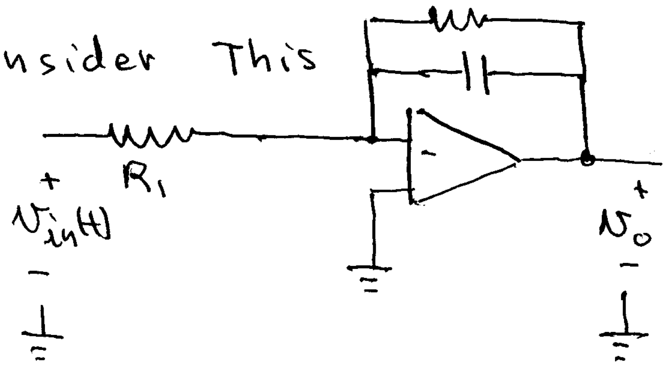
Exponential Excitation

Consider This

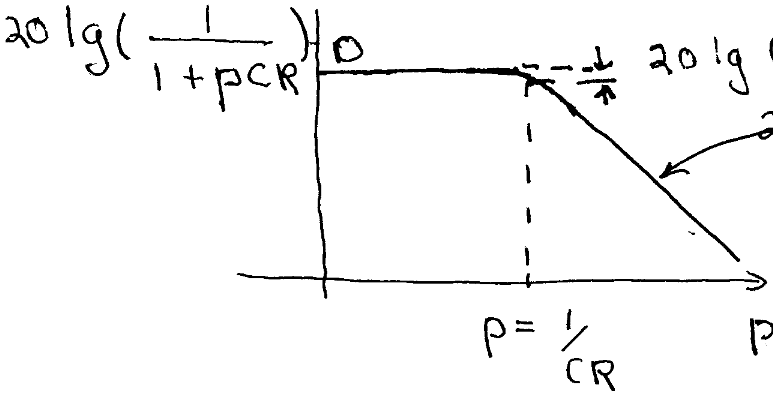


$v_{in} = e^{st}$ a growing exponential excitation

$$\frac{v_o}{\frac{1}{pC} \parallel R} = \frac{v_{in}}{R}$$

$$v_o = \frac{1}{R} \frac{R}{1+pCR} v_{in}(t)$$

$$= \frac{1}{1+pCR} v_{in}(t) \quad -(1)$$



$20 \lg\left(\frac{1}{(1+1)}\right) = -20 \lg 2 = -3\ \underline{dB}$

$20 \lg \frac{1}{pCR} = 20(-\lg(CR) - \lg p)$

$= 0$ when $p = \frac{1}{RC}$

decade = factor of 10

$\lg 10 = 1$

So -20 dB / decade.

what is p - consider the differential

equation from (1) $(1+pCR)\, v_o(t) = v_{in}(t)$

with $v_{in}(t) = \overset{\text{constant}}{A}\, e^{st}$ → a growing exponential

Assuming $v_o(t) = \underset{\overset{\parallel}{B}}{\text{Constant}} + \underset{\overset{\parallel}{D}}{\text{Constant}_1}\, e^{st}$

for $v_o(t)$, the D.E. implies

$$\left(1 + RC\frac{d}{dt}\right)v_o(t) = Ae^{st} \quad -(2)$$

$$B + De^{st} + RCpDe^{st} = Ae^{st}$$

Thus $B = 0 \qquad D = \dfrac{A}{1 + sRC}$

Consequently the above plot is telling one that as $p = s$ increase beyond $1/RC$, the voltage across the capacitor decreases below the excitation amplitude. The voltage to an increasing extend is dropped across the resistor.

Example 2 let $v_{in}(t) = De^{j\omega t}$ → complex

Now we just substitute $p = j\omega$

(If you don't believe this just put it into Eq(2) with the Ae^{pt} replaced by $Ae^{j\omega t}$)

Thus $D = \dfrac{A}{1 + j\omega RC} = \dfrac{A}{\sqrt{1 + (\omega RC)^2}}\, e^{-j\tan^{-1}(\omega RC)}$

when we change to polar notation for the complex number $1 + j\omega RC$

These are precicely the parameters plotted in problem set 3 problem 2