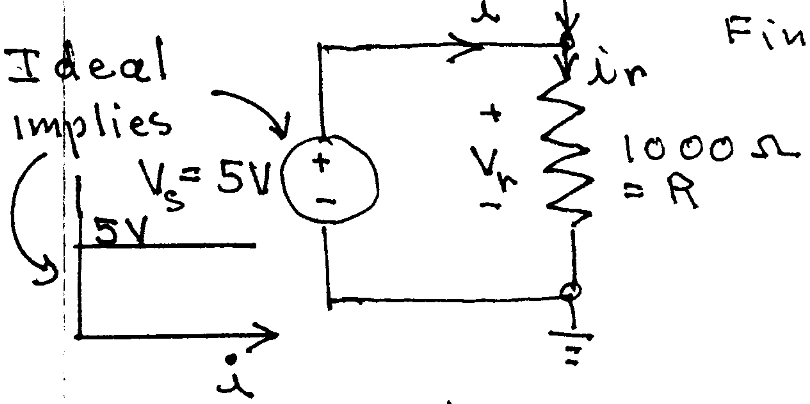
Single Node $i = i_r$

Example Calculation With Resistor



Find the current

KVL

$V_r = V_s = 5$ Volts

Branch relation

$V_r = i R$

$= i \times 1000$

$\therefore \; i = \frac{V_r}{1000} = 5 \times 10^{-3}$ A $= 5$ mA

What is the power dissipated

Power delivered $= 5$ V $\times 5$ mA

$= 25 \times 10^{-3}$ Watts ← J/sec

$= i^2 R$

Energy dissipated in 1 hr

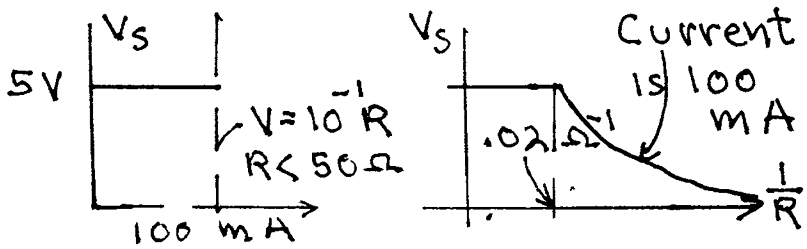
$= 25 \times 10^{-3} \times 3600$ Watt hr

$= .25 \times 360$ Watt hr

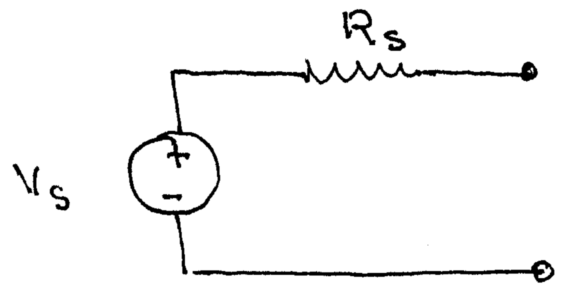
$= 90$ Watt hr $= .09$ kWatt hr

If the voltage source is limited to <u>100 mA</u>, how small can R be and still experience 5 V ; ans. $\frac{5V}{100 mA} = 50\Omega$

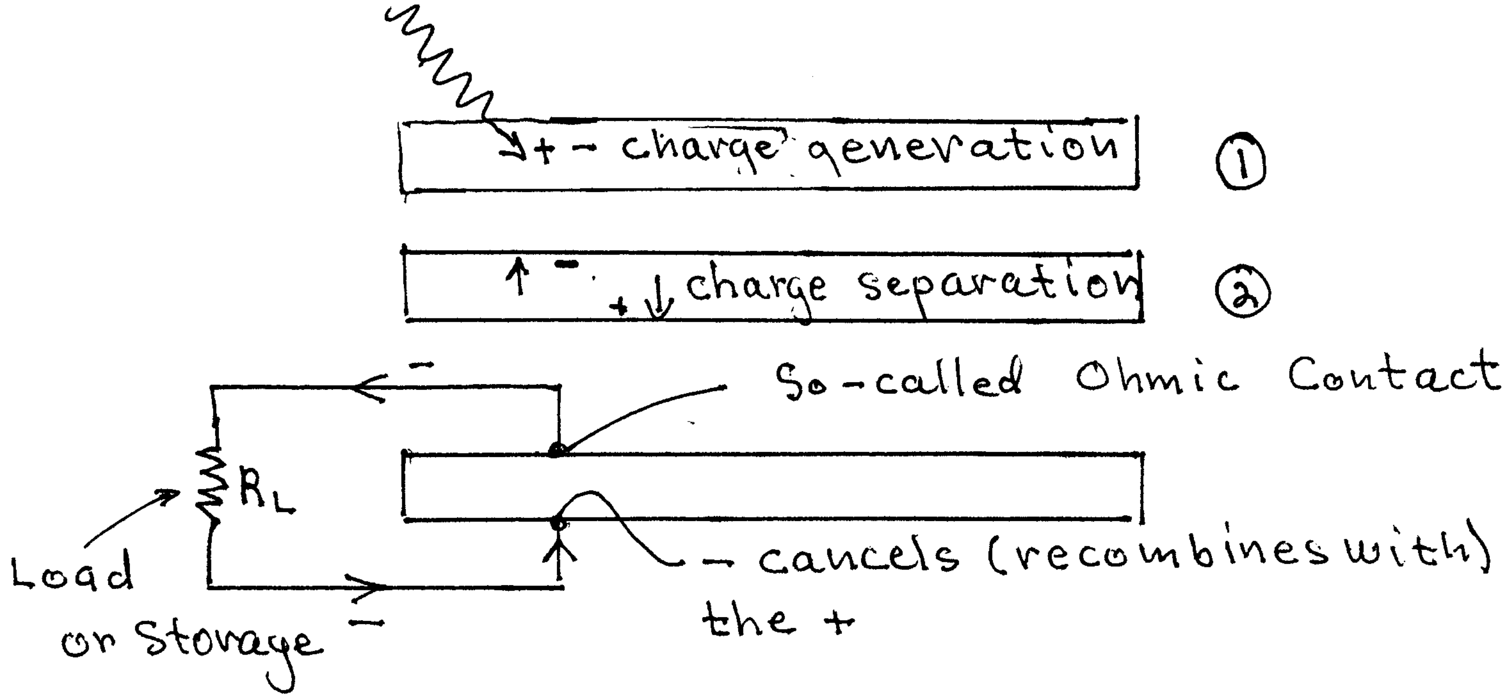
Below this the source acts as a current source and the voltage falls



