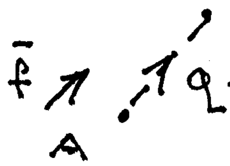
Variables we will deal with.

a) Voltage - The work <u>we do in moving</u> a unit charge from point A to point B



$q = 1$ Coulomb (an electron is 1.602×10^{-19} C.)

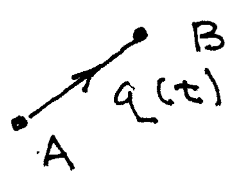
$$\text{Work} = \underset{\text{newtons}}{(\text{force we exert})} \times \underbrace{(B - A)}_{\text{meters}}$$

$= V_{BA} =$ Volts at B <u>with respect to</u> A. The voltage is always relative to some chosen reference point.

The work done by "the field" is thus $-V_{BA}$

b) Current - rate of flow of charge

$$i = \frac{dq(t)}{dt}$$



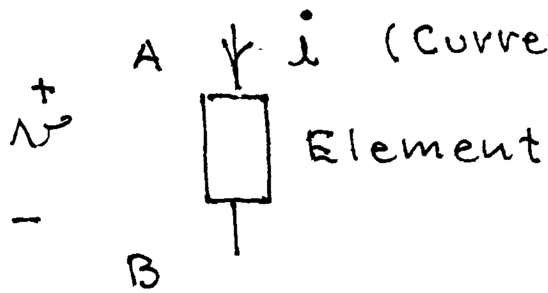
$$i_{BA} = \frac{dq(t)}{dt}$$

(what if $q(t)$ depends on where I am on AB → then I had better specify the position as well! $i_{BA}(\bar{r}) = \frac{d}{dt} q(t, \bar{r})$. This <u>generally</u> does not happen for our purposes)

Example 1.2 $\quad q(t) = 0.01 \sin(200t)$

$$i_{BA} = -0.01 \times 200 \cos(200t)$$
$$= -i_{AB}$$

Direction $\quad$ Choose Fig 1.14

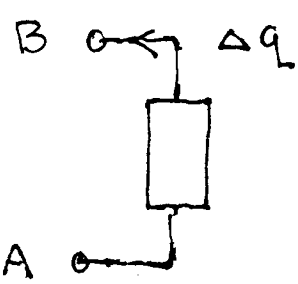


(Current positive when charge flows as shown (into A))

v is positive when $v_{AB} > 0$

<u>power</u>?

Power = rate at which work is done



$$\text{force} \times \text{distance} \times \frac{1}{\Delta t}$$

$$\Delta q \left(\frac{\text{force}}{\text{Coulomb}} \times \text{distance} \right) \frac{1}{\Delta t}$$

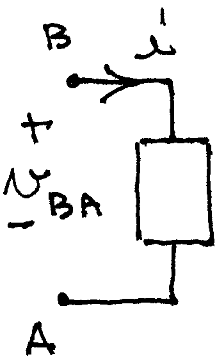
$$= \frac{\Delta q}{\Delta t} (V_{BA})$$

This is the work we do in overcoming the field to move charges from A to B. Its the work we put in.. the field

The work done by the field is the negative of this. This is the power of interest in electronics. Thus:

Power = rate at which work is done by the field is

$$= -\frac{\Delta q}{\Delta t} V_{BA} = \frac{(-\Delta q)}{\Delta t} V_{Ba}$$



$$= i\, v_{BA}$$

when the current reference is as shown. This implies field puts energy into the element. This can be dissipated or stored

Thus page 15 $\quad p = v\, i$ (volts, amps)

$$= \frac{\text{joules}}{\text{coulomb}} \times \frac{\text{coulomb}}{\text{sec}} = \frac{\text{joules}}{\text{sec}}$$

$$\triangleq \text{Watts}$$

Basic Circuit Elements

Element	Symbol	Linear Relation	In General
Resistor	i, $+v-$, R	$v = R\, i$	$v = R(p)\, i$ with $p = \frac{d}{dt}$; $\frac{1}{p}$
Inductor	i, $+v-$, Lp	$v = Lp\, i$	Can we write? $p\left(\frac{1}{p}\right) = 1$
Capacitor	i, $+v-$, $\frac{1}{Cp}$	$v = \frac{1}{C}\frac{1}{p}\, i$	Yes, since $\frac{d}{dt}\int_0^t v\, dt = v$

which is $\underbrace{p \times \frac{1}{p}\, v}$ =

Resistors

Current density   Elec Field

Basic σ conductivity (More/advanced $\vec{J} = \sigma \vec{E}$)

Units $\frac{Amps}{m^{\cancel{2}}} \times \frac{\cancel{m}}{Volt} = \frac{Amps}{Volt\, m} = \frac{1}{Ohm-m}$ $\frac{Amp}{m^2}$ $\frac{V}{m}$

