C280, Computer Vision

Prof. Trevor Darrell

trevor@eecs.berkeley.edu

Lecture 2: Image Formation

Administrivia

- We're now in 405 Soda...
- New office hours: Thurs. 5-6pm, 413 Soda.
- I'll decide on waitlist decisions tomorrow.
- Any Matlab issues yet?
- Roster...

Physical parameters of image formation

- Geometric
 - Type of projection
 - Camera pose
- Optical
 - Sensor's lens type
 - focal length, field of view, aperture
- Photometric
 - Type, direction, intensity of light reaching sensor
 - Surfaces' reflectance properties
- Sensor
 - sampling, etc.

Physical parameters of image formation

- Geometric
 - Type of projection
 - Camera pose
- Optical
 - Sensor's lens type
 - focal length, field of view, aperture
- Photometric
 - Type, direction, intensity of light reaching sensor
 - Surfaces' reflectance properties
- Sensor
 - sampling, etc.

Perspective and art

- Use of correct perspective projection indicated in 1st century B.C. frescoes
- Skill resurfaces in Renaissance: artists develop systematic methods to determine perspective projection (around 1480-1515)



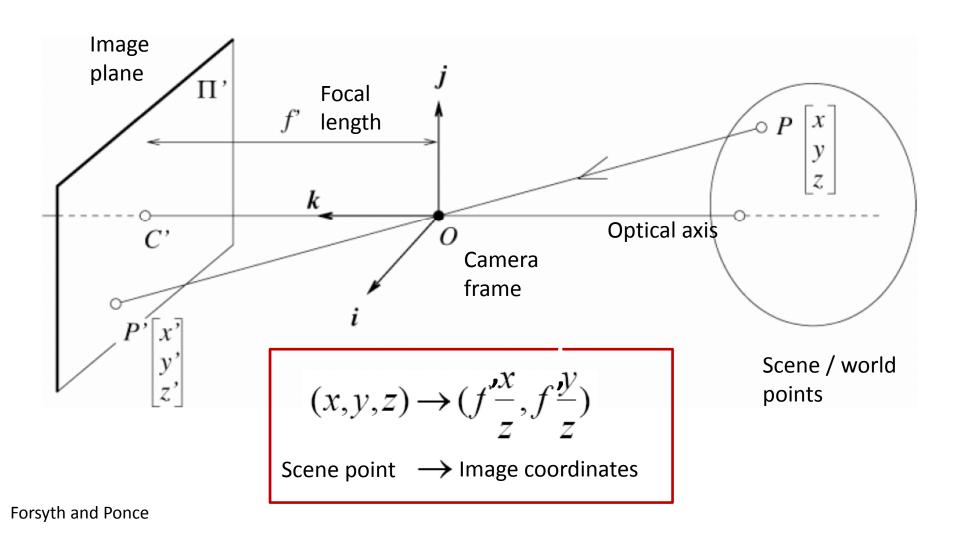
Raphael



Durer, 1525

Perspective projection equations

3d world mapped to 2d projection in image plane



Homogeneous coordinates

Is this a linear transformation?

no—division by z is nonlinear

Trick: add one more coordinate:

$$(x,y) \Rightarrow \left[egin{array}{c} x \\ y \\ 1 \end{array} \right]$$

homogeneous image coordinates

$$(x,y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
 $(x,y,z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

homogeneous scene coordinates

Converting from homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection Matrix

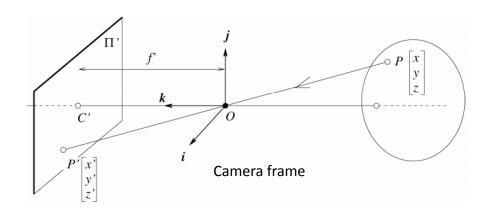
Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f' & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f' \end{bmatrix} \Rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$
divide by the third coordinate to convert back to non-homogeneous coordinates

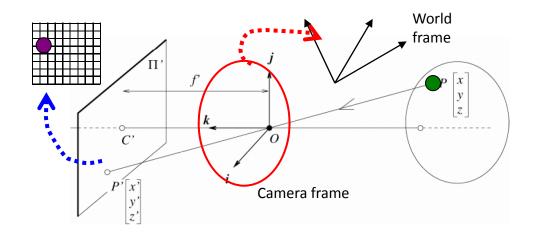
Complete mapping from world points to image pixel positions?

Perspective projection & calibration

- Perspective equations so far in terms of *camera's* reference frame....
- Camera's intrinsic and extrinsic parameters needed to calibrate geometry.



Perspective projection & calibration



Extrinsic:

Camera frame ←→World frame

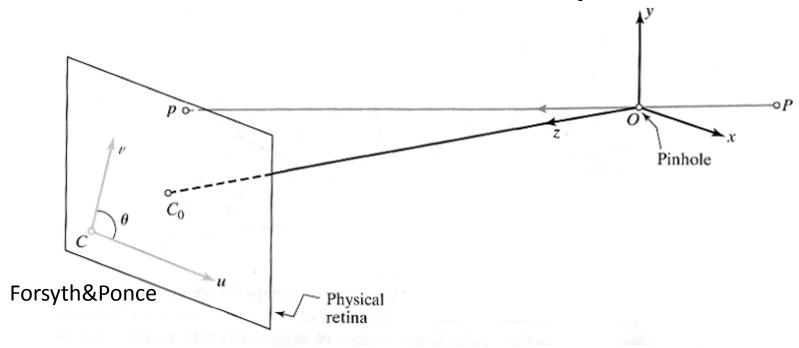
Intrinsic:

Image coordinates relative to camera

←→ Pixel coordinates

3D point (4x1)

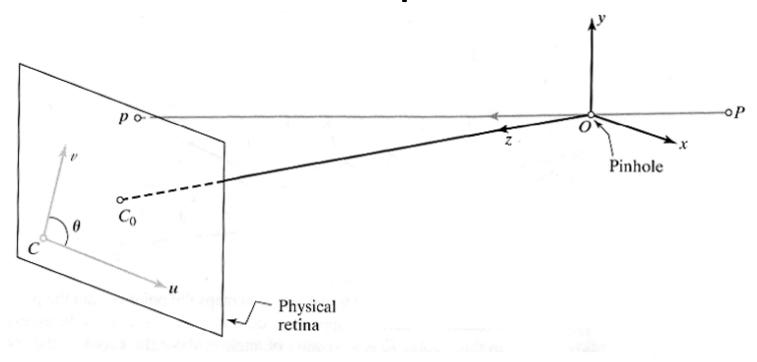
Intrinsic parameters: from idealized world coordinates to pixel values



Perspective projection

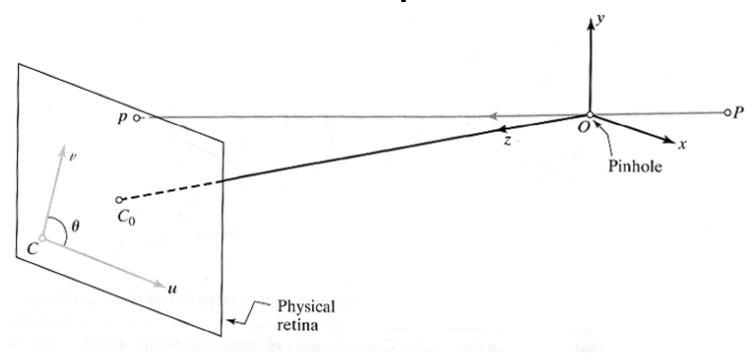
$$u = f \frac{x}{z}$$

$$v = f \frac{y}{z}$$



But "pixels" are in some arbitrary spatial units

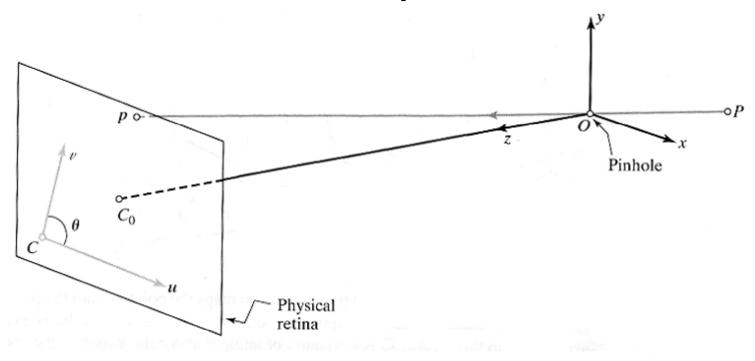
$$u = \alpha \frac{x}{z}$$
$$v = \alpha \frac{y}{z}$$