Non Ideal Voltage Source



Solar Generator (p-n junction)



Solar radiation ≃ 1 kw/m^2
Efficiency ≃ 20%

Problems :
a) loss of carrier energy during separation
b) recombining of electron and + in the material
c) Absorption of sunlight depends upon color
d) Ohmic Contacts
e) R_s

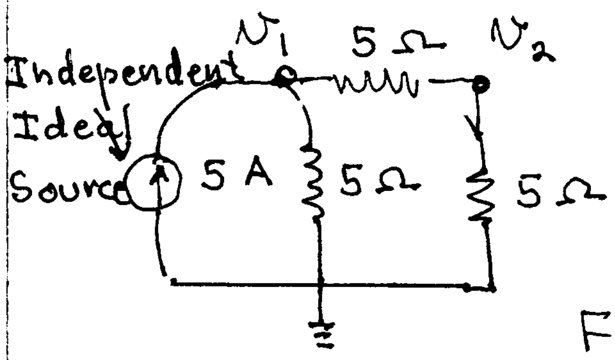
Loop and Node Systems of Equations

Loop → Kirchoff's Voltage "Law"

Node → Kirchoff's Current "Law" (KCL)

KCL → conservation of charge
Nodes do not store charge
Thus sum of currents into or out of a node has to be zero.

Ideal for circuits with Nodes and current sources. Let unknows be node voltages - pick one as ground (our zero) So $n-1$ independent equations. for $n-1$ node voltages,



$$5 = \frac{v_1}{5} + \frac{(v_1 - v_2)}{5}$$

$$\frac{v_2}{5} + \frac{v_2 - v_1}{5} = 0$$

Find v_2

Simultaneous Equations

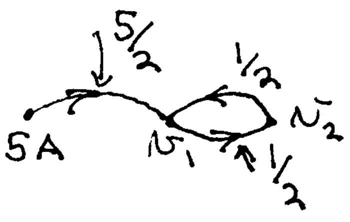
Graphical Solution Technique

$$G = \frac{1}{\Delta} \sum_{\substack{\text{All} \\ \text{Forward} \\ \text{paths}}} G_k \Delta_k$$

Variant of Cramers Rule

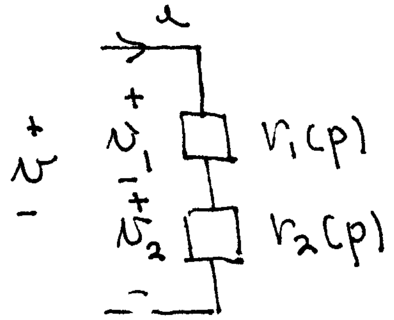
$$v_1 \left(\frac{1}{5} + \frac{1}{5}\right) = \frac{v_2}{5} + 5$$

$$v_2 \left(\frac{1}{5} + \frac{1}{5}\right) = \frac{v_1}{5}$$



$$v_2 = 5G = 5 \times \frac{5}{4} \cdot \frac{1}{1 - \frac{1}{4}} = \frac{25}{3}$$ Volts

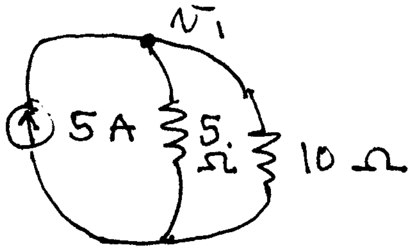
Other way combining resistors.



$$v = v_1 + v_2$$
$$= i_1 r_1(p) + i_2 r_2(p)$$
$$= i\,(r_1(p) + r_2(p))$$
$$= i\,(r(p))$$

Thus $r(p) = r_1(p) + r_2(p)$

So circuit is equivalent to

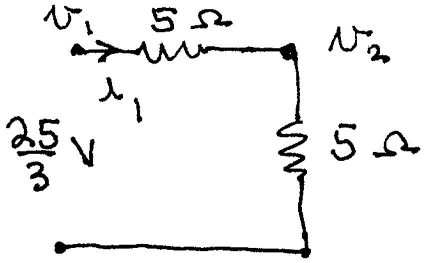


prove resistors in parallel add inversely! So equivalent to

$$\frac{1}{\frac{1}{5} + \frac{1}{10}} = \frac{50}{15} = \frac{10}{3}\,\Omega$$

$$\therefore\ v_1 = 5\text{ A} \times \frac{10}{3}\,\Omega = \frac{50}{3}\text{ V}$$

Now we unfold it.