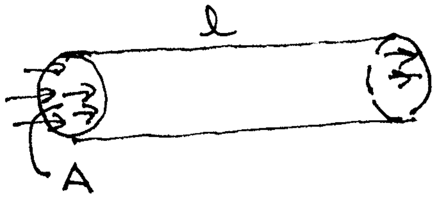
$\frac{1}{Ohm}$ is called Sieman

$\rho \triangleq \frac{1}{\sigma}$ called resistivity (units are Ohm-cm)

Table 2-1 of Maharbiz gives a table of resistivities

(Eg Al 2.62×10^{-8} Ω-m   Si 2.3×10^{3} Ω-m)

Going from ρ to R (resistance)



Total current is (Amps)
current density $\left(\frac{Amps}{m^2}\right) \times$
Area (m^2)

Thus σ gets multiplied by A to obtain current. Consequently its (ρ/A) to obtain "total resistivity"

Also Volts is "Electric Field multiplied by distance, l" so

I = Total current = $\sigma \times A \times$ Electric Field (Volts/m)

$$I = \sigma \times A \times \frac{Volts}{l} = \left(\frac{\sigma A}{l}\right) Volts$$

Consequently from this

$$\text{Volts } (V) = \left(\frac{l}{\sigma A}\right) I \triangleq R' I$$

2.2
1.4
88
22
308

Thus

$$R = \frac{l}{\sigma A}$$

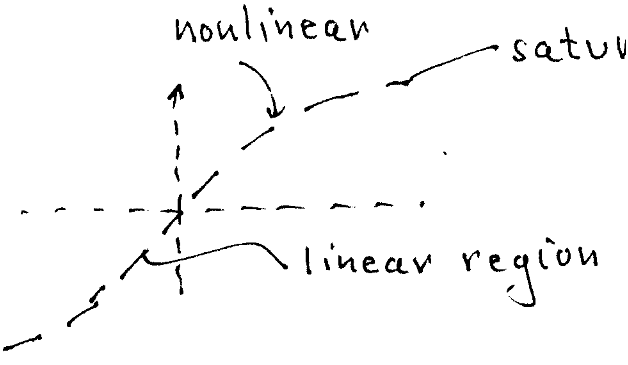
Example   Si Rod   d = 1mm, 5 cm long

$$R = \rho \frac{l}{A} = 2.3 \times 10^3 \times \frac{.05}{\pi (0.5 \times 10^{-3})^2} \simeq 3 \times 10^2 (M\Omega)$$

Various types of resistors

Thermistor
Piezoresistor
Rheostat
Potentiometer

$v = iR$ → plot



power Watts = Volts × Amps

(How do we establish this? Force = charge × Field
coulomb (charge unit) V/m

Energy = Force × distance
= Force × (meters)

Thus [Force × (m)] = $(\text{charge} \times \frac{V}{\cancel{m}}) \times \cancel{m}$

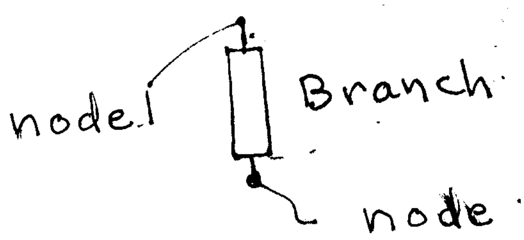
Power = Energy/time = (charge/time) × V
definition
= current × V
= $i\,v$)

To calculate power then

power = $i\,v = i\,(i\,R) = i^2 R$

can work out Example 2-1 d.c. Motor

A circuit element is abstracted as



- A node in a circuit is a conducting connection of two or more elements

- A branch is a trace between two nodes

- A path - trace of continuous sequence of branches for which a node is encount[ered] only once

- A loop - path ending and starting on the same node
  - Independent - one that contains at least one branch not contained in another loop
  - Mesh - loop that encloses no other loop

Basic Theorem b = No Branches ℓ = No of independent loops n = No of nodes

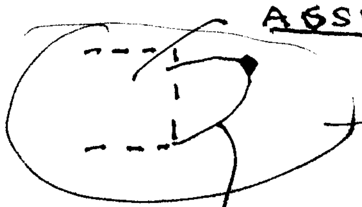
$$b = n + \ell - 1$$ - prove by induction

Proof (Induction)

$b = 1$
$n = 1$
$l = 1$

$b = n + l - 1$ **True**

<u>Assume True</u> $(b_n = n_n + l_n - 1)$



$b = b_n + 2$
$n = n_n + 1$

$\therefore (b_n + 2) = (n_n + 1) + \underbrace{(l_n + 1)}_{= l} - 1$

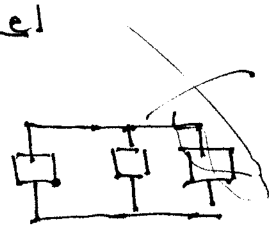
add 2 loops and node

Thus $b = n + l - 1$ — true

Thus true in general

<u>Series</u> And <u>Parallel</u>

series!



parallel!