Maybe pixels are not square

$$u = \alpha \frac{x}{z}$$

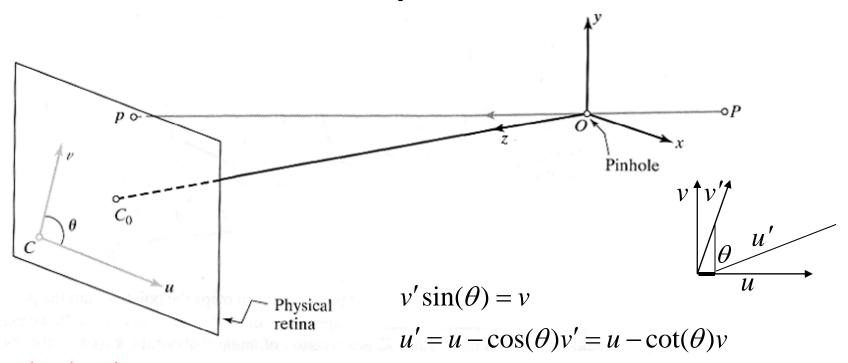
$$v = \beta \frac{y}{z}$$



We don't know the origin of our camera pixel coordinates

$$u = \alpha \frac{x}{z} + u_0$$

$$v = \beta \frac{y}{z} + v_0$$

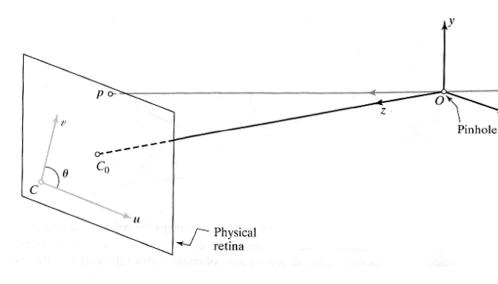


May be skew between camera pixel axes

$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$

$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$

Intrinsic parameters, homogeneous coordinates



$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$
$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$

Using homogenous coordinates,

we can write this as:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha \cot(\theta) & u_0 & 0 \\ 0 & \frac{\beta}{\sin(\theta)} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

or:

In pixels
$$\rightarrow$$
 $\vec{p} = \mathbf{K}$

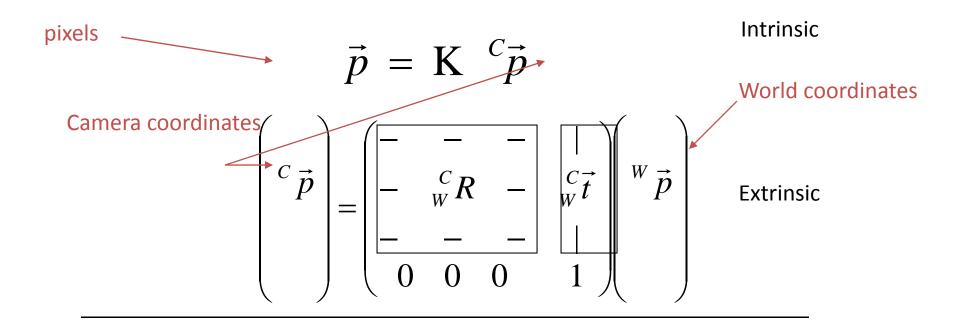
Extrinsic parameters: translation and rotation of camera frame

$$\vec{p} = {}^{C}_{W}R \quad \vec{p} + {}^{C}_{W}\vec{t}$$

Non-homogeneous coordinates

Homogeneous coordinates

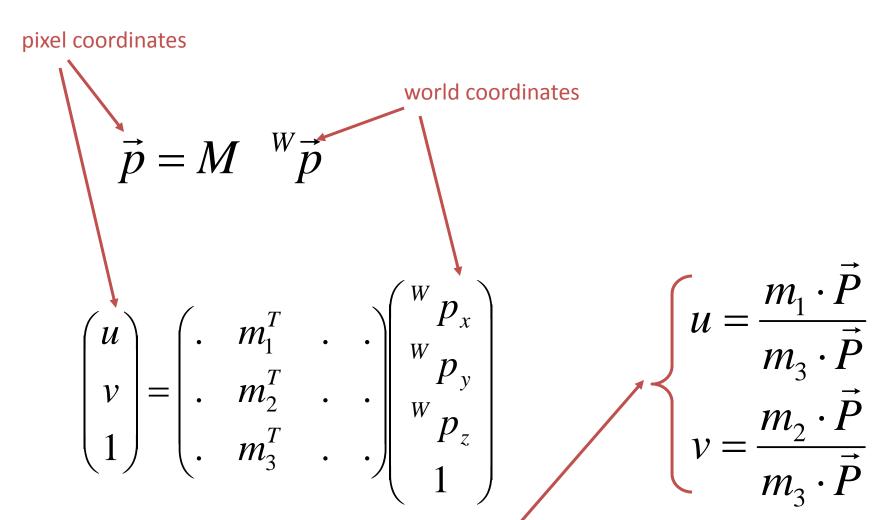
Combining extrinsic and intrinsic calibration parameters, in homogeneous coordinates



$$\vec{p} = K \left(\begin{pmatrix} C & C & C \\ W & 0 & 0 & 0 \end{pmatrix} \right) \quad \vec{p}$$

$$\vec{p} = M \quad \vec{p}$$

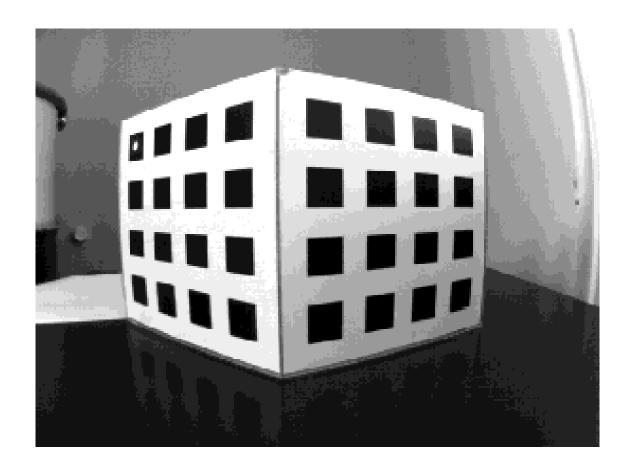
Other ways to write the same equation



Conversion back from homogeneous coordinates

leads to:

Calibration target



The Opti-CAL Calibration Target Image

Find the position, u_i and v_i , in pixels, of each calibration object feature point.

http://www.kinetic.bc.ca/CompVision/opti-CAL.html

From before, we had these equations relating image positions,

u,v, to points at 3-d positions P (in homogeneous coordinates):

$$u = \frac{m_1 \cdot \vec{P}}{m_3 \cdot \vec{P}}$$
$$v = \frac{m_2 \cdot \vec{P}}{m_3 \cdot \vec{P}}$$