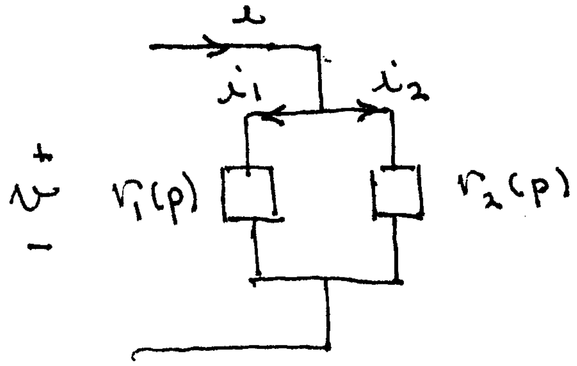
$$i_1 = \frac{v_1}{10} = \frac{50}{30}\text{ Amps}$$

$$\therefore\ v_2 = 5 \times i_1 = \frac{25}{3}\text{ Volts}$$

We have actually use KVL

KVL − The potential at any point is uniquely defined. Thus the sum of the voltage drops across elements around a loop must be zero

Current Division

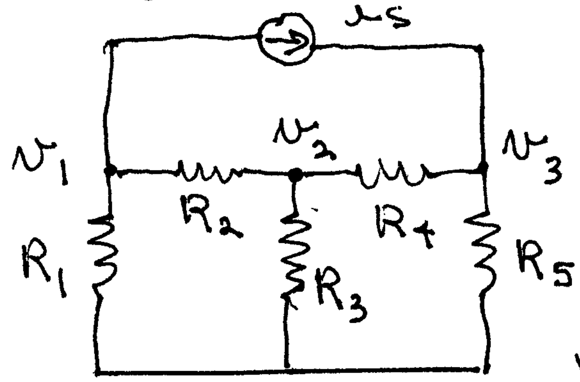


$$i = i_1 + i_2$$
$$= \frac{v}{r_1(p)} + \frac{v}{r_2(p)}$$
$$= \frac{v}{r(p)}$$

$$\therefore \quad v = i \Big/ \left(\frac{1}{r_1(p)} + \frac{1}{r_2(p)} \right)$$

$$i_2 = \frac{v}{r_2(p)} = i \left(\frac{1/r_2(p)}{1/r_1(p) + 1/r_2(p)} \right)$$

Example page 65



Once we have node voltages we have all variables since we can use branch relations to obtain currents

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

$$\frac{v_3}{R_5} - i_s + \frac{v_3 - v_2}{R_4} = 0$$

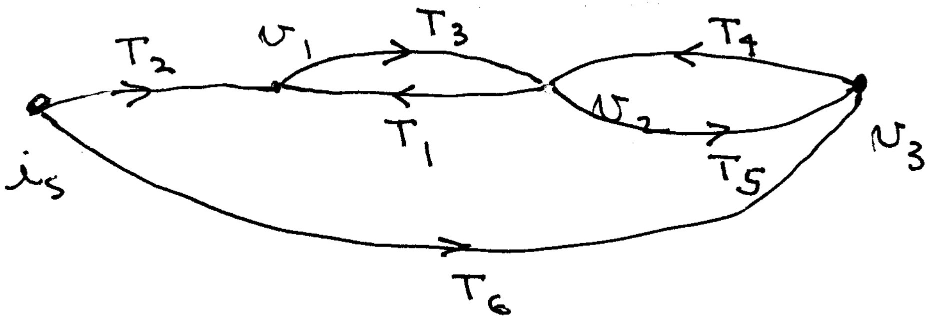
Solving. Write equations as

$$v_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = v_2/R_2 - i_s \rightarrow \text{write as } v_1 = T_1 v_2 + T_2 i_s$$

$$v_2\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) = v_1/R_3 + v_3/R_4 \rightarrow \text{write as } v_2 = T_3 v_1 + T_4 v_3$$

$$v_3\left(1/R_5 + 1/R_4\right) = v_2/R_4 + i_s \rightarrow \text{write as } v_3 = T_5 v_2 + T_6 i_s$$

Draw a graph



Definitions

$$T_2 = -\frac{1}{1/R_1 + 1/R_2}$$

$$T_1 = \frac{1/R_2}{1/R_1 + 1/R_2}$$

$$T_3 = \frac{1/R_2}{1/R_2 + 1/R_3}$$

$$T_4 = \frac{1/R_4}{1/R_2 + 1/R_3}$$

$$T_5 = \frac{1/R_4}{1/R_5 + 1/R_4}$$

$$T_6 = \frac{1}{1/R_5 + 1/R_4}$$

solve for v_3

$$v_3 = \frac{\left(T_2 T_3 T_5 + T_6(1 - T_1 T_3)\right) i_s}{1 - T_3 T_1 - T_4 T_5}$$

