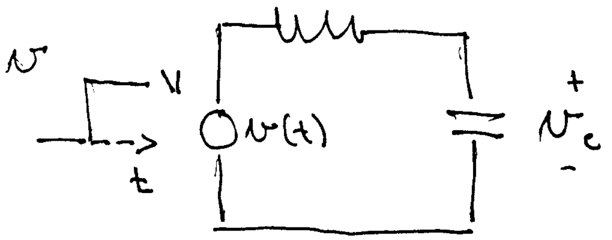
Exponential Response of a Capcitor Resistor Circuit.



$$v_c(t) = \frac{1/pC}{1/pC + R}\, v(t)$$

$$= \frac{1}{pCR}\,\frac{1}{1 + 1/pCR}\cdot v(t) \quad - (1)$$

expand $\frac{1}{1 + 1/pCR}$ using $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

with $x = \frac{1}{pCR}$

$$\frac{1}{p} = \int dt$$

integral, double integral, (integral of (double integral)) ...

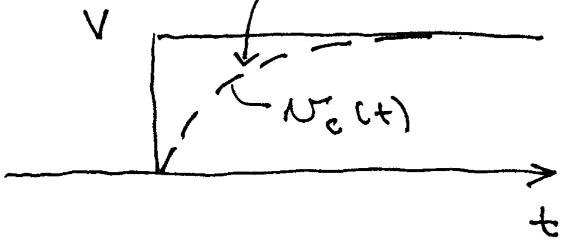
$$\therefore\ v_c(t) = \left(\frac{1}{pCR} - \frac{1}{(pCR)^2} + \frac{1}{(pCR)^3} + \cdots\right) v(t)$$

$v(t) = V$ a constant so

$$v_c(t) = \frac{V}{CR}\left(t - \frac{1}{CR}\frac{t^2}{2} + \frac{t^3}{3\cdot 2}\frac{1}{(CR)^2} \cdots\right)$$

$$= V\left(1 - \left(1 - t/RC - \frac{1}{2}\left(t/RC\right)^2 + \frac{1}{3!}\left(t/RC\right)^3 + \cdots\right)\right)$$

(2) $$v_c(t) = V\left(1 - e^{-t/RC}\right)$$



Differential Equation

From (1)

(3) — $(pCR + 1)\, v_c(t) = v(t)$

$$CR\frac{dv_c}{dt} + v_c(t) = v(t) = V$$

Substitute Eq (2)

$$CR\frac{dv_c}{dt} = +V e^{-t/RC}$$

Thus we immediately see this satisfies (3) with $v_c(0) = 0$ (initially uncharged)