Problem 1

\[ a) \quad \begin{array}{c}
\Sigma F_x = F \cos \theta = ma_x \\
\Sigma F_y = N + F \sin \theta - mg = ma_y
\end{array} \]

\[ b) \quad \boxed{a_x = \frac{F \cos \theta}{m}} \quad (1') \]

\[ c) \quad \Delta v_x = a_x \Delta t \quad \Rightarrow \quad \Delta t = \frac{\Delta v_x}{a_x} = \frac{v_f - v_0}{\frac{F \cos \theta}{m}} \quad \text{(starts at rest)} \]

Assuming \( v_0 = 0 \),

\[ t = \frac{mv_f}{F \cos \theta} \quad (2) \]

\[ d) \quad x = v_0 t + \frac{1}{2} a t^2 \]

\[ x = \frac{1}{2} a t^2 - (4) \]

\[ x = \frac{1}{2} \left( \frac{F \cos \theta}{m} \right) \left( \frac{mv_f}{F \cos \theta} \right)^2 = \frac{\frac{1}{2} mv_f^2}{F \cos \theta} \]

\[ x = \frac{\frac{1}{2} mv_f^2}{F \cos \theta} \]

(If you remember work-energy, this makes sense:)

\[ W = \Delta KE \]

\[ \int F \cdot dr = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \]

\[ FX \cos \theta = \frac{1}{2} mv_f^2 \]
c) The block's vertical acceleration is zero, as defined by the problem.

f) \[ F_y = N + F \sin \alpha - mg = 0 \]
\[ \Rightarrow N = mg - F \sin \alpha \]

9) Correction: "For what angle \( \alpha \) will the magnitudes of \( \vec{N} \) and \( mg \) equal?"

\[ N = mg - F \sin \alpha \]
Requirement that
\[ N = mg \]
\[ \Rightarrow 0 = -F \sin \alpha \]
\[ \Rightarrow \alpha = \pi, \pi \pi \]
i.e., the force must be horizontal

Problem 2

\[ \begin{array}{c}
\text{a) } \begin{array}{c}
\text{X-comp:} \\
F \cos \alpha - F_{\text{fric}} = m a_x \end{array} \\
\text{Y-comp:} \\
N + F \sin \alpha - mg = 0 \\
\text{mg} \\
\text{N} \\
\text{Frict} \\
\text{Frict} \\
\text{x-comp} \\
\text{y-comp} \\
\end{array} \\
\begin{array}{c}
\text{b) } \begin{array}{c}
F \cos \alpha - F_{\text{fric}} = m a_x \\
F \cos \alpha - m a_x = m a_x \\
\Rightarrow N + F \sin \alpha - mg = 0 \Rightarrow N = mg - F \sin \alpha \\
F \cos \alpha - M a_x (mg - F \sin \alpha) = m a_x \\
\Rightarrow a_x = F \left[ \cos \alpha - M a_x \sin \alpha \right] - M a_x \end{array} \\
c, d) \text{Same as #1, but plug in new } a_x. \text{ (for d, plug in } t \text{ from c)} \\
e - g) \text{Same as #1 (Horizontal components are independent of vertical)}
\end{array} \]
Problem 3

There are two unknowns here ($T_1$, $T_2$). We therefore must consider two systems. We have 2 options for this:

1. System 1 is $m_b$, System 2 is $m_a$.

   \[ \Sigma F_y = T_1 - m_b g = 0 \]
   \[ \Rightarrow T_1 = m_b g \]

   \[ \Sigma F_y = T_2 - T_1 - m_a g = 0 \]
   \[ \Rightarrow T_2 = T_1 + m_a g \]

2. System 1 is $m_b$, System 2 is both masses.

   Note: We do not consider $T_1$ for system 2 because this action-reaction pair is now an internal force to system 2. Only external forces are considered on a system whose mass is not changing.

   System 1:
   \[ \Sigma F_y = T_1 - m_b g = 0 \]
   \[ \Rightarrow T_1 = m_b g \]

   System 2:
   \[ \Sigma F_y = T_2 - (m_a + m_b) g = 0 \]
   \[ \Rightarrow T_2 = (m_a + m_b) g \]

b) Now, instead of being in static equilibrium, the net force results in an acceleration.