Alternate and more common

$$\begin{vmatrix} \frac{1}{R_1}+\frac{1}{R_2}, & -\frac{1}{R_2}, & 0 \\ -\frac{1}{R_2}, & \frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}, & -\frac{1}{R_4} \\ 0 & -\frac{1}{R_4}, & \frac{1}{R_4}+\frac{1}{R_5} \end{vmatrix} \begin{Vmatrix} v_1 \\ v_2 \\ v_3 \end{Vmatrix} = \begin{Vmatrix} i_s \\ 0 \\ i_s \end{Vmatrix}$$

solution

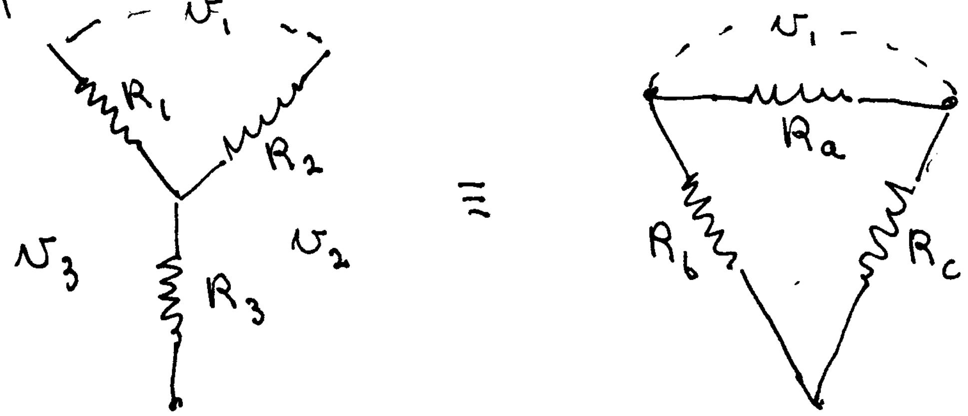
$$v_3 = \frac{\Delta_{13}}{\Delta} i_s + \frac{\Delta_{33}}{\Delta} i_s$$

Δ = determinant

Δ_{13} = 13 cofactor (row, column)

General $v_j = \frac{\Delta_{\text{cause},j}}{\Delta} \times (\text{cause})$

Example: The Delta - Y Transformation



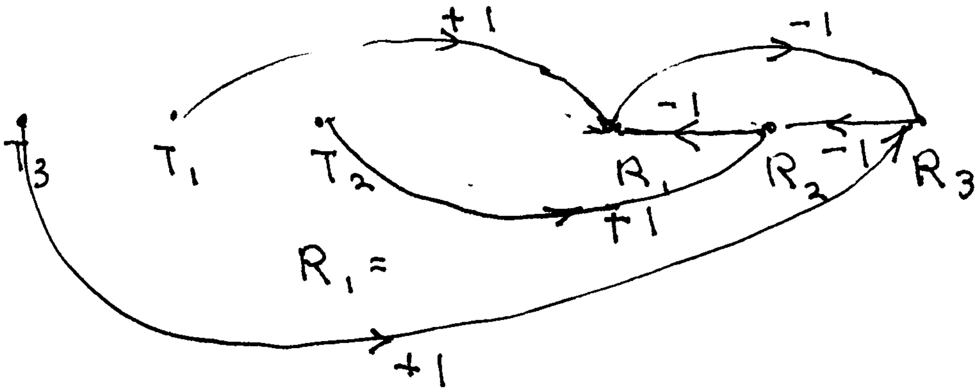
$v_1 = (R_1 + R_2)\, i_1 \qquad v_1 = \left(\frac{R_b + R_c}{R_a + R_b + R_c}\right) R_a \, i_1$

$$\therefore R_1 + R_2 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = T_1$$

cyclically permute

$$R_2 + R_3 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = T_2$$

$$R_3 + R_1 = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = T_3$$



Cramers Rule

$$R_1 = (-T_2 + T_1 + T_3)/2$$
$$= (R_a R_b)/(R_a + R_b + R_c)$$
$$R_2 = (-T_3 + T_1 + T_2)/2$$
$$= R_c R_a/(R_a + R_b + R_c)$$
$$R_3 = R_b R_c/(R_a + R_b + R_c)$$

Note $R_1 R_2 + R_2 R_3 + R_3 R_1$

$$= \frac{(R_a^2 R_b R_c + R_c^2 R_a R_b + R_b^2 R_c R_a)}{(R_a + R_b + R_c)^2}$$

$$= \frac{R_a (R_a R_b R_c + R_c^2 R_b + R_b^2 R_c)}{(R_a + R_b + R_c)^2}$$

$$= R_a R_b R_c \frac{(R_a + R_c + R_b)}{(R_a + R_c + R_b)^2}$$

$$= R_a R_b R_c / (R_a + R_b + R_c)$$

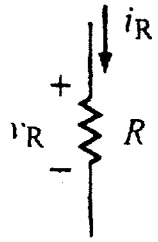
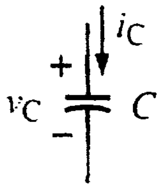
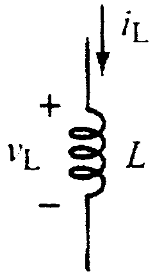
$$= R_a R_3 = R_b R_2 = R_c R_1$$

Thus $R_a = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$

$R_b = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$

$R_c = \dfrac{''}{R_1}$

Table 1-4: Passive circuit elements and their symbols.