So for each feature point, i, we have:

$$(m_1 - u_i m_3) \cdot \vec{P}_i = 0$$

$$(m_2 - v_i m_3) \cdot \vec{P}_i = 0$$

Stack all these measurements of i=1...n points

$$(m_1 - u_i m_3) \cdot \vec{P}_i = 0$$
$$(m_2 - v_i m_3) \cdot \vec{P}_i = 0$$

into a big matrix:

$$\begin{pmatrix}
P_1^T & 0^T & -u_1 P_1^T \\
0^T & P_1^T & -v_1 P_1^T \\
\cdots & \cdots \\
P_n^T & 0^T & -u_n P_n^T \\
0^T & P_n^T & -v_n P_n^T
\end{pmatrix}
\begin{pmatrix}
m_1 \\
m_2 \\
m_3
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
\vdots \\
0 \\
0
\end{pmatrix}$$

In vector form:
$$\begin{pmatrix} P_{1}^{T} & 0^{T} & -u_{1}P_{1}^{T} \\ 0^{T} & P_{1}^{T} & -v_{1}P_{1}^{T} \\ \cdots & \cdots \\ P_{n}^{T} & 0^{T} & -u_{n}P_{n}^{T} \\ 0^{T} & P_{n}^{T} & -v_{n}P_{n}^{T} \end{pmatrix} \begin{pmatrix} m_{1} \\ m_{2} \\ m_{3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

Showing all the elements:
$$\begin{pmatrix} P_{1x} & P_{1y} & P_{1z} & 1 & 0 & 0 & 0 & -u_1P_{1x} & -u_1P_{1y} & -u_1P_{1z} & -u_1 \\ 0 & 0 & 0 & 0 & P_{1x} & P_{1y} & P_{1z} & 1 & -v_1P_{1x} & -v_1P_{1y} & -v_1P_{1z} & -v_1 \\ & & & & & & & & & & & \\ P_{nx} & P_{ny} & P_{nz} & 1 & 0 & 0 & 0 & -u_nP_{nx} & -u_nP_{ny} & -u_nP_{nz} & -u_n \\ 0 & 0 & 0 & 0 & P_{nx} & P_{ny} & P_{nz} & 1 & -v_nP_{nx} & -v_nP_{ny} & -v_nP_{nz} & -v_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

W. Freeman

$$\begin{pmatrix} P_{1x} & P_{1y} & P_{1z} & 1 & 0 & 0 & 0 & -u_1 P_{1x} & -u_1 P_{1y} & -u_1 P_{1z} & -u_1 \\ 0 & 0 & 0 & 0 & P_{1x} & P_{1y} & P_{1z} & 1 & -v_1 P_{1x} & -v_1 P_{1y} & -v_1 P_{1z} & -v_1 \\ \vdots & & & & & & & & & & & & \\ P_{nx} & P_{ny} & P_{nz} & 1 & 0 & 0 & 0 & 0 & -u_n P_{nx} & -u_n P_{ny} & -u_n P_{nz} & -u_n \\ 0 & 0 & 0 & P_{nx} & P_{ny} & P_{nz} & 1 & -v_n P_{nx} & -v_n P_{ny} & -v_n P_{nz} & -v_n \end{pmatrix} \begin{pmatrix} m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{24} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

$$Q m = 0$$

We want to solve for the unit vector m (the stacked one) that minimizes

 $|Qm|^2$

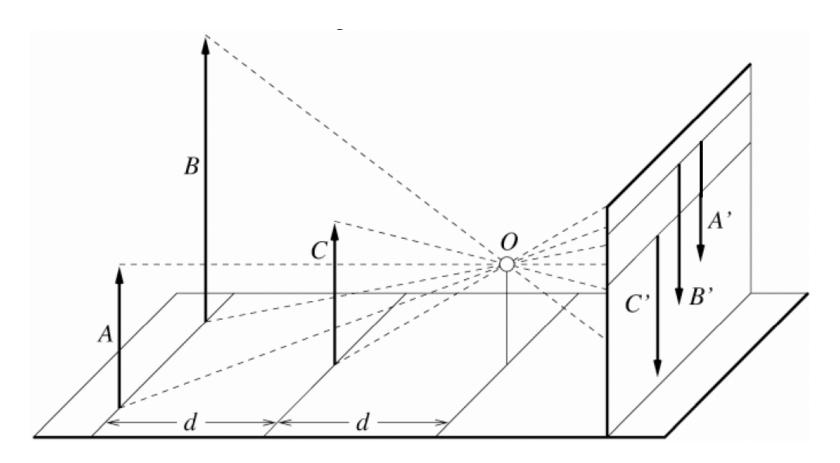
The minimum eigenvector of the matrix Q^TQ gives us that (see Forsyth&Ponce, 3.1), because it is the unit vector x that minimizes $x^T Q^TQ x$.

Once you have the M matrix, can recover the intrinsic and extrinsic parameters as in Forsyth&Ponce, sect. 3.2.2.

$$\mathcal{M} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix}$$

Recall, perspective effects...

Far away objects appear smaller



Perspective effects

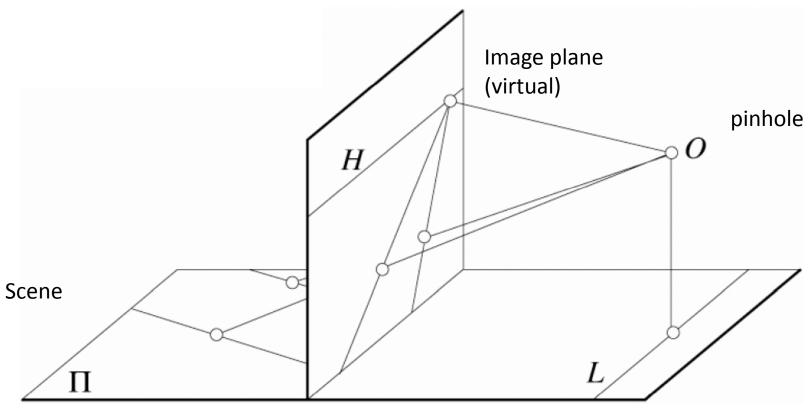


Perspective effects



Perspective effects

- Parallel lines in the scene intersect in the image
- Converge in image on horizon line

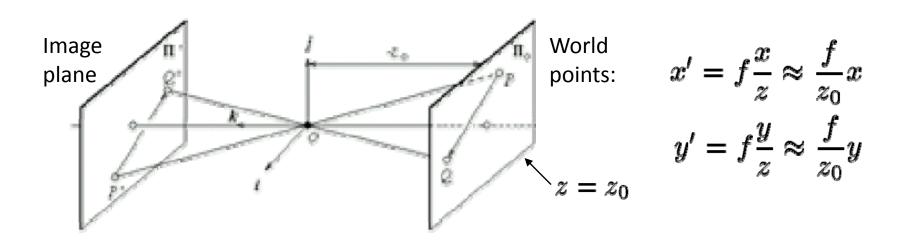


Projection properties

- Many-to-one: any points along same ray map to same point in image
- Points \rightarrow ?
 - points
- Lines \rightarrow ?
 - lines (collinearity preserved)
- Distances and angles are / are not? preserved
 - are not
- Degenerate cases:
 - Line through focal point projects to a point.
 - Plane through focal point projects to line
 - Plane perpendicular to image plane projects to part of the image.

