k$, $m$, $x$, $g$, $h$, and $y$ as needed but do not use the unknown distance $\ell$. 