Element	Symbol	i–v Relationship
Resistor		$v_R = Ri_R$
Capacitor		$i_C = C \frac{dv_C}{dt}$
Inductor		$v_L = L \frac{di_L}{dt}$

Define $\frac{d}{dt} \triangleq p$

treat it as any number

$i_C = C \frac{dv_C}{dt} = C p v_C$

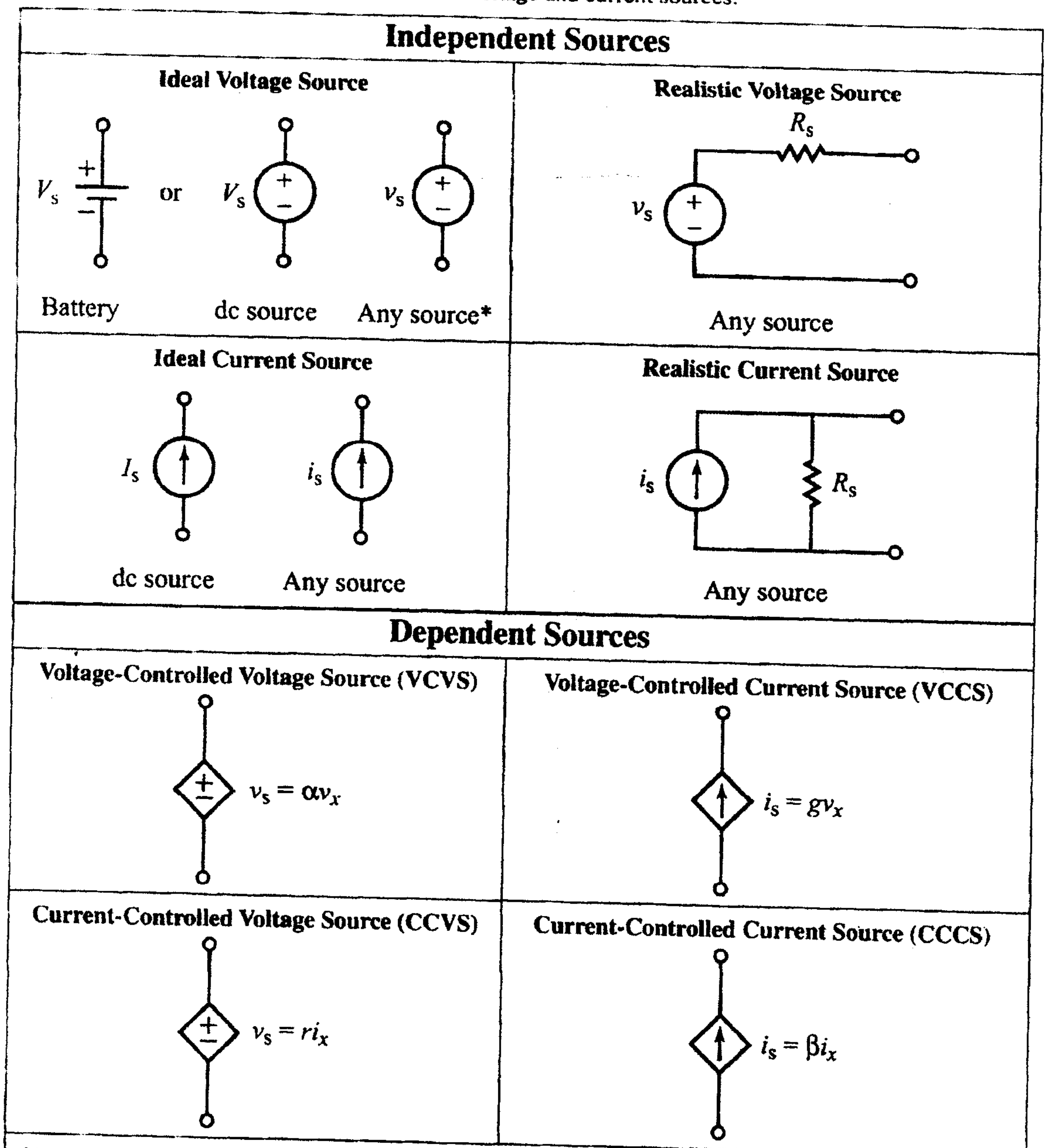
so $v_C = \left(\frac{1}{Cp}\right) i_C$