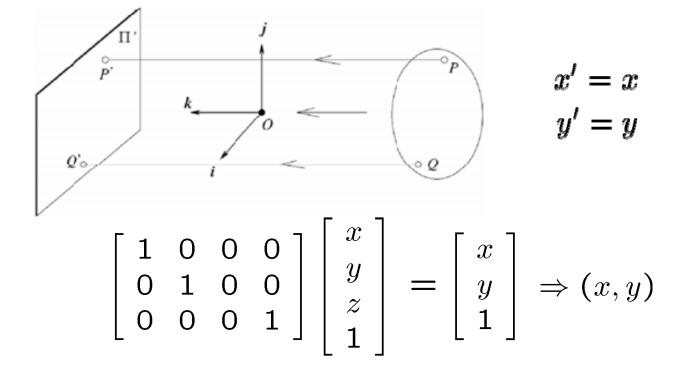
Weak perspective

- Approximation: treat magnification as constant
- Assumes scene depth << average distance to camera

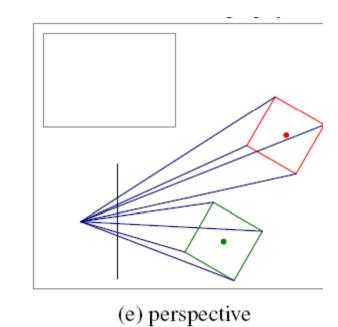


Orthographic projection

- Given camera at constant distance from scene
- World points projected along rays parallel to optical access



(c) scaled orthography



$$x = [s\mathbf{I}_{2\times 2}|0] \ p.$$

$$x = \mathcal{P}_z(\mathbf{p}) = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}.$$

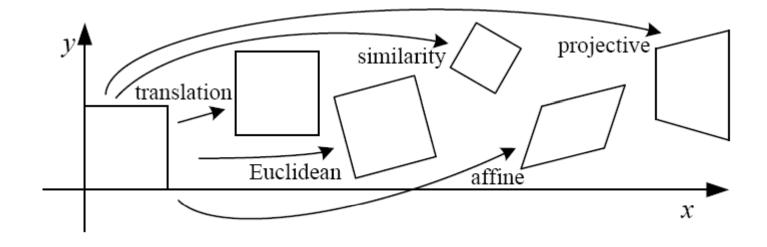


Figure 2.4: Basic set of 2D planar transformations

2D

Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$egin{bmatrix} ig[egin{array}{c c} I & t \end{bmatrix}_{2 imes 3} \end{array}$	2	orientation $+ \cdots$	
rigid (Euclidean)	$\left[egin{array}{c c} R & t\end{array} ight]_{2 imes 3}$	3	lengths + · · ·	\Diamond
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2 \times 3}$	4	angles +···	\Diamond
affine	$\left[egin{array}{c} A \end{array} ight]_{2 imes 3}$	6	parallelism + · · ·	
projective	$\left[egin{array}{c} ilde{m{H}} \end{array} ight]_{3 imes 3}$	8	straight lines	

3D

Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{3 imes 4}$	3	orientation $+ \cdots$	
rigid (Euclidean)	$igg[egin{array}{c c} R & t \end{bmatrix}_{3 imes 4}$	6	lengths + · · ·	\Diamond
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{3 \times 4}$	7	angles +···	\Diamond
affine	$\left[egin{array}{c} oldsymbol{A} \end{array} ight]_{3 imes4}$	12	parallelism + · · ·	
projective	$\left[egin{array}{c} ilde{m{H}} \end{array} ight]_{4 imes 4}$	15	straight lines	

Other types of projection

- Lots of intriguing variants...
- (I'll just mention a few fun ones)

360 degree field of view...



Basic approach

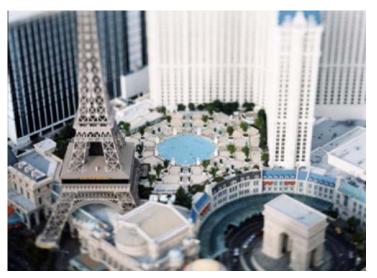
- Take a photo of a parabolic mirror with an orthographic lens (Nayar)
- Or buy one a lens from a variety of omnicam manufacturers...
 - see http://www.cis.upenn.edu/~kostas/omni.html

Tilt-shift



http://www.northlight-images.co.uk/article_pages/tilt_and_shift_ts-e.html





Titlt-shift images from Olivo Barbieri and Photoshop imitations

tilt, shift





Tilt-shift perspective correction



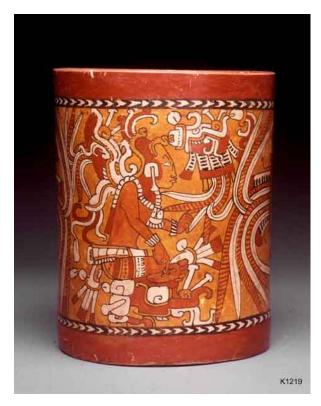
Three photos of the 1858 Robert M. Bashford House Madison, Dane County, Wisconsin, placed on the National Register of Historic Places in 1973.

In the first photo, the camera has been leveled, but no shift lens was used. The top of the house isn't in the picture at all. The second shows what results when the same camera without a shift lens is tilted to get the whole house. The house looks like it is falling over backwards.

The third view, from the same angle, but this time with a shift, or PC, lens gives the results wanted.



Rotating sensor (or object)

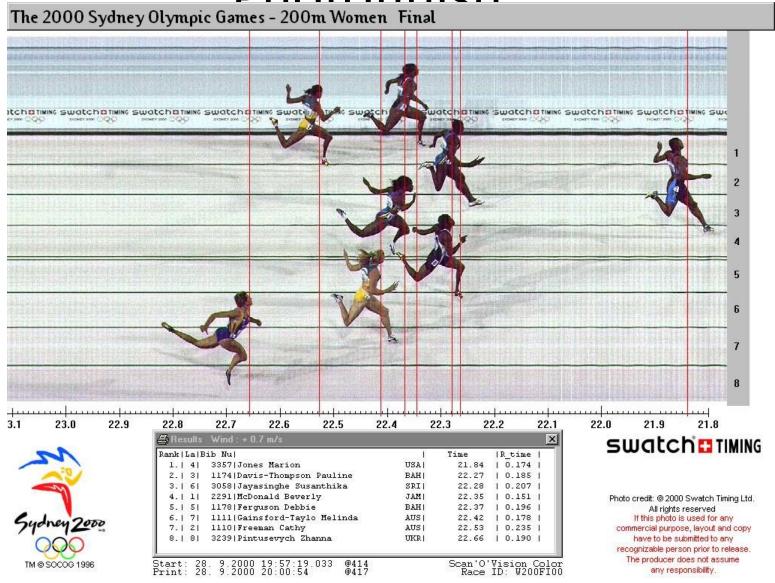




Rollout Photographs © Justin Kerr http://research.famsi.org/kerrmaya.html

Also known as "cyclographs", "peripheral images"

Photofinish



Physical parameters of image formation

- Geometric
 - Type of projection
 - Camera pose
- Optical
 - Sensor's lens type
 - focal length, field of view, aperture
- Photometric
 - Type, direction, intensity of light reaching sensor
 - Surfaces' reflectance properties
- Sensor
 - sampling, etc.

Pinhole size / aperture

How does the size of the aperture affect the image we'd get?

