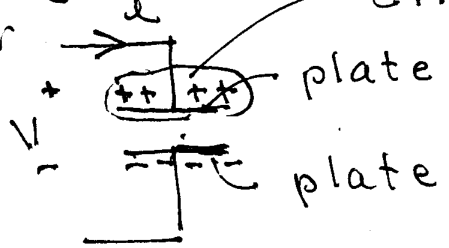
Capacitor & Inductor: They do not dissipate energy - They store it!

Capacitor



$i = \frac{dQ}{dt}$

Q proportional to V

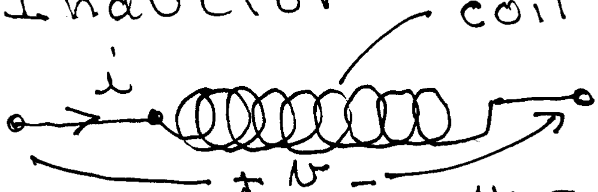
$Q \propto V$ $\quad C$ = capacitance [Farads]

$Q = CV$

Thus $i = \frac{dQ}{dt} = C\frac{dV}{dt} = CpV(t)$

$V(t) = \frac{1}{pC}\, i(t) = r_C(p)\, i(t)$

Inductor



Faraday $v = \frac{d(\text{Flux})}{dt}$

But Flux proportional to i

Flux $= Li$ $\quad L$ = inductance [Henries]

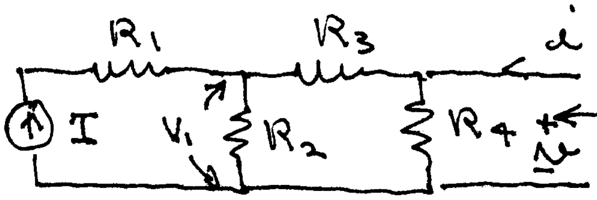
Thus $v = L\frac{di}{dt} = pL\, i(t)$

$= r_L(p)\, i(t)$

Treat $r_C(p) = \frac{1}{pC}$ and $r_L(p) = pL$ as any resistor (Do the same algebra)

Examples of Thevenin and Norton

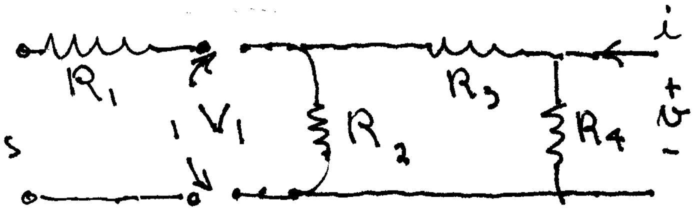
(1)



Find Thev! and Norton

a) Find R_{eq}
Open I

Circuit becomes



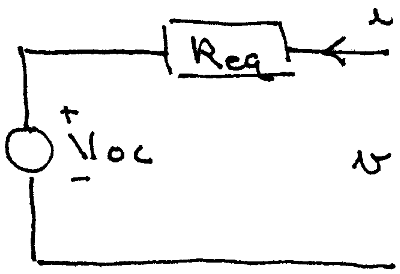
Combine R_2 R_3 & R_4 to obtain R_{eq}
(looking in from v)

$$R_{eq} = R_4 \| (R_3 + R_2) = \frac{R_4(R_3+R_2)}{R_2+R_3+R_4}$$

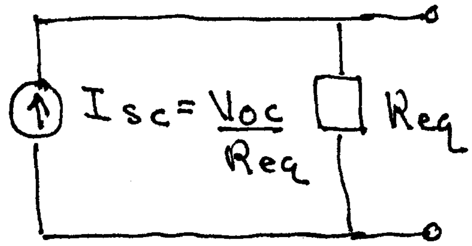
b) Use voltage divider to find Voc

$$V_{oc} = V_1 \left(\frac{R_4}{R_4+R_3}\right)$$

But $V_1 = I \, R_2 \| (R_3+R_4) = I \, \frac{R_2(R_3+R_4)}{R_2+R_3+R_4} \frac{R_4}{R_4+R_3}$



Norton



$$I_{sc} = \frac{V_{oc}}{R_{eq}}$$

Can also find $I_{sc} = I\left(\frac{R_2}{R_2+R_3}\right)$ (current divider)

Then $R_{eq} = \frac{V_{oc}}{I_{sc}}$

$$= \frac{(R_2+R_3)}{\cancel{R_2}\,\cancel{I}} \times \cancel{I}\,\frac{\cancel{R_2}}{R_2+R_3+R_4} \times \frac{R_4}{R_4+R_3}(R_4+R_3)$$