$= r_C(p)\, i_C$

$v_L = L \frac{di_L}{dt} = L p i_L = r_L(p)\, i_L$

so $i_L = \frac{1}{Lp} v_L$

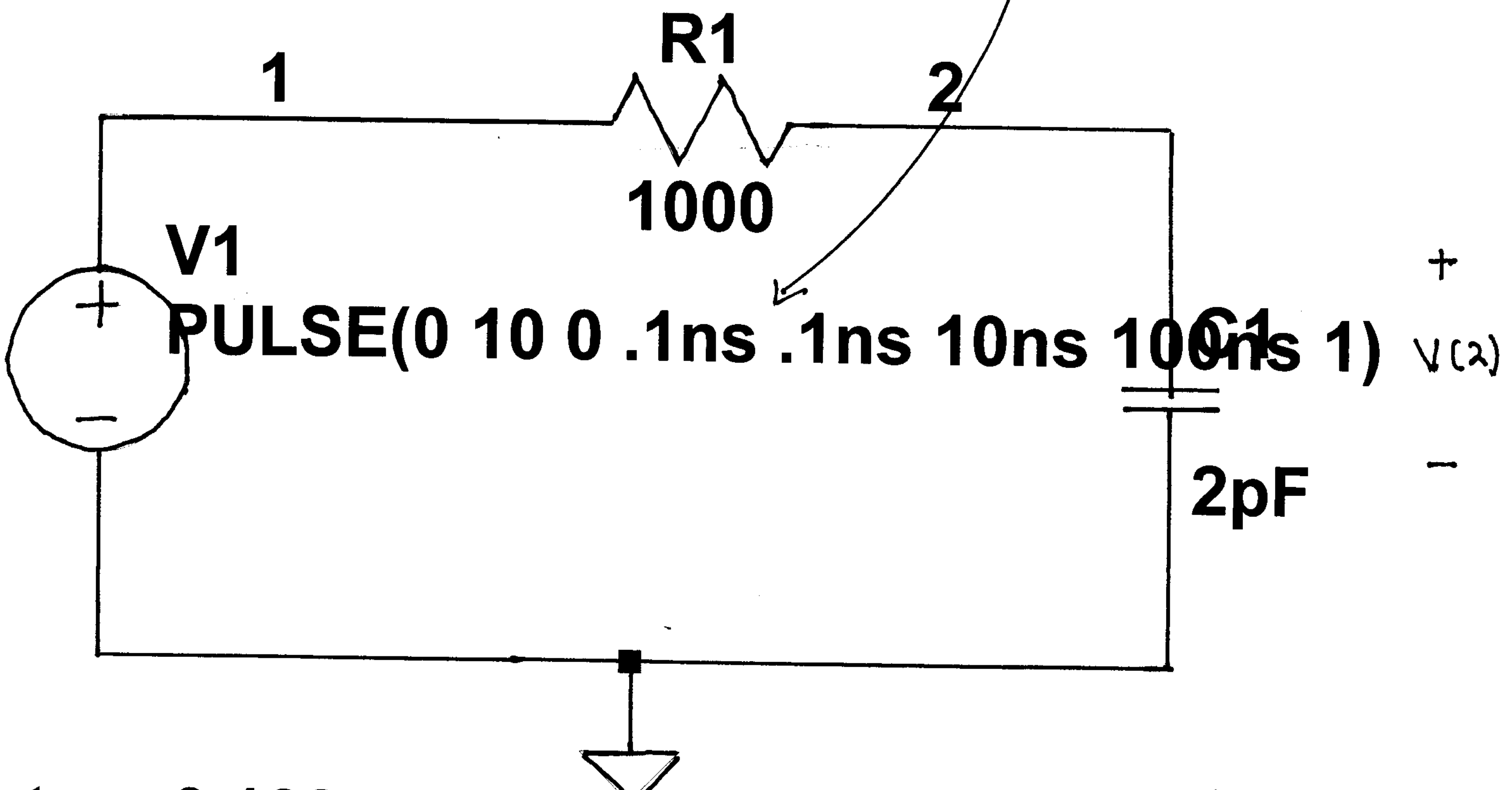
Table 1-3: Voltage and current sources.



Note: α, g, r, and β are constants; v_x and i_x are a specific voltage and a specific current elsewhere in the circuit. *Lowercase v and i represent voltage and current sources that may or may not be time varying, whereas uppercase V and I denote dc sources.

Fun With LT Spice.

Pulse starting at 0 Volts ramping up to 10 Volts with a rise-time .1 nsec, fall time of .1 nsec, 10 nsec. in duration a period of 100 nsec. 1 cycle plotted. This is the spice netlist statement



.tran 0 100ns 0 10ns ← transient analysis from 0 to 100 nsec for a pulse from 0 to 10 nsec

