EE 232 Lightwave Devices **Miterm-2 Solutions**
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Commonly used constants

$h\_bar := 1.05459 \cdot 10^{-34}$ (J-sec) $q := 1.6 \cdot 10^{-19}$ (Coul)

$m0 := 9.11 \cdot 10^{-31}$ (kg) $c := 3 \cdot 10^{8}$ (m/sec)

$nm := 10^{-9}$ (m) $\varepsilon 0 := 8.854 \cdot 10^{-12}$ (F/m)

$\mu m := 10^{-6}$ (m) $eV := 1.6 \cdot 10^{-19}$ (Joul)

Global Parameters:

$Ep := 24 \cdot eV$ $Ep = 3.84 \times 10^{-18}$

$Mb2 := \frac{m0}{6} \cdot Ep$ $Mb2 = 5.83 \times 10^{-49}$ ($kg^2\ m^2 / sec^2$)

$Eg := 1 \cdot eV$ $nr := 3$ $\varepsilon r := nr^2$ $\varepsilon r = 9$

1.(a)

$me := 0.1 \cdot m0$ $mh := m0$ $eV = 1.6 \times 10^{-19}$ $Lz := 10nm$ $Lz = 1 \times 10^{-8}$

$$Ee(n) := \frac{h\_bar^2}{2 \cdot me} \cdot \left[\left(\frac{\pi}{Lz}\right)^2 \cdot n^2\right] \qquad \frac{Ee(2) - Ee(1)}{eV} = 0.113 \text{ [eV]}$$

$$Eh(m) := -\frac{h\_bar^2}{2 \cdot mh} \cdot \left(\frac{\pi}{Lz}\right)^2 \cdot m^2 \qquad \frac{Eh(1) - Eh(2)}{eV} = 0.011 \text{ [eV]}$$

$Eg := 1eV$

$$E := Eg + Ee(1) - Eh(1) \qquad \frac{E}{eV} = 1.041 \text{ [eV]}$$

(b) $\mu 21 := \frac{-16}{9 \cdot \pi^2} \cdot q \cdot Lz$

Maximum absorption is achieved when the Fermi energy starts to cross Eh2

$$\rho h\_2d := \frac{mh}{\pi \cdot h\_bar^2 \cdot Lz}$$

$P := \rho h\_2d \cdot (Eh(1) - Eh(2))$ $P = 4.712 \times 10^{24}$ [m^{-3}]

$\omega := \frac{Eh(1) - Eh(2)}{h\_bar}$ $\Gamma := 10^{-2} \cdot eV$

$$\alpha\_peak := \frac{\omega}{nr \cdot c \cdot \varepsilon 0} \cdot \frac{(|\mu 21|)^2}{\frac{\Gamma}{2}} \cdot P \qquad \alpha\_peak = 1.052 \times 10^{6} \text{ [m}^{-1}\text{]}$$

(c) $\omega := \dfrac{Ee(1) - Eh(1) + Eg}{h\_bar}$ $Ep := 24 \cdot eV$

$C0 := \dfrac{\pi \cdot q^2}{nr \cdot c \cdot \varepsilon 0 \cdot m0^2 \cdot \omega}$ $mr := \left(\dfrac{1}{me} + \dfrac{1}{mh}\right)^{-1}$

$\rho r\_2d := \dfrac{mr}{\pi \cdot h\_bar^2 \cdot Lz}$ $Mb2 := \dfrac{m0}{6} \cdot Ep$

$gm := C0 \cdot Mb2 \cdot \rho r\_2d$ $gm = 1.064 \times 10^6$ $[m^{-1}]$

Maximum absorption corresponding to Eh1 --> Ee1 is achieved when the separation of quasi Fermi levels, or bias voltage, is smaller than Ee1-Eh1+Eg = 1.041 V

2.