   System 1:
   \[ \Sigma F_y = T_1 - m_b g = M_b a \]
   \[ \Rightarrow T_1 = m_b (g + a) \]

   System 2:
   \[ \Sigma F_y = T_2 - (m_a + m_b) g = (m_a + m_b) a \]
   \[ \Rightarrow T_2 = (m_a + m_b) (g + a) \]
Problem 4

a) \[ y = - \tan \theta \]

b) \[ \text{x-component}\]
   \[ T \sin \theta = ma \]
   \[ \text{y-component}\]
   \[ T \cos \theta - mg = 0 \]
   \[ \Rightarrow T \cos \theta = mg \quad (2') \]

\[ t \tan \theta = \frac{Vh}{ymg} \]

\[ \tan \theta = \frac{q}{g} \quad \Rightarrow \theta = \tan^{-1} \frac{q}{g} \]

d) \[ a = g \tan \theta \]

\[ V = V_i + at \]

\[ V_f = a \cdot t_d = g \cdot t \cdot tan \theta \]

\[ = 5.66 \frac{m}{s} \]

e) \[ a = g \tan \theta \]

\[ \frac{1}{T} \]

constant easy to measure.

One easy measurement and calculation allows us to determine acceleration.

Problem 5

a) Because the string attaches the blocks, we can use the same trick as #1 to make \( T \) an internal force.

b) \[ \text{x-component}\]

\[ -F_{\text{fric}} + m_b g = (m_a + m_b) a \]

\[ -m_a N + m_b g = (m_a + m_b) a \]

\[ -m_b mg + m_a g = (m_a + m_b) a \]

\[ \Rightarrow a = g \frac{(m_b - m_a) / (m_a + m_b)}{1} \]
Now, we must transform our problem back to the original.

\[ \vec{a}_a = \frac{g(m_b - m_b m_a)}{m_a + m_b} \hat{x} \]
\[ \vec{a}_b = \frac{-g(m_b - m_b m_a)}{m_a + m_b} \hat{y} \]

**Problem 6:**

a) [Diagram of a system with forces and acceleration components]

b) Acceleration down the slope, i.e., in the \( x' \)-direction.

\[ x' \)-direction
\[ m_g \sin \theta = m_a \]
\[ \Rightarrow a_{x'} = \frac{g \sin \theta}{m_a} \]

Note that this acceleration is regardless of mass.

**Problem 7:**

Consider both:

a) [Diagram of a system with forces and acceleration components]

\[ \Sigma F_x = F = (M+m)a \]
\[ \Rightarrow a = \frac{F}{M+m} \]

The horizontal acceleration of the block must match this for the block to stay.

Consider the block only:

\[ \Sigma F_x = N \sin \theta = ma \]
\[ \Sigma F_y = N \cos \theta - mg = 0 \]
\[ \Rightarrow N = \frac{mg}{\cos \theta} \]

\[ \Rightarrow a = g \tan \theta \]
\[ F = (M+m)g \tan \theta \]
b) Now add friction:

We want the values of $F$. This will be a range, because friction wants to keep the block in place.

\[ \Sigma F_x = N \sin \theta \pm \mu_s N \cos \theta = m a_x \]

\[ \Sigma F_y = N \cos \theta = \mu_s N \sin \theta - mg = 0 \]

\[ N (\sin \theta \pm \mu_s \cos \theta) = m \frac{F}{m + m} \]

\[ \Rightarrow N = \frac{mg}{\cos \theta \pm \mu_s \sin \theta} \]

\[ mg \left( \frac{\sin \theta \pm \mu_s \cos \theta}{\cos \theta \pm \mu_s \sin \theta} \right) = m \frac{F}{m + m} \]

\[ F = (m + m) \left( \frac{\sin \theta \pm \mu_s \cos \theta}{\cos \theta \pm \mu_s \sin \theta} \right) \]

This gives the max. and min. values. Everything in between keeps the block on the incline.

**Problem 8:**

a) \[ F(t) = \cos(2 \pi t) \hat{x} + 2 \sin(2 \pi t) \hat{y} \]

b) \[ \vec{V}(t) = \vec{F}(t) = -2 \pi \sin(2 \pi t) \hat{x} + 4 \pi \cos(2 \pi t) \hat{y} \]

c) \[ \vec{a}(t) = \vec{V}(t) = -4 \pi^2 \cos(2 \pi t) \hat{x} - 8 \pi^2 \sin(2 \pi t) \hat{y} \]

d) \[ F(t) = m \cdot \vec{a}(t) = m \left( \vec{v} \right) \]

e) The force is not centripetal because it has a tangential component for all $t$ but $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, ...$