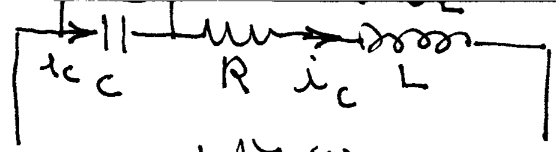
Second Order Circuit



$+ \; v_{in}(t) \; -$

$$v_c(t) = \frac{1/pC}{1/pC + pL + R} \, v_{in}(t)$$

$$= \frac{1}{1 + p^2LC + RpC} \, v_{in}(t)$$

$$v_{in}(t) = A\cos\omega t \quad \rightarrow \text{ use } \quad \frac{A}{2} e^{j\omega t}$$

$$v_c(t) = \text{Re} \, \frac{1}{1 - \omega^2 LC + Rj\omega C} \, A e^{j\omega t}$$

take real

$$= \frac{1}{\sqrt{(1-\omega^2LC)^2 + (R\omega C)^2}} \, A \, \text{Re}\left(e^{j\omega t - j\varphi}\right)$$

$$\varphi = \tan^{-1} \omega RC / (1 - \omega^2 LC)$$

(1) $$\boxed{v_c(t) = \frac{A}{\left((1-\omega^2LC)^2 + (R\omega C)^2\right)^{1/2}} \cos(\omega t - \varphi)}$$

<u>Forced Response</u>

<u>Natural Response</u> (Homogeneous solution)

This occurs when $v_c(t) \neq 0$ even if $v_{in}(t) = 0$

This can only happen when $1 + p^2LC + RpC = 0$

or $p^2 + p\,R/L + 1/LC = 0 \Rightarrow p = -\frac{1}{2}\frac{R}{L} \pm \sqrt{\frac{1}{4}\left(\frac{R}{L}\right)^2 - \frac{1}{LC}}$

$$= \begin{cases} s_1 \rightarrow \text{with + sign} \\ s_2 \rightarrow \text{with − sign} \end{cases}$$

$v_{c_h} = B e^{s_1 t} + C e^{s_2 t}$ satisfies this.

since $\frac{1}{1 + p^2LC + RpC} e^{s_1 t} = \frac{1}{1 + s_1^2 LC + R s_1 C} e^{s_1 t}$ and

similarly for $e^{s_2 t}$

Total solution is the sum of

$$v_c(t) + v_{ch}(t) = v_{c\,Tot}(t)$$

Example: Find $v_{c_T}(t)$ when $v_c(0^+) = 0$, $i_L(0^-) = 0$

when $v_{in}(t) = A\cos\omega t$

Solution

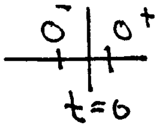
Since $v_c(0^+) = \dfrac{A}{((1-\omega^2LC)^2 + (R\omega C)^2)^{1/2}}\cos(\phi)$

$v_{ch}(0^+)$ must be $-v_c(0^+)$

Thus $\boxed{B + C = -v_c(0^+)}$ → need another condition (observe $i_c = C\dfrac{dv_c}{dt}$; $v_L = L\dfrac{di_c}{dt}$)

If v_L is finite $\dfrac{di_c}{dt}$ can change at $t=0$ **BUT**

$$i_c(0^+) - i_c(0^-) = \int_{0^-}^{0^+}\left(\frac{v_L}{L}\right)dt$$



→ 0 as 0^+ approaches 0^-. Thus

$i_c = i_L$ can't change instantaneously and $i_c(0^+) = 0$

$$i_c\Big|_{t=0} = C\frac{dv}{dt} = \frac{A}{((1-\omega^2LC)^2 + (R\omega C)^2)^{1/2}}(-\omega\sin\phi) + BS_1 + CS_2 = 0$$

Thus we determine B and C from

$$B + C = -\frac{A}{((1-\omega^2LC)^2 + (R\omega C)^2)^{1/2}}\cos\phi$$

$$BS_1 + CS_2 = +\frac{\omega A}{((1-\omega^2LC)^2 + (R\omega C)^2)^{1/2}}\sin\phi$$

We note that the homogeneous (or natural part of the solution always decays because $p = -\frac{1}{2}\frac{R}{L}$ for S_1 and S_2

LT Spice plot of v_c

LT Spice plots the magnitude and phase of the steady-state! That is it plots (from page 1)

$$dB = 20 \lg\left[\frac{A}{((1-\omega^2 LC)^2 + (R\omega C)^2)^{1/2}} \Big/ A\right]$$

magnitude of $v_c(t)$ at low frequencies

$$= 20 \lg\left[\frac{\text{magnitude}(\omega)}{\text{magnitude}(0)}\right]$$

this eliminates A the excitation amplitude

versus $\lg(\omega)$

It also plots the phase φ

$$-\varphi = -\tan^{-1} \omega RC/(1-\omega^2 LC)$$

versus $\lg \omega$

<u>We thus see</u> $dB = 0$ for ω small

and $dB = 20 \lg \frac{1}{\omega^2 LC} = -40 \lg \omega - 20 \lg LC$ for ω large. It is a line of slope -40 dB/decade

Also φ starts at 0 for small ω and approaches $\varphi \to -\tan^{-1}\infty = -\pi/2$ for $\omega = \frac{1}{\sqrt{LC}}$

and $\varphi \to -\tan^{-1} 0 = -\pi$ for $\omega \to \infty$

The height of the magnitude at $\omega = \frac{1}{\sqrt{LC}}$

(Eq 1) is given by $\frac{1}{R\frac{1}{\sqrt{LC}}C} = \frac{1}{\frac{R\sqrt{LC}}{L}} = \frac{1}{R/\omega_0 L} = \frac{\omega_0 L}{R}$

with $\omega_0 = \frac{1}{\sqrt{LC}}$. $\frac{\omega_0 L}{R} = Q$ is called the quality factor of the circuit