which is the same as above,

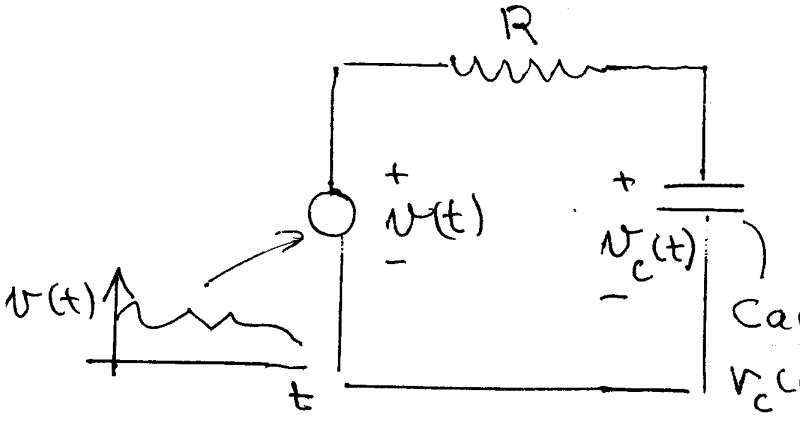
② Replace R_4 by an inductor, find R_{eq} V_{oc} and I_{sc}.

ans. simply substitute $R_4 = pL$ in the above

Note In general then I is time varying (otherwise $v_L = Lp\, i_L = 0$, the inductor is a short (d.c.))

Simple Derivation using $p = \frac{d}{dt}$ "operator" when we have storage elements (L and C)

Consider



Find the DE for $v_c(t)$

$r_c(p) = \frac{1}{pC}$: Treat p as any number

Use the voltage divider relation

$$v_c(t) = \frac{r_c(p)}{r_c(p) + R} v(t)$$

$$= \frac{1/pC}{1/pC + R} v(t)$$

Now clear the p's

a) multiply through by pC on the R.H.S.

$$v_c(t) = \frac{1}{1 + pCR} v(t)$$

b) multiply both sides by $(1 + pCR)$

$$(1 + pCR) v_c(t) = v(t)$$

c) let $p = \frac{d}{dt}$ so the final DE

is $$v_c(t) + CR \frac{dv_c}{dt} = v(t)$$

One of the LT Spice pictures shows the solution when $v(t)$ is a rectangular pulse

Examples From Hambley

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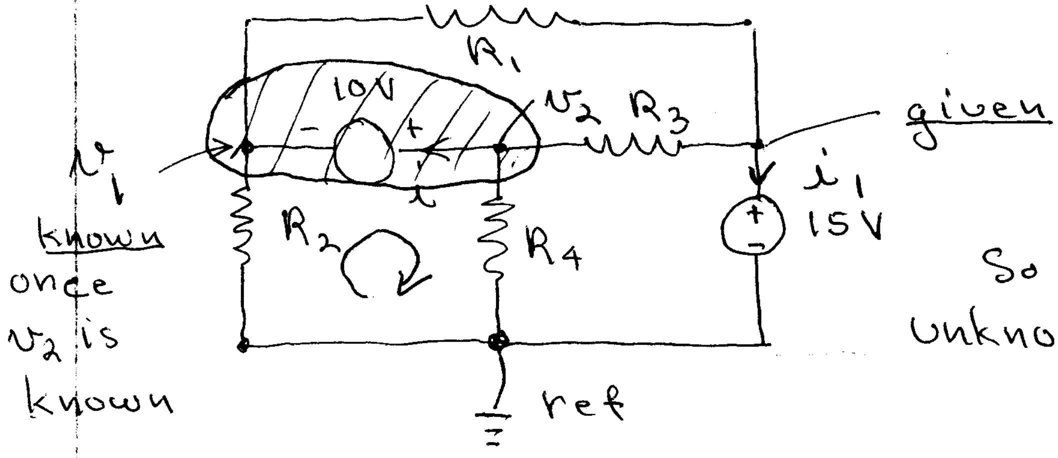
Node Analysis With a Voltage Source

Super nodes.



So only one unknown - v

(1) $\dfrac{v_2 - 15}{R_3} + \dfrac{v_2}{R_4} + i = 0$

gives i once v_2 is known

(2) $\dfrac{v_1}{R_2} - i + \dfrac{v_1 - 15}{R_1} = 0$

(3) KVL $\rightarrow$ $v_1 + 10 = v_2$

(1) plus (2) to eliminate i — supernode equation (4)

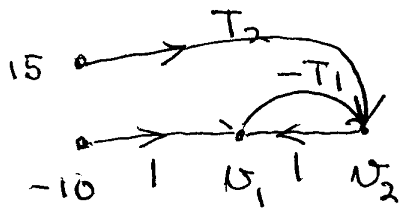
$$v_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right) + v_1\left(\frac{1}{R_2}+\frac{1}{R_1}\right) - \frac{15}{R_3} - \frac{15}{R_1} = 0$$

Use (4) + (3) to obtain v_2 and v_1

<u>Graph</u>

$$v_1 = -10 + v_2$$

$$v_2 = -v_1 \underbrace{\frac{(1/R_2 + 1/R_1)}{(1/R_3 + 1/R_4)}}_{T_1} + 15 \underbrace{\frac{(1/R_3 + 1/R_1)}{(1/R_3 + 1/R_4)}}_{T_2}$$



$$v_1 = \frac{-10 + 15 \times T_2}{1 + T_1}$$

$$v_2 = [-T_2 \times 15 + T_1(10)]/(1+T_1)$$