Binary Adder.

$A + B$

2 bits + carry from previous column to produce S a sum plus K the carry!

Truth Table

A	B	C	S	K
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

compare

S

C \ AB	00	01	11	10
0	0	1	0	1
1	1	0	1	0

K

C \ AB	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$\bar{A}BC$

$A\bar{B}C$

AB

$$K = AB + CA + BC$$

$$S = (\bar{A}B + A\bar{B})\bar{C} + (\bar{A}\bar{B} + AB)C$$

Note $\bar{K} = \underbrace{\overline{AB}\;\overline{CA}\;\overline{BC}}_{\text{NAND Gates}}$

$\bar{K}$

C \ AB	00	01	11	10
0	1	1	0	1
1	1	0	0	0

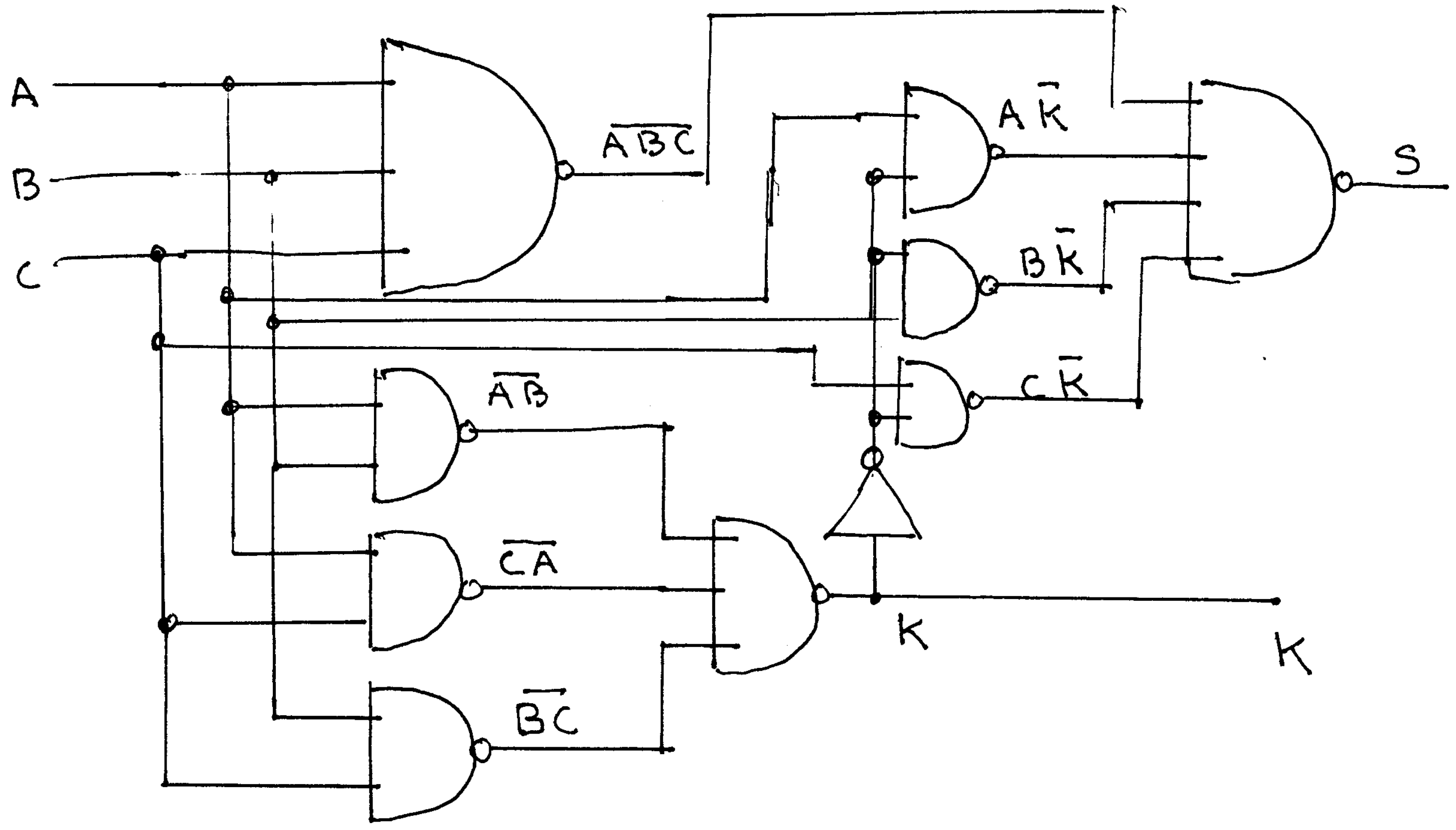
$A+B+C$ kills this $= 1$ for the rest

ABC

$$\therefore S = ABC + (A+B+C)\bar{K}$$

$$= \overline{\overline{ABC}\;\overline{A\bar{K}}\;\overline{B\bar{K}}\;\overline{C\bar{K}}}$$

Implement with NAND Gates



NAND Gate Implementation

2'nd implementation

$S = (A \oplus B)\bar{C} + C(A \oplus \bar{B}) = (A \oplus B) \oplus C$

$= (A \oplus B)\bar{C} + \overline{(A \oplus B)}C$

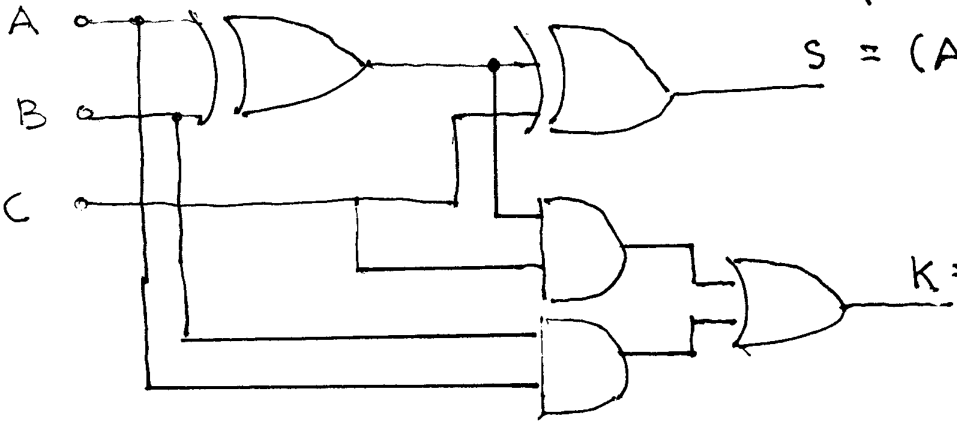
Note 2'nd term is equal to $C\,\overline{(A\bar{B} + B\bar{A})}$

$= C\ \overline{A\bar{B}}\ \overline{B\bar{A}} = C\,(\bar{A} + B)(\bar{B} + A)$

$= C(\bar{A}\bar{B} + BA + \underbrace{\bar{A}A}_{0} + \underbrace{B\bar{B}}_{0})$

$K = AB + AC + BC$

$= AB + C(A \oplus B)$ (see Karnaugh map previous page)



$S = (A \oplus B) \oplus C$

But must implement the exclusive OR

$K = AB + C(A \oplus B)$

Wikipedia Gives This