UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Scences

EEECS210, Spring 2010 **T. K. Gustafson**

NAME(PLEASE PRINT CLEARLY):
S.I.D. :

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Problem Set No. 3 (Midterm) (Open discussion encouraged)

Based upon Chapters 5, 6 ,7 and 9 of Jackson and other references given
Antennas, Electron polariton surface modes, and classical-quantum radiation

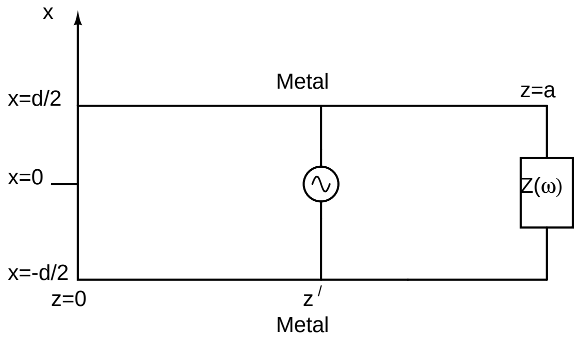
Problem Number 1) Perturbational Frequency Shifts
Suppose a small variation $\delta\omega$ produces variations $\delta\mathbf{E}$ and $\delta\mathbf{H}$ in fields.
a) Starting from Maxwell's equations, show:

$$\int_S (\mathbf{E} \ \times (\delta\mathbf{H}) - (\delta\mathbf{E}) \ \times \mathbf{H}) \cdot d\mathbf{S} = j\delta\omega \int_V (\mu H^2 - \epsilon E^2) dV$$

This is taken from RWVD problem 11.11c 3'rd Ed . A development of this can be extracted from "Waves and Fields in Optoelectronics", H. A. Haus Prentice Hall page 309
This can be used to determine the frequency shift of a cavity resonance when the surface losses due to penetration of the fields in the walls is small but important.

Problem Number 2) Idealized Nanocavity-radiator Excited by a Current Source
This can be modelled as a transmission line problem in the following manner:

The current source could represent a localized tunneling contribution (junction) ($i(z,t) = I_g\delta(z - z')e^{j(\omega t)}$). An interesting supposition is that this is optically excited and ω is an optical frequency. (The width has to be of the order of 1 - 2 nm at this point) At $z = 0$ there is a high reflectivity termination (conductor). At $z = a$ the junction is "open" as represented by a terminating impedance. This would be due to radiation from the current excited on the $z = a$ surface for ($x < -d/2$ and $x > d/2$). Having this reactive is of course an approximation.
The mode we wish to consider is the coupled surface electronic polariton mode for the cavity. This has an E_x in the x-direction of the form $E_x = \tilde{E}cosh(\gamma x)exp(j[\omega t - kz])$ for $|x| < d/2$ and $E_x = \tilde{E}'exp[j(\omega t - kz) - \gamma'(|x| - d/2)]$ for $|x| > d/2$. There is also an E_z but no H_z.



For low frequencies $\gamma --> \infty$ and $\gamma' -->$ *skin depth* which approaches zero when the conductivity goes to ∞. The z-field also goes to zero in this limit. Thus this is the idealized TEM mode in the low frequency limit for high conductivity.
Thus working through the low frequency case (e.m. limit) forms a useful basis for understanding the strong polariton limit.
Define $V(z,t) = -\int_{-d/2}^{+d/2} E_x dx$ and current in the z-direction $I(z,t)$ (z-directed $\int (polarizationcurrent) dx$ crossing 1 nm in the y-direction). Then a transmission line analog can be used.
a) What are L and C for this case?
b) Since I(z,t) is a delta function this is a Green's function problem. Obtain two solutions for $\tilde{V}(z)$ when the termination is lossles (i.e. $Z_L = jX$)
$(ans.I$

$$V = \Sigma_{n=1}^{\infty} \frac{j\omega L\tilde{I}_g sin(k_n z_<) sin(k_n z_>)}{(\lambda_n - {k_o}^2)(\frac{a}{2} - \frac{sin(2k_n a)}{4k_n})}$$

(Hint : use and eigenmode approach)
where $\lambda_n = k_n^2$ and $tan k_n a = -k_n X/\omega L$)

(ans II

$$V = jZ_c I_g \frac{sin k_o z_< [XY_c cos k_o(a - z_>) + sin k_o(a - z_>)]}{sin k_o a + XY_c cos k_o a}$$

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(Ref: Field Theory of guided waves Collin second ed IEEE press page 163)
b) Why is the second solution of a) the one of primary interest for the higher frequency polariton mode when $k_n a$ is small?
c) Can you extend this solution (the second one) to include loss ($jX --- > jX + R_{rad}$) where it is implied that the resistance come from radiation.
d) The most interesting case come when the full polariton character of the mode is displayed. What is the Green's function in this case?
e) Finally, the load impedance arises from the single surface mode in the $\pm x$ direction at z= a. For a half wave-length antenna (planar) antenna can you deduce Z_L by applying what we

did in lecture?

Problem three) Bi-conical antenna

a) Find the solutions for the field of a TEM wave with components E_θ H_ϕ for a symmetrical biconical antenna line with circular cross-sections. Integrate H_ϕ around one conductor to find the radial current on one conductor and integrate E_θ from one conductor to the other to obtain the potential difference V. Show that $V/I = Z_C = \frac{1}{\pi}\sqrt{\frac{\mu_o}{\epsilon_o}} ln(cot(\theta/2))$ where θ is the apex angle.

b) The solution in a) of course has $E_r = 0$. Can you generalize this solution to take the finite field penetration in the metal conical surfaces into account?

Problem 4) Radiation Theory

a) Show that the scalar and vector potentials up to second order in the magnitude of the $\mathbf{k}$ are given by

$$\phi = \frac{e^{i\omega t - ikr}}{r}\left\{ik(1-\frac{i}{kr})\hat{r}\cdot\mathbf{P} - \frac{k^2}{2}\left[\hat{r}\cdot\mathbf{Q}\cdot\hat{r} - \frac{i}{kr}(3\hat{r}\cdot\mathbf{Q}\cdot\hat{r} - Q_s\right]\right\}$$

$$\mathbf{A} = \frac{e^{i\omega t - ikr}}{r}\left\{ik\mathbf{P} - ik(1-\frac{i}{kr})\hat{r}\times\mathbf{M} - \frac{k^2}{2}(1-\frac{i}{kr})(\hat{r}\cdot\mathbf{Q}\right\}$$

where $\mathbf{P} = \int \rho \mathbf{r} dV$, $\mathbf{Q} = \int \rho \mathbf{r}\mathbf{r} dV$ and $\mathbf{M} = \frac{1}{2}\int \mathbf{r}\times\mathbf{I}$ and $Q_s = \int \rho r^2 dV$ $\mathbf{I}$ and ρ being current and charge density.

b) Show that for atomic (or other) transitions in which $\Delta m = 1$, where m is the m in the associated L polynomials that:

$$Re\mathbf{P}e^{i\omega t} = 1/\sqrt{2}P(\hat{x}\ cos(\omega t) \mp \hat{y}\ sin(\omega t))$$

(Rather than Jackson a good reference is The Theory of Atomic Spectra by Condon and Shortley by Cambridge Press)