HW #4
Due October 23 (Tuesday) in class

1. The homework problem is an extension of the class lecture on the DC characteristics of semiconductor lasers for both below and above threshold. When we derive the light-versus-current (L-I) characteristics, we ignore the spontaneous emission term. Here, we show that the laser output is basically amplified spontaneous emission, and that when we consider the spontaneous emission term, the carrier concentration is very close to, but never reaches, the threshold carrier concentration derived in class. The L-I curve at threshold increases smoothly rather than abruptly.
The rate equations we discussed in class are:
$$\frac{dN}{dt} = \eta_i \frac{I}{qV_{active}} - \frac{N}{\tau} - \frac{c}{n_r} g(N)\cdot S$$
$$\frac{dS}{dt} = \frac{c}{n_r}\Gamma \cdot g(N)\cdot S - \frac{S}{\tau_p} + \beta \cdot R_{sp}$$
where
N is carrier concentration, S is the photon density, both in $1/\mathrm{m}^3$;
V_{active} is the volume of the active layer (i.e., width x length x thickness);
τ is the total carrier lifetime, $\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$, τ_r is radiative recombination lifetime, and τ_{nr} is nonradiative recombination lifetime;
$\frac{c}{n_g}$ is the speed of light in semiconductor;
$g(N) = a\cdot(N - N_{tr})$ is the gain coefficient. Here, we use linear gain approximation.
Γ is the optical confinement factor;
$\tau_p = \dfrac{1}{\frac{c}{n_r}(\alpha_m + \alpha_i)}$ is the photon lifetime;
β is the spontaneous emission factor, i.e., the fraction of the spontaneous emission coupled to the waveguide mode; and
$R_{sp} = \frac{N}{\tau_r} = \eta_r \frac{N}{\tau}$ is the spontaneous rate.

   a. In steady state, show that $S(N) = \dfrac{\beta \cdot R_{sp}(N)}{\frac{1}{\tau_p} - \frac{c}{n_r}\Gamma \cdot g(N)}$ for both below threshold and above threshold.
   b. In steady state, show that $I(N) = \dfrac{qV_{active}}{\eta_i}\left(\frac{N}{\tau} + \frac{c}{n_r} g(N)\cdot S(N)\right)$ for both below and above threshold.

c. Show that the output power $P(N) = \hbar\omega \cdot \frac{V_{active}}{\Gamma} \alpha_m \frac{c}{n_r} \cdot S(N)$.

d. Since both output power *P(N)* and current *I(N)* are both functions of *N*, we can plot the light-versus-current (L-I) curve using *N* as a parameter. Use the following parameters for the plot:

$R_1 = R_2 = 30\%, N_{tr} = 10^{24}\ \text{1/m}^3, a = 10^{-20}\ \text{m}^2, \Gamma = 50\%, \alpha_i = 1000\ \text{m}^{-1}$

$L = 300\ \mu\text{m},\ w = 1\ \mu\text{m},\ \text{t} = 0.1\ \mu\text{m},\ \tau = 1\ \text{nsec}$

$\eta_\text{i} = 100\%, \eta_r = 90\%$

$\beta = 10^{-3}$

$\lambda = 1550\ \text{nm},\ n_r = 3.5$ (effective refractive index)

Plot both L-I curve (i.e., *P(N)* versus *I(N)*). Choose the range of *N* such that the *I(N)* goes from 0 to 3 I_{th}.

**** **Please note that the $N < N_{th}$ even above threshold, though it is getting closer and closer to N_{th} above threshold. To show clearly the behavior around threshold, you need to use a very fine interval for *N*, or alternatively, you can use non-uniform intervals with much denser points near N_{th}.**

**** You need to take sufficient number of data points. So using Matlab or other math solver is recommended.

e. Plot the carrier concentration *N* as a function of current *I* for the same range of *I* as in part d.

f. Plot *L-I* and *N-I* again using logarithmic scale for the vertical axes.

2. To show effect of β on the *L-I* characteristics, plot *L-I* for $\beta = 10^{-2}$, 10^{-3}, and 10^{-4} in the same graph. Choose the ranges of *N* such that *I(N)* goes from 0 to 3 I_{th}. Likewise, plot *N-I* curve for the three β values. Show your plots in log-linear and linear-linear plots for both families of curves.