(1) E_{h2} → E_{h1} : intersubband, μ_{21} is along z direction ➔ TM

(2) E_{h2} → E_{e1} : interband transition. Overlap integral of QW envelop function $I_{h2}^{e1} = 0$ ➔ transition is forbidden

(3) E_{h2} → E_{e2} : interband. $I_{h2}^{e2} = 1$, allowed transition. Assume HH band ➔ TE

(4) E_{h1} → E_{e1} : interband. $I_{h1}^{e1} = 1$, allowed transition. Assume HH band ➔ TE

(5) E_{h1} → E_{e2} : interband transition. Since $I_{h1}^{e2} = 0$ ➔ transition is forbidden

(6) E_{e1} → E_{e2} : intersubband, μ_{21} is along z direction ➔ TM

(a) Surface illumination → TE polarization only.
  (1) and (6) matrix element = 0 because intersubband responds to TM only.
  (2) and (5) matrix element = 0 because the transitions are forbidden

(b) TM polarization
  (2) and (5) matrix element = 0 because the transitions are forbidden
  (3) and (4) matrix element = 0 if HH band is assumed (the matrix element of light hold band is not zero)

3 (a)

$n\_clad := 3$ $n\_core := 4$ $d := 0.1\mu m$

$\lambda := 1\mu m$ $V := \dfrac{2 \cdot \pi}{\lambda} \cdot d \cdot \sqrt{n\_core^2 - n\_clad^2}$ $V = 1.662$

V < π --> single mode

(b) $\Gamma := \dfrac{V^2}{2 + V^2}$ $\Gamma = 0.58$

(c) $Lz := 10nm$ $\Gamma\_QW := \Gamma \cdot \dfrac{Lz}{d}$ $\Gamma\_QW = 0.058$

4 $R := 30\%$ $R1 := R$ $R2 := R$ $a := 10^{-16} \cdot 10^{-4}$ $N\_tr := 10^{18} \cdot 10^{6}$

$\lambda p := 1.24 \cdot \mu m$ $\Gamma := 1\%$ $\alpha i := 1 \cdot 100$ $Lz := 10 \cdot nm$ $Lz = 1 \times 10^{-8}$

$f := \frac{c}{\lambda p}$ $f = 2.419 \times 10^{14}$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.52 \times 10^{15}$

$C0 := \frac{\pi \cdot q^2}{nr \cdot c \cdot \varepsilon 0 \cdot m0^2 \cdot \omega}$ $C0 = 8 \times 10^{9}$

$\alpha m(L) := \frac{1}{2 \cdot L} \cdot \ln\left(\frac{1}{R1 \cdot R2}\right)$ $\alpha i := 1 \cdot 100$

(a) $m\_e := 0.1 \cdot m0$ $m\_h := 0.2 \cdot m0$

$m\_r := \frac{m\_e \cdot m\_h}{m\_e + m\_h}$ $m\_r = 6.073 \times 10^{-32}$

$\rho r\_2D := \frac{m\_r}{\pi \cdot h\_bar^2 \cdot Lz}$ $\rho r\_2D = 1.738 \times 10^{44}$

$Ep := 24 \cdot eV$ $Ep = 3.84 \times 10^{-18}$

$Mb2 := \frac{m0}{6} \cdot Ep$ $Mb2 = 5.83 \times 10^{-49}$ (kg^2 m^2 / sec^2)

Tensile strained --> TM polarized

$M2\_TM := 2 \cdot Mb2$

$g\_m := C0 \cdot \rho r\_2D \cdot M2\_TM$ $g\_m = 1.622 \times 10^{6}$ m^{-1}

(b) $\alpha m\_max := \Gamma \cdot g\_m - \alpha i$ $\alpha m\_max = 1.612 \times 10^{4}$

$\alpha m(50 \cdot \mu m) = 2.408 \times 10^{4}$

$L := 100 \cdot \mu m$ Given

$\alpha m(L) = \alpha m\_max$

$L\_min := Find(L)$ $\frac{L\_min}{\mu m} = 74.709$ [μm]

(c) Number of QW = N

Modal gain for N QW = N * Γ * g_QW

$gth(N) := \frac{\alpha m(10\mu m) + \alpha i}{N \cdot \Gamma}$

Maximum N such that gth(N) < gm

$\frac{gth(1)}{gm} = 11.328$

Minimum number of QW = 12