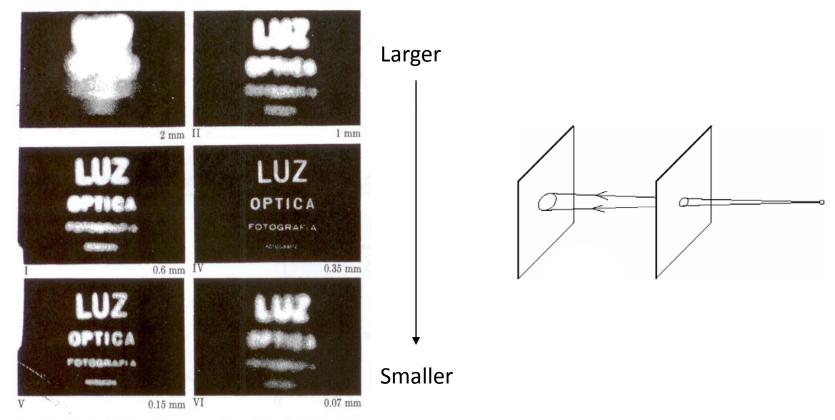
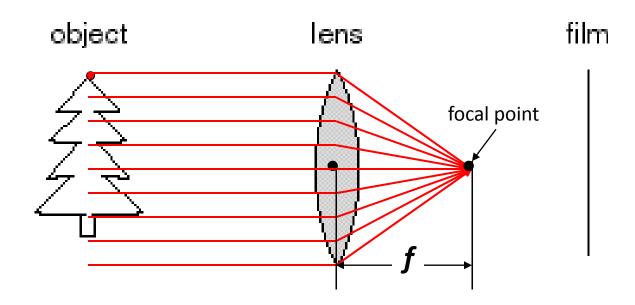


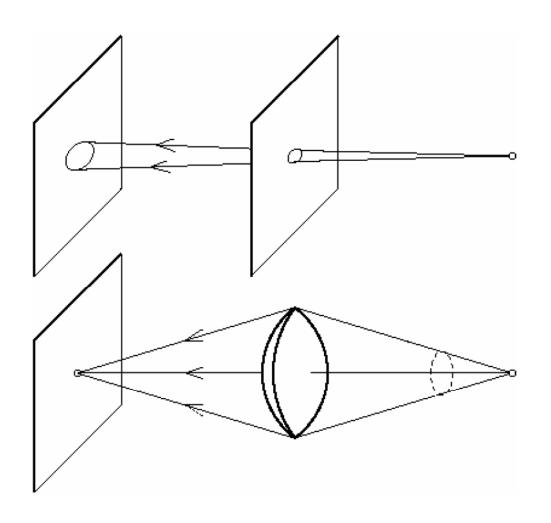
Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]

Adding a lens

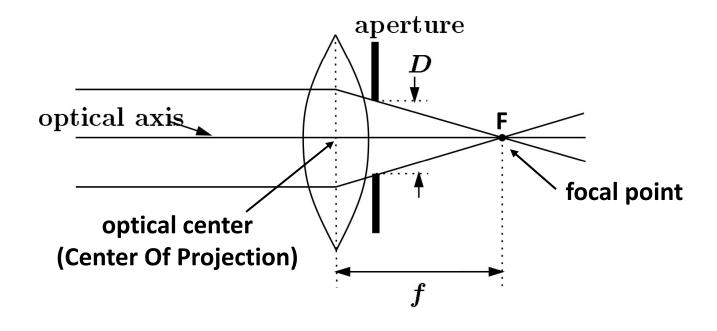


- A lens focuses light onto the film
 - Rays passing through the center are not deviated
 - All parallel rays converge to one point on a plane located at the $\it focal\ length\ f$

Pinhole vs. lens



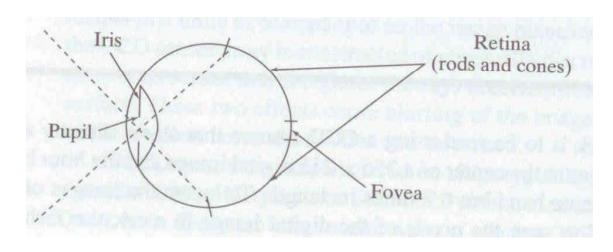
Cameras with lenses



- A lens focuses parallel rays onto a single focal point
- Gather more light, while keeping focus; make pinhole perspective projection practical

Human eye

Rough analogy with human visual system:

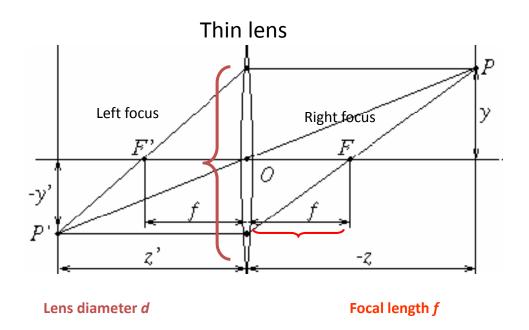


Pupil/Iris – control amount of light passing through lens

Retina - contains sensor cells, where image is formed

Fovea – highest concentration of cones

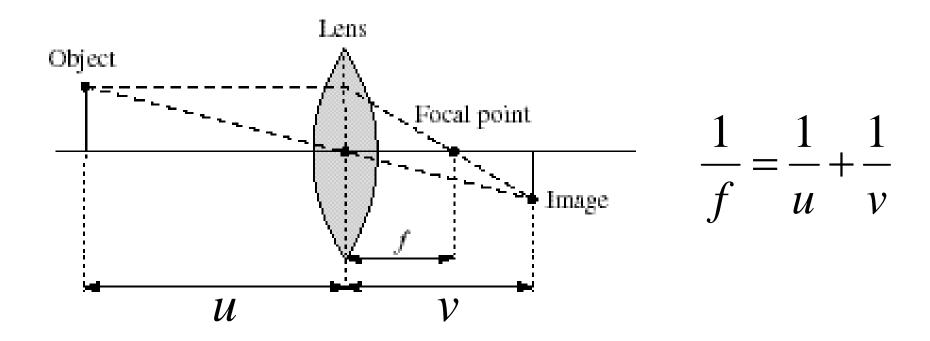
Thin lens



Rays entering parallel on one side go through focus on other, and vice versa.

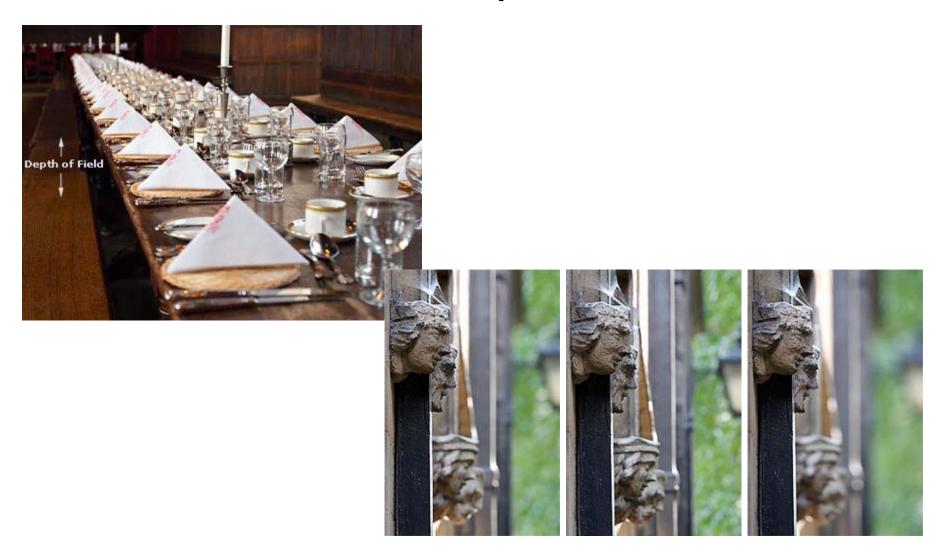
In ideal case – all rays from P imaged at P'.