<u>Planar</u> – If we can possibly draw the circuit in 2d.

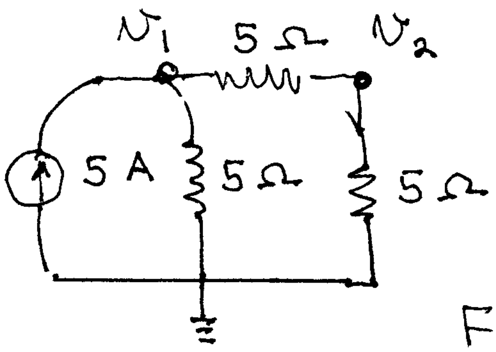
Loop and Node Systems of Equati

Loop → Kirchoff's Voltage "Law"

Node → Kirchoff's Current "Law" (KCL)

KCL → conservation of charge
Nodes do not store charge
Thus sum of currents into or out of a node has to be zero.

Ideal for circuits with Nodes and current sources. Let unknows be nod voltages - pick one as ground (our zer So $n-1$ independent equations. for $n-1$ node voltages,



$$5 = \frac{v_1}{5} + \frac{(v_1 - v_2)}{5}$$

$$\frac{v_2}{5} + \frac{v_2 - v_1}{5} = 0$$

Find v_2

Simultaneous Equations

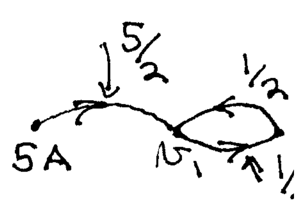
Graphical Solution Technique

$$v_1\left(\frac{1}{5}+\frac{1}{5}\right) = \frac{v_2}{5} + 5$$

$$v_2\left(\frac{1}{5}+\frac{1}{5}\right) = \frac{v_1}{5}$$

$$\boxed{G = \frac{1}{\Delta}\sum_{\substack{\text{All}\\ \text{Forward}\\ \text{paths}}} G_k \Delta_k}$$

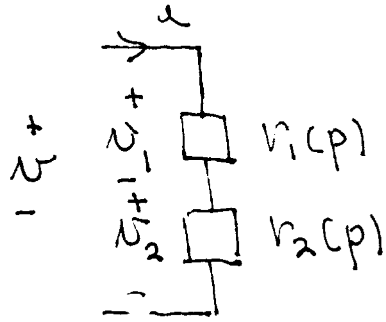
Variant of Cramers Rule



$$v_2 = 5G = 5\times\frac{5}{4}\ \frac{1}{1-\frac{1}{4}} = 2$$

Vo

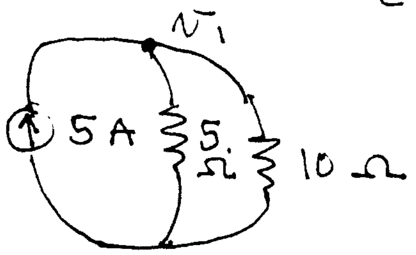
Other way combining resistors.



$$v = v_1 + v_2$$
$$= i_1 r_1(p) + i_2 r_2(p)$$
$$= i\,(r_1(p) + r_2(p))$$
$$= i\,(r(p))$$

Thus $r(p) = r_1(p) + r_2(p)$

So circuit is equivalent to

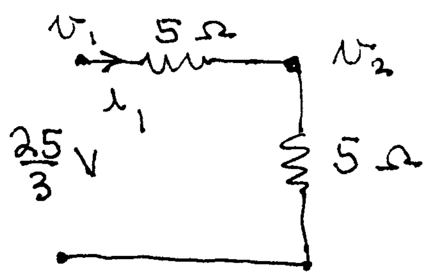


prove resistors in parallel add inversely! So equivalent to

$$\frac{1}{\frac{1}{5} + \frac{1}{10}} = \frac{50}{15} = \frac{10}{3}\,\Omega$$

$$\therefore\ v_1 = 5\,A \times \frac{10}{3}\,\Omega = \frac{50}{3}\,V$$

Now we unfold it.



$$i_1 = \frac{v_1}{10} = \frac{5\cancel{0}}{3\cancel{0}}\ \text{Amp}$$

$$\therefore\ v_2 = 5 \times i_1 = \frac{25}{3}\ \text{Volt}$$

We have actually use KVL

KVL - The potential at any point is uniquely defined. Thus the sum of th voltage drops across elements around a loop must be zero