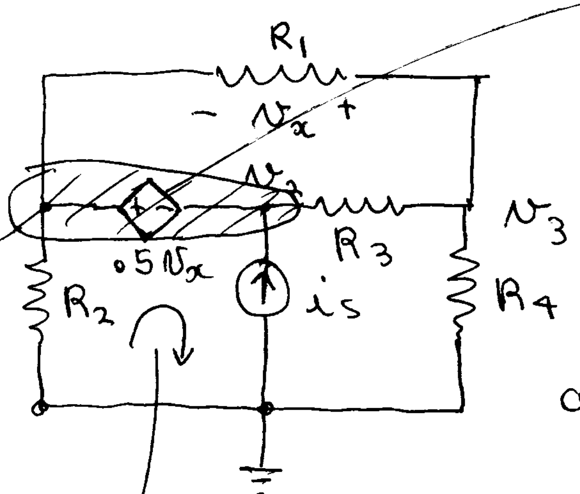
Example 2.10 Node-Voltage with a dependent voltage source

dependent source



① Note that $-v_x = (v_1 - v_3)$

v_x is a branch voltage.

Thus once v_1 & v_3 are known v_x is known

Analysis basically the same as previously

$$v_x = v_3 - v_1$$

KVL (a) $v_2 = v_1 - .5(v_3 - v_1)$ ①

$$= \tfrac{3}{2} v_1 - .5 v_3$$

KCL $\frac{v_1}{R_2} + \frac{v_2 - v_3}{R_3} - i_s = 0$ ②

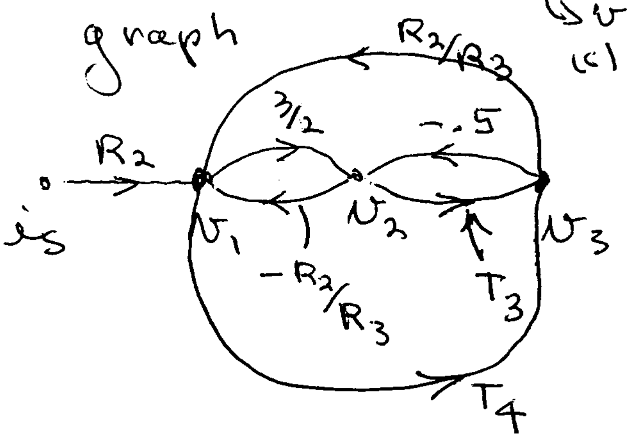
eliminate v_2

write as (b) $v_1\left(\frac{1}{R_2}\right) = -\frac{v_2}{R_3} + \frac{v_3}{R_3} + i_s$

KCL at ③ $\frac{v_3}{R_4} + \frac{(v_3 - v_2)}{R_3} + \frac{(v_3 - v_1)}{R_1} = 0$

(c) $v_3\left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{R_1}\right) - \frac{v_2}{R_3} - \frac{v_1}{R_1} = 0$

or graph



$$T_3 = \frac{1/R_3}{1/R_4 + 1/R_3 + 1/R_1}$$

$$T_4 = \frac{1/R_1}{1/R_4 + 1/R_3 + 1/R_1}$$

$$v_3 = \frac{(T_4 R_2 + R_2(3/2) T_3)}{1 - \frac{R_2}{R_3} T_4 + \frac{3}{2}\frac{R_2}{R_3} + .5 T_3 - .5 \frac{R_2}{R_3} T_4 - \frac{3}{2}\frac{R_2}{R_3} T_3)}$$

Check Using ~~Cramers~~ Rule

(1) $v_3 = T_3 v_2 + T_4 v_1$

(2) $v_1 = -\frac{R_2}{R_3} v_2 + \frac{R_2}{R_3} v_3 + i_s R_2$

(3) $v_2 = \frac{3}{2} v_1 - .5 v_3$

$$\begin{vmatrix} -T_4 & -T_3 & 1 \\ -1 & +\frac{R_2}{R_3} & -\frac{R_2}{R_3} \\ -\frac{3}{2} & 1 & .5 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix} = \begin{vmatrix} 0 \\ i_s R_2 \\ 0 \end{vmatrix}$$

$$\Delta = -T_4 \left(\frac{R_2}{R_3} \cdot \frac{1}{2} + \frac{R_2}{R_3} \right) + T_3 \left(.5 - \frac{3}{2} R_2/R_3 \right) + \left(1 + \frac{3}{2} R_2/R_3 \right)$$

For v_3 $\Delta_{23} = +T_4 + \frac{3}{2} T_3$

Note

If one wants voltages KCL and Branch Relations are Prefered

KCL gives currents which are expressed in terms of voltages with Branch relations

For currents KVL is favored which with Branch relations yields currents