Thin lens equation



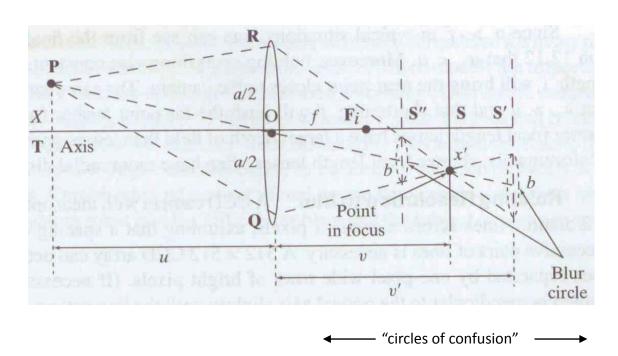
 Any object point satisfying this equation is in focus

Focus and depth of field



Focus and depth of field

 Depth of field: distance between image planes where blur is tolerable

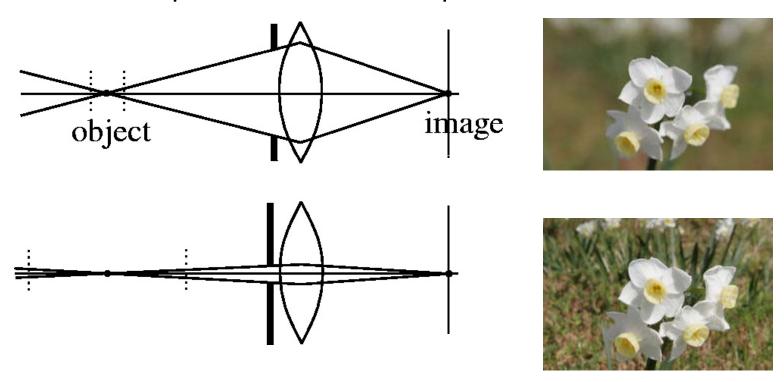


Thin lens: scene points at distinct depths come in focus at different image planes.

(Real camera lens systems have greater depth of field.)

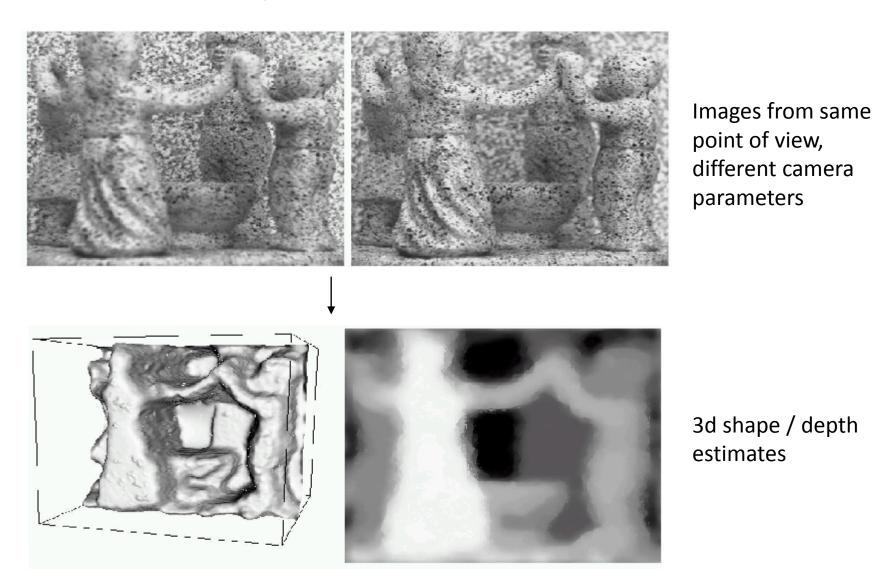
Focus and depth of field

How does the aperture affect the depth of field?



 A smaller aperture increases the range in which the object is approximately in focus

Depth from focus



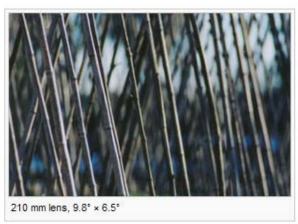
Field of view

Angular
 measure of
 portion of 3d
 space seen by
 the camera



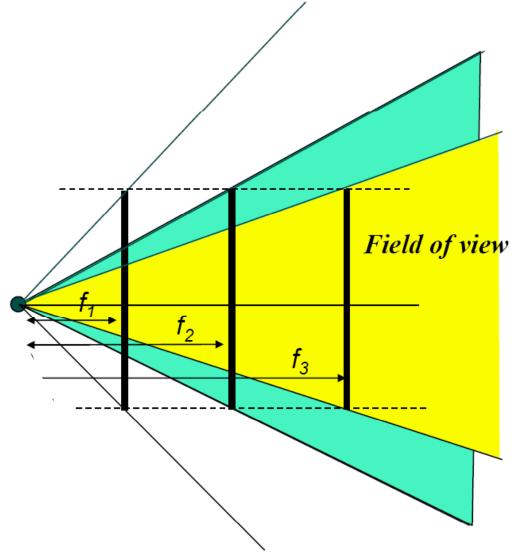




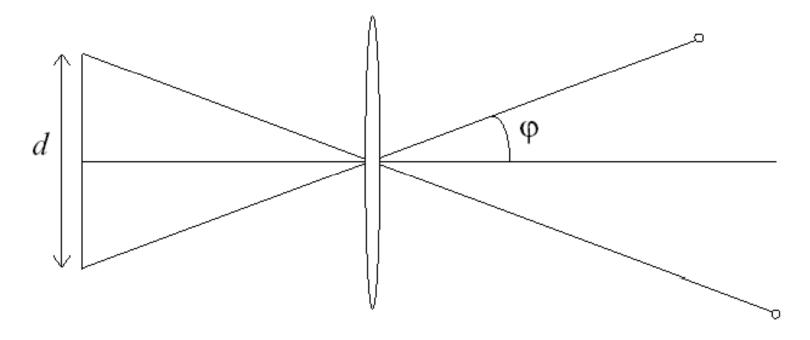


Field of view depends on focal length

- As f gets smaller, image becomes more wide angle
 - more world points project onto the finite image plane
- As f gets larger, image becomes more telescopic
 - smaller part of the world projects onto the finite image plane



Field of view depends on focal length



Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}(\frac{d}{2f})$$

Smaller FOV = larger Focal Length

Vignetting



http://www.ptgui.com/examples/vigntutorial.html



http://www.tlucretius.net/Photo/eHolga.html

Vignetting

"natural":

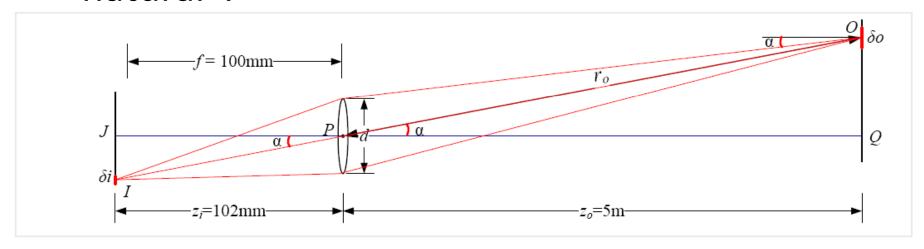


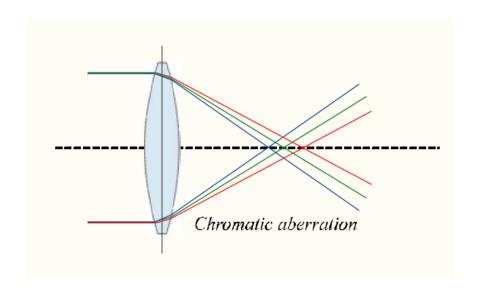
Figure 2.23: The amount of light hitting a pixel of surface area δi depends on the square of the ratio of the aperture diameter d to the focal length f, as well as the fourth power of the off-axis angle α cosine, $\cos^4 \alpha$.

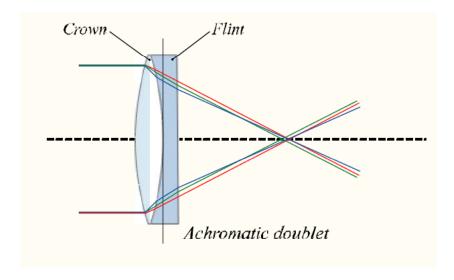
"mechanical": intrusion on optical path

Chromatic aberration



Chromatic aberration





Physical parameters of image formation

- Geometric
 - Type of projection
 - Camera pose
- Optical
 - Sensor's lens type
 - focal length, field of view, aperture
- Photometric
 - Type, direction, intensity of light reaching sensor
 - Surfaces' reflectance properties
- Sensor
 - sampling, etc.

