1. Prove that we can assume without loss of generality that all amplitudes in a quantum computation are real numbers, by showing how to simulate a quantum circuit consisting of CNOT, Hadamard and say $\pi/8$-phase gate (which maps $|1\rangle$ to $e^{i\pi/8}|1\rangle$) by a quantum circuit which is only constant factor larger, with gates of your choice and such that all amplitudes in the simulating circuit are real.

2. Consider a bipartite quantum state $|\psi\rangle \in \mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B}$. Show that if A performs an arbitrary unitary operation on her part of $|\psi\rangle$ and then B measures his qubits in the standard basis then the result of B's measurement is independent of A's actions.

   Conclude that there is no measurement that B can perform to tell which unitary operation A performed on her qubits.

3. Suppose Alice and Bob share two Bell states. Can Alice use teleportation to send an arbitrary quantum state on two qubits to Bob? How many classical bits does she need to send to Bob?

   Now suppose that Alice and Bob share n Bell states. Can Alice teleport an arbitrary quantum state on n qubits to Bob? How would this work if the n qubits are highly entangled?

4. Recall that the bias of a coin can be estimated to within ϵ with confidence $1 - \delta$ by flipping it $O(\frac{log 1/\delta}{\epsilon^2}$ times and taking the sample mean as the estimate. This can be used to show that the error probability of a BPP machine can be reduced to inverse exponential in n at a cost of a polynomial factor increase in running time. Prove that the same kind of boosting of the probability of correctness can be achieved also for a BQP machine.

5. Consider the following quantum circuit on n qubits: it consists of a Hadamard transform on the n qubits, followed by another Hadamard transform. Follow the proof of $BQP \subseteq P^{\#P}$ on this example: let the input be a basis state input $|x\rangle$; for each possible output state $|y\rangle$, what is the total contribution to $|y\rangle$ by computational paths with positive amplitudes, and what is the total negative contribution?