The "Monty Hall" Problem

Mr. Monty Hall was the master of ceremonies for a TV game show during which a contestant was offered a choice of one from three closed doors. Behind one door was a car the contestant would win if this door were chosen. Behind each of the other two doors was a goat. After the contestant had chosen a door but before it was opened, Monty Hall would open one of the other two doors to reveal a goat, and the contestant would then be allowed again to choose a door, now one from two closed doors. The contestant could adhere to the original choice, or reconsider and choose the other door. Which choice was more likely to win a car?

At first sight, most people think that the car was as likely to be behind either door as the other, so a change of choice did not alter the contestant's chances of winning the car. This is wrong. To see why, let us suppose that, out a very large number of contestants, K chose the car's door at first, M chose one goat's door at first, and N chose the other goat's door at first. If the doors seemed sufficiently indistinguishable that each was as likely as either of the others to be a contestant's first choice, we would expect $K \approx M \approx N$. After Monty Hall opened one of the goats' doors, some contestants changed their choices; suppose k out of K , m out of M and n out of N changed their choices. Since every contestant belonged to one of the three groups of K , M or N but did not yet know which, we may reasonably expect roughly equal fraction(s) $k/K \approx m/M \approx n/N$.

$\gamma := (0 + m + n)/(k + m + n)$ is the fraction of contestants who changed their choice and won a car. The fraction of those who didn't change but won a car is $\pi := (K - k)/(K - k + M - m + N - n)$. If, as we expect, count(s) $K \approx M \approx N$ and fraction(s) $k/K \approx m/M \approx n/N$, regardless of what value the fraction(s) may have, we find that $\gamma \approx 2/3$ and $\pi \approx 1/3$. In other words, …

Choosing the other door roughly doubled the likelihood of winning a car.

This seems counter-intuitive only to those who disregard Monty Hall's intervention in the game; he knew which door concealed a car. The contestant knew only that the door chosen first was twice as likely to be a goat's as a car's, *and this ratio persisted after Monty Hall opened a goat's door.* Of the two doors available for a second choice, the one not chosen first was twice as likely to conceal a car. These two doors would present equally attractive choices only to contestants who forgot their first choice. History deserves respect.