Result for V(2) next page

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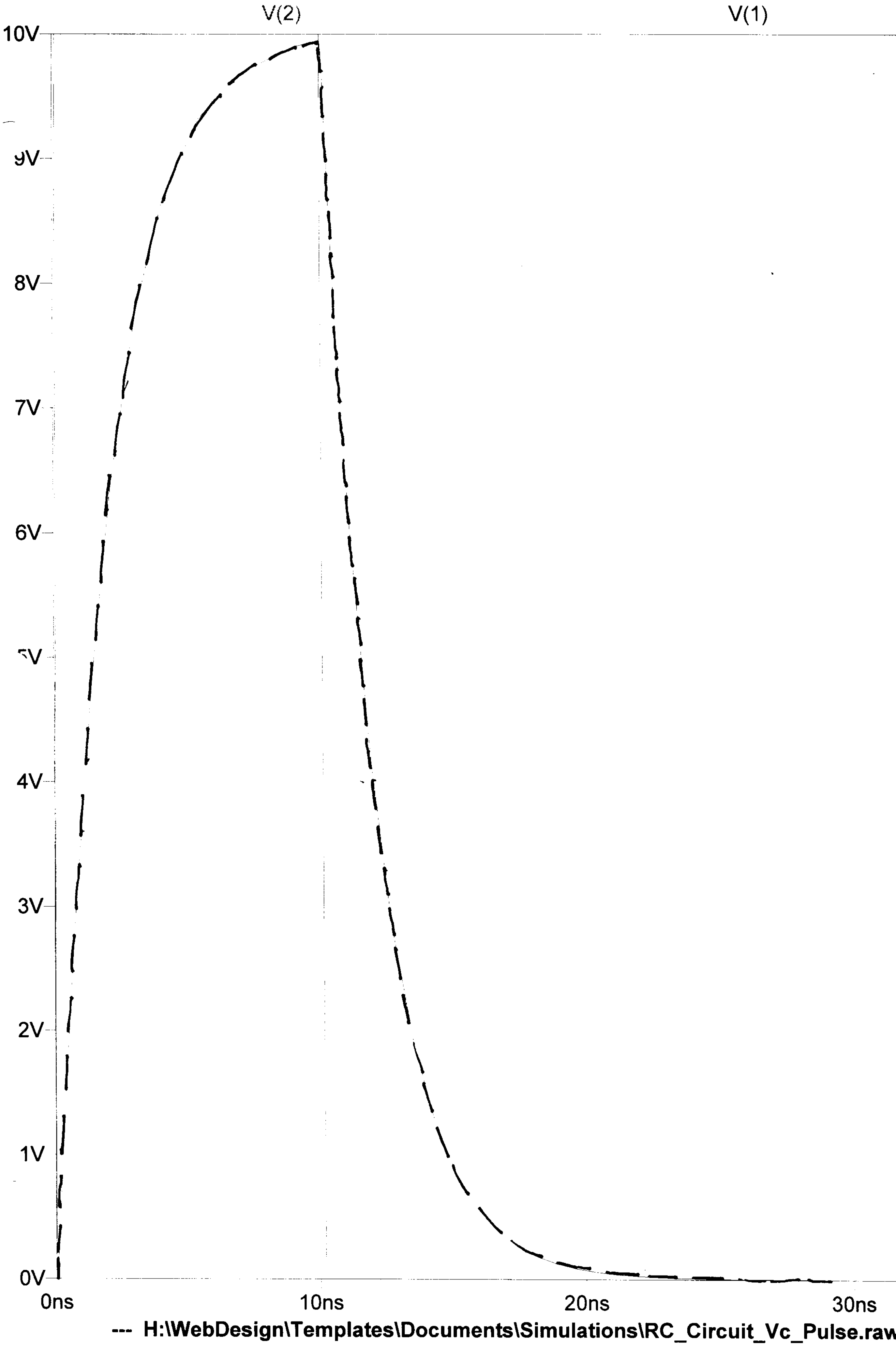