Environment map

$$L(\hat{\boldsymbol{v}};\lambda),$$



http://www.sparse.org/3d.html

BDRF

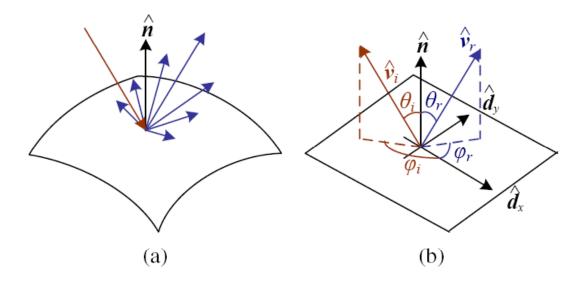


Figure 2.15: (a) Light scattering when hitting a surface. (b) The bidirectional reflectance distribution function (BRDF) $f(\theta_i, \phi_i, \theta_r, \phi_r)$ is parameterized by the angles the incident \hat{v}_i and reflected \hat{v}_r light ray directions make with the local surface coordinate frame $(\hat{d}_x, \hat{d}_y, \hat{n})$.

For an isotropic material, we can simplify the BRDF to

$$f_r(\theta_i, \theta_r, |\phi_r - \phi_i|; \lambda)$$
 or $f_r(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}}; \lambda)$,

Diffuse / Lambertian

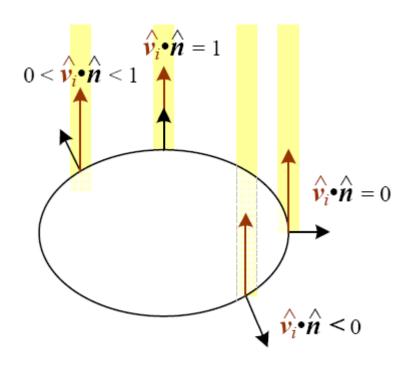


Figure 2.16: This close-up of a statue shows both diffuse (smooth shading) and specular (shiny highlight) reflection, as well as the darkening in the grooves and creases due to reduced light visibility and interreflections. (Photo courtesy of Alyosha Efros.)

While light is scattered uniformly in all directions, i.e., the BRDF is constant,

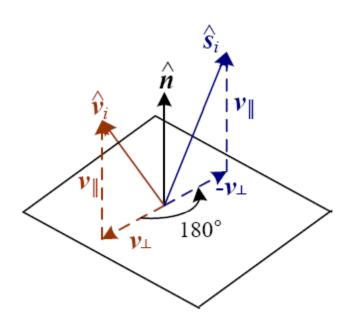
$$f_d(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}}; \lambda) = f_d(\lambda),$$

Foreshortening



The diminution of returned light caused by foreshortening depends on $\hat{v}_i \cdot \hat{n}$, the cosine of the angle between the incident light direction \hat{v}_i and the surface normal \hat{n} .

Specular reflection



Phong

Diffuse+specular+ambient:

$$f_d(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}}; \lambda) = f_d(\lambda),$$

$$f_s(\theta_s; \lambda) = k_s(\lambda) \cos^{k_e} \theta_s,$$

$$f_a(\lambda) = k_a(\lambda) L_a(\lambda).$$

$$L_r(\hat{\boldsymbol{v}}_r;\lambda) = k_a(\lambda)L_a(\lambda) + k_d(\lambda)\sum_i L_i(\lambda)[\hat{\boldsymbol{v}}_i \cdot \hat{\boldsymbol{n}}]^+ + k_s(\lambda)\sum_i L_i(\lambda)(\hat{\boldsymbol{v}}_r \cdot \hat{\boldsymbol{s}}_i)^{k_e}.$$

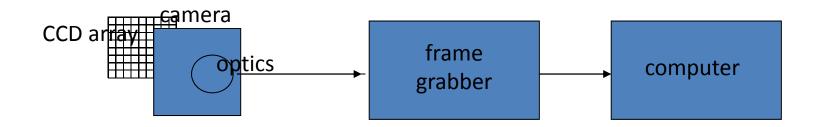
Physical parameters of image formation

- Geometric
 - Type of projection
 - Camera pose
- Optical
 - Sensor's lens type
 - focal length, field of view, aperture
- Photometric
 - Type, direction, intensity of light reaching sensor
 - Surfaces' reflectance properties
- Sensor
 - sampling, etc.

Digital cameras

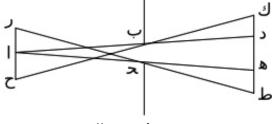
- Film → sensor array
- Often an array of charge coupled devices
- Each CCD is light sensitive diode that converts photons (light energy) to electrons





Historical context

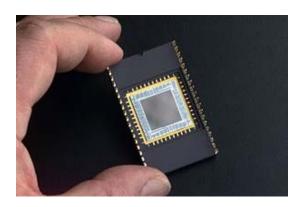
- Pinhole model: Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Principles of optics (including lenses):
 Alhacen (965-1039 CE)
- Camera obscura: Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- First photo: Joseph Nicephore Niepce (1822)
- Daguerréotypes (1839)
- **Photographic film** (Eastman, 1889)
- Cinema (Lumière Brothers, 1895)
- Color Photography (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- First consumer camera with CCD: Sony Mavica (1981)
- First fully digital camera: Kodak DCS100 (1990)



Alhacen's notes

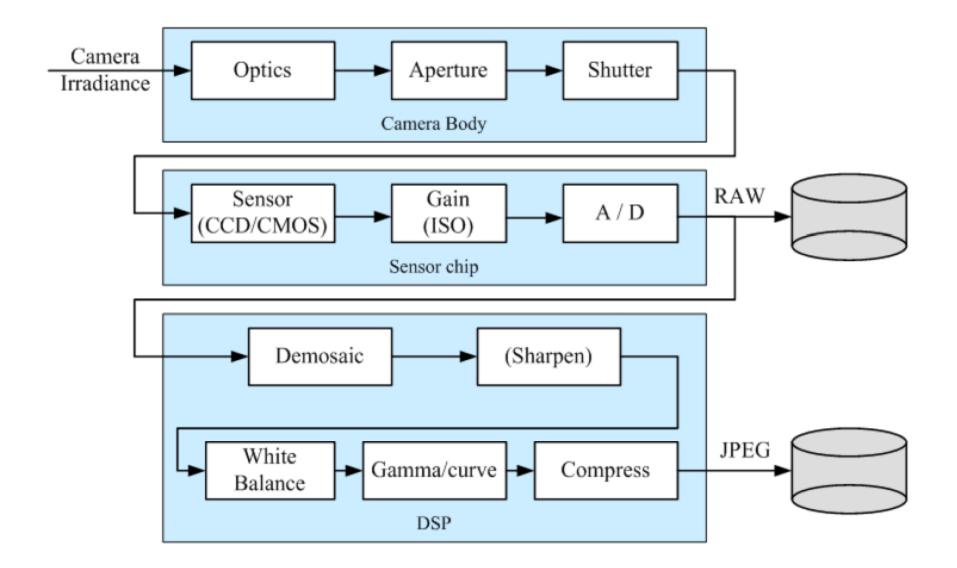


Niepce, "La Table Servie," 1822

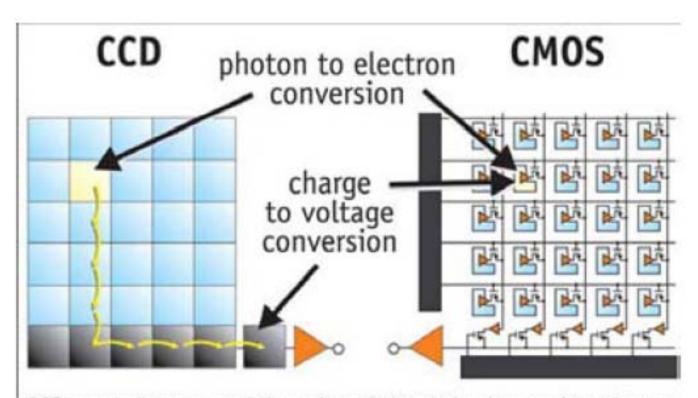


CCD chip

K. Grauman



Digital Sensors



CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

Resolution

- sensor: size of real world scene element a that images to a single pixel
- image: number of pixels
- Influences what analysis is feasible, affects best representation choice.



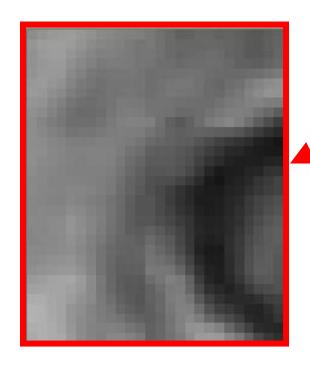


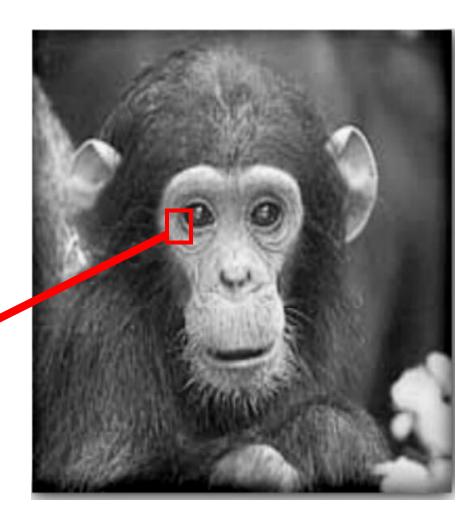


[fig from Mori et al]

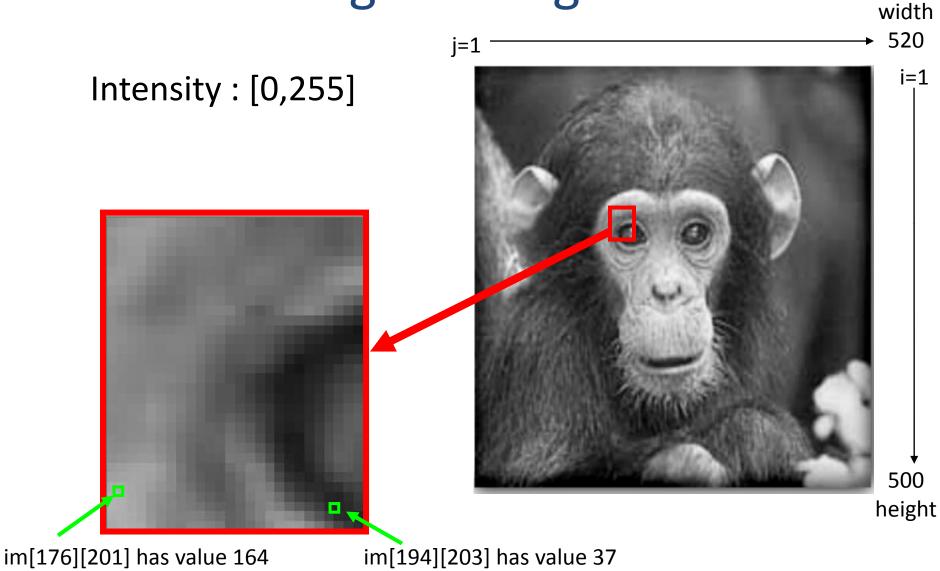
Digital images

Think of images as matrices taken from CCD array.



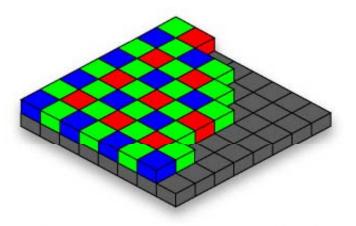


Digital images



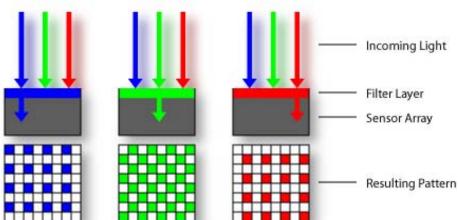
Color sensing in digital cameras

Bayer grid

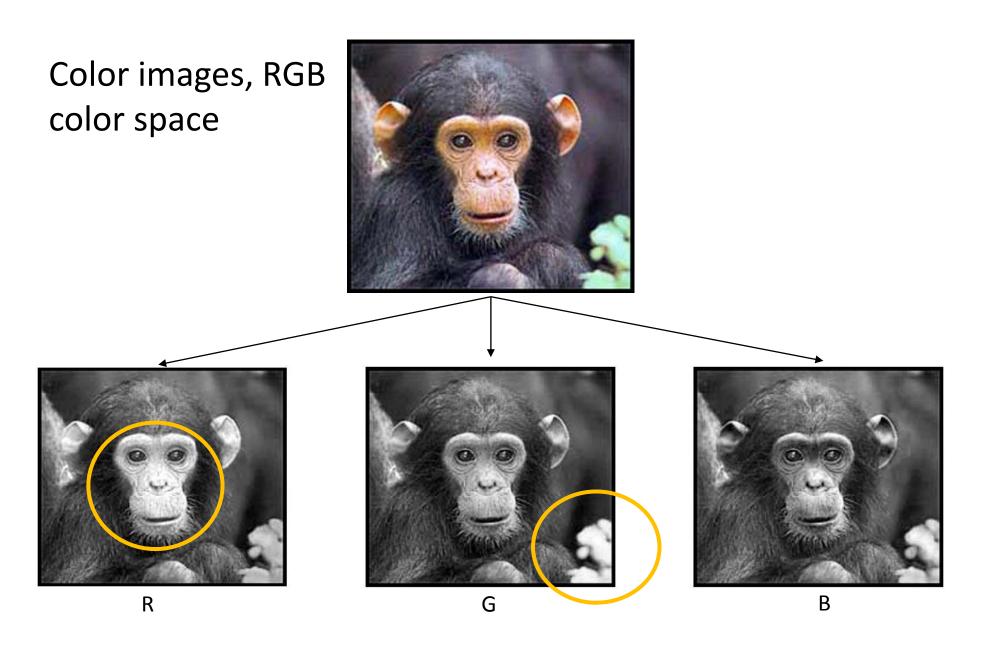


Estimate missing components from neighboring values (demosaicing)





Source: Steve Seitz



Much more on color in next lecture...

Physical parameters of image formation

- Geometric
 - Type of projection
 - Camera pose
- Optical
 - Sensor's lens type
 - focal length, field of view, aperture
- Photometric
 - Type, direction, intensity of light reaching sensor
 - Surfaces' reflectance properties
- Sensor
 - sampling, etc.

Summary

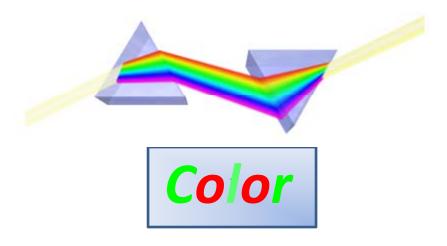
- Image formation affected by geometry, photometry, and optics.
- Projection equations express how world points mapped to 2d image.
- Homogenous coordinates allow linear system for projection equations.
- Lenses make pinhole model practical
- Photometry models: Lambertian, BRDF
- Digital imagers, Bayer demosaicing

Parameters (focal length, aperture, lens diameter, sensor sampling...) strongly affect image obtained.

Slide Credits

- Bill Freeman
- Steve Seitz
- Kristen Grauman
- Forsyth and Ponce
- Rick Szeliski
- and others, as marked...

Next time



Readings:

- Forsyth and Ponce, Chapter 6
- Szeliski, 2.3.2