V(2)
V(1)
10V
9V
8V
7V
6V
5V
4V
3V
2V
1V
0V
0ns
10ns
20ns
30ns
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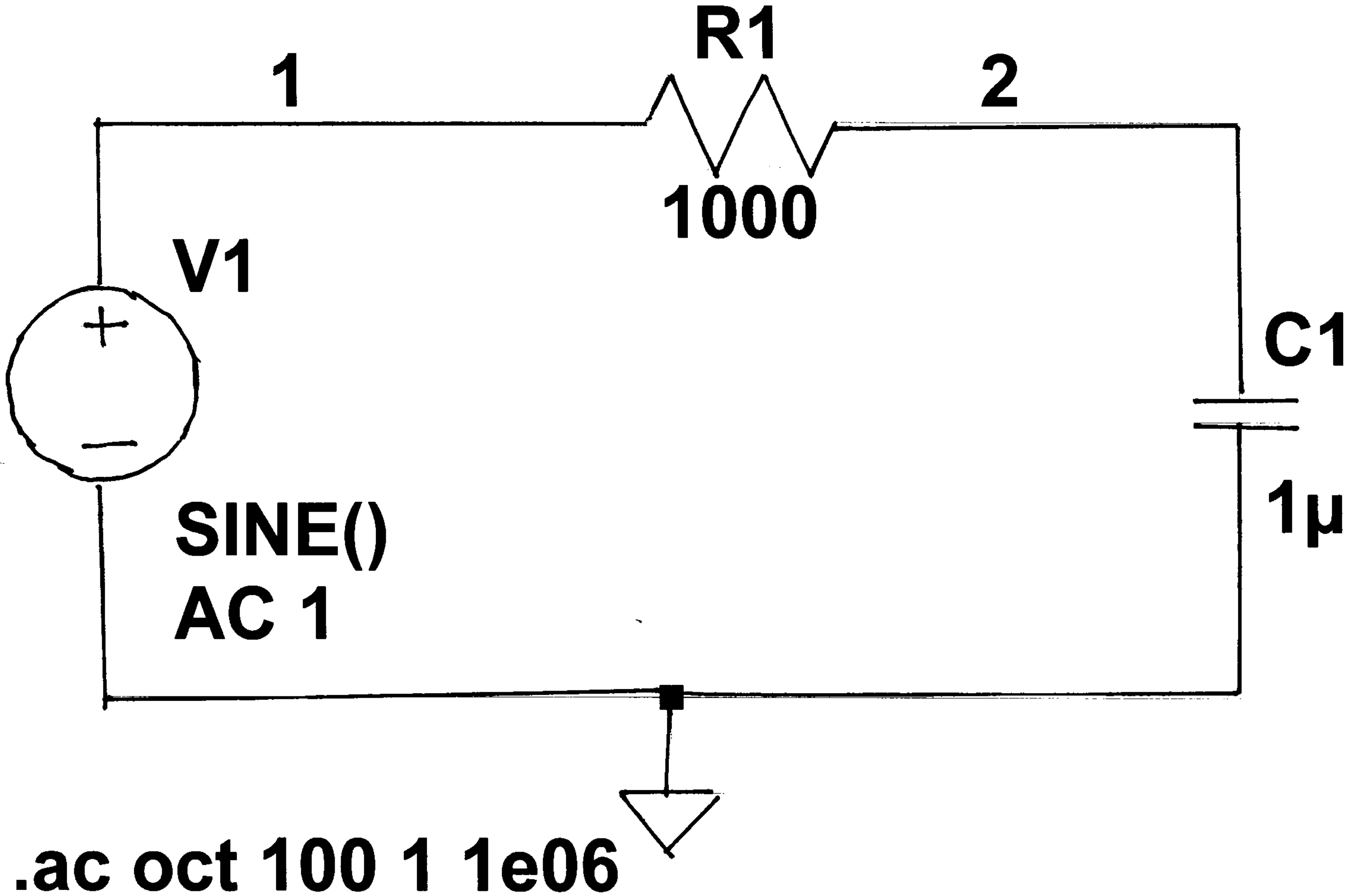
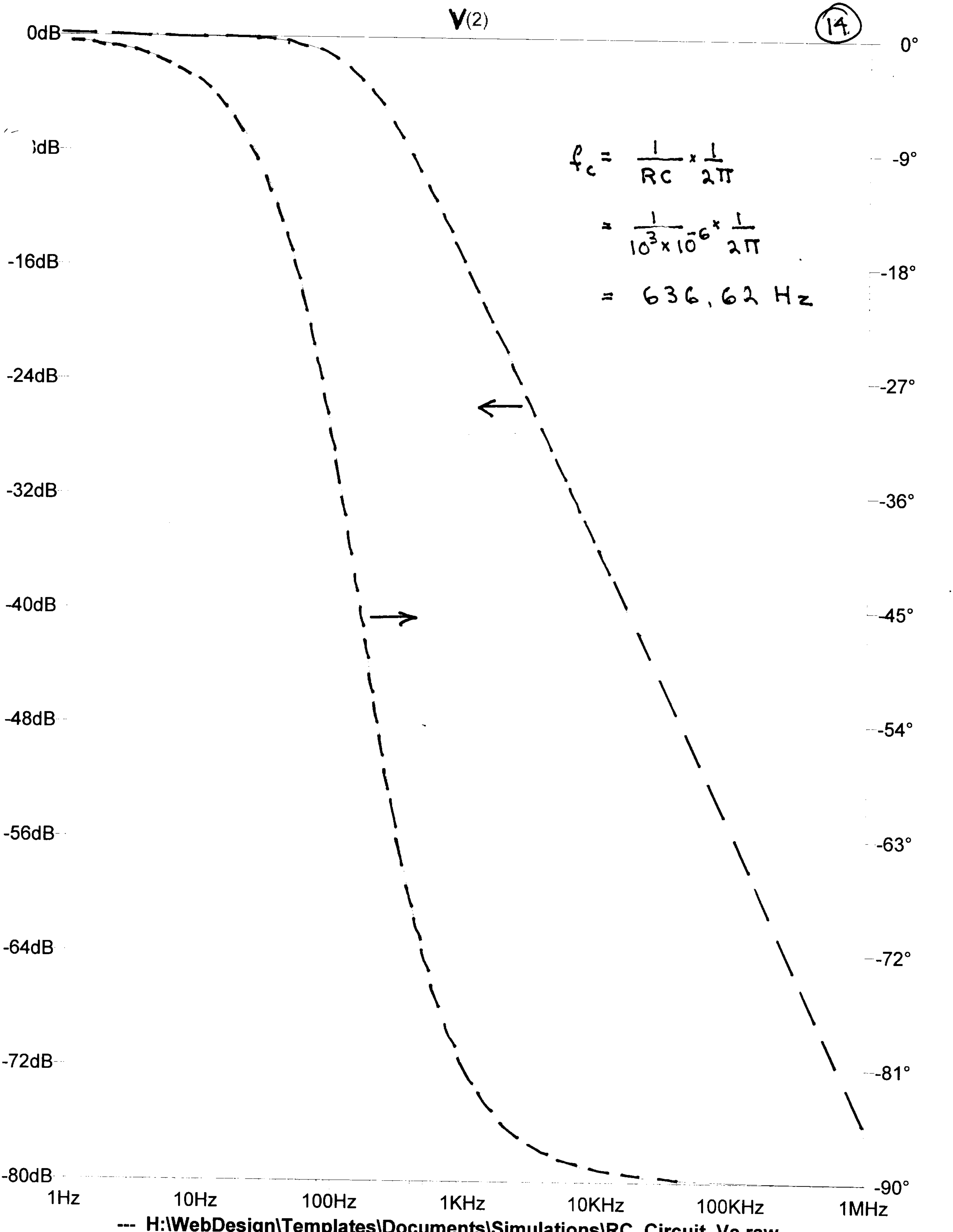
Try This For Fun With LT Spice

R-C Circuit LTSpice
sinwave excitation
frequency scanned from
100 Hz to 10^6 Hz




V(2)
0dB
-16dB
-24dB
-32dB
-40dB
-48dB
-56dB
-64dB
-72dB
-80dB
0°
-9°
-18°
-27°
-36°
-45°
-54°
-63°
-72°
-81°
-90°
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz
$f_c = \frac{1}{RC} \times \frac{1}{2\pi}$
$= \frac{1}{10^3 \times 10^{-6}} \times \frac{1}{2\pi}$
$= 636,62\ Hz$
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