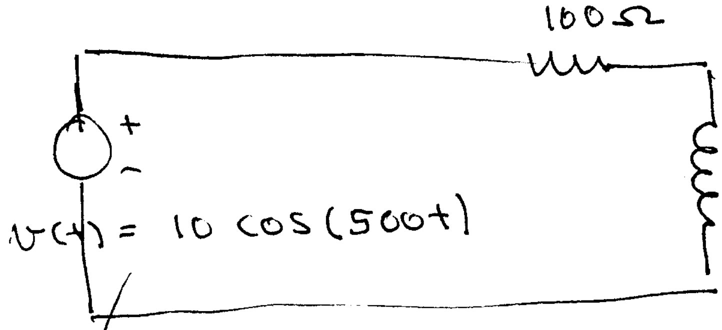
Problem 5.41 -



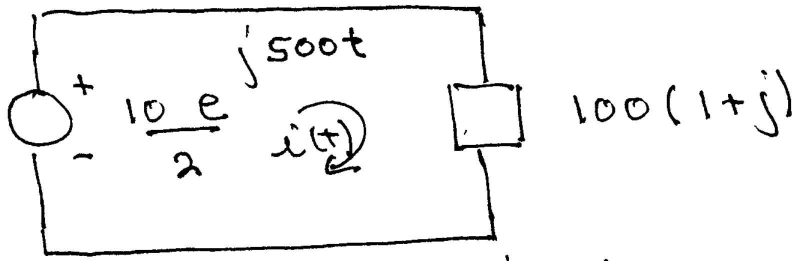
$.2H \rightarrow j\omega L = j\,500 \times .2 = j100$

phasor

write

$$v(t) = \frac{10\, e^{j500t}}{2} + c.c.$$

find response



$$i(t) = 2\,\mathrm{Re}\; \frac{10}{2} e^{j500t} / (100(1+j))$$

$$= 2\,\mathrm{Re}\; \frac{10}{2} e^{j500t} / (100\sqrt{2}\, e^{j\tan^{-1}1})$$

$$= 2\,\mathrm{Re}\; \frac{10}{2} \times \frac{1}{100} \times \frac{1}{\sqrt{2}}\, e^{j500t}\, e^{-j\frac{\pi}{4}}$$

phasor current I

phasor voltage

(every thing but the 2's (since they cancel and the $e^{j\omega t} = e^{j500t}$)

$$\boxed{i(t) = \frac{1}{10\sqrt{2}} \cos\left(500t - \frac{\pi}{4}\right)}$$

This is the Steady-State Response

∴ phasor current

$$\frac{10}{100}\times\frac{1}{\sqrt{2}}e^{-j\pi/4}$$

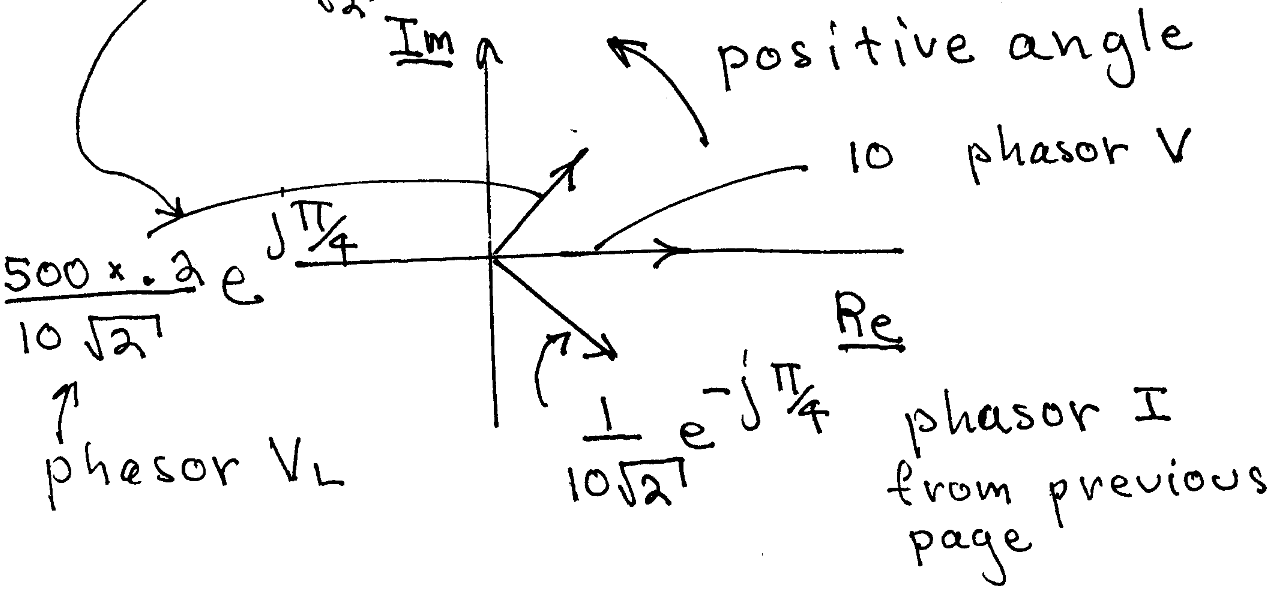
$$v_L(t) = L\frac{di}{dt}$$

$$= L\frac{d}{dt}\,\mathrm{Re}\,\frac{1}{10\sqrt{2}}e^{j500t}e^{-j\pi/4}$$

$$= \mathrm{Re}\left(\underbrace{j500}_{\frac{d}{dt}\text{ gives this}}\, e^{j500t}\,\underbrace{e^{-j\pi/4}\frac{L}{10\sqrt{2}}}_{\text{phasor } V_L}\right)$$

$$= \mathrm{Re}\,\left(\underbrace{\frac{500L}{10\sqrt{2}}e^{-j\pi/4}e^{+j\pi/2}}_{\text{phasor } V_L}\right)e^{j500t}$$

$$v_L(t) = \frac{10}{\sqrt{2}}\cos(+\pi/4 + 500t)$$



Note that $i(0) \neq 0$. $i(0) = \frac{1}{10\sqrt{2}}\cos\frac{\pi}{4}$

We need the homogeneous solution (or natural solution to have $i(0) = 0$

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Natural solution

① $\dashrightarrow$ $i(t) = \dfrac{v(t)}{R+pL}$ $\qquad v(t) = 10\cos 500t$

If $v(t) = 0$ $\quad i(t) \neq 0$ when $p = -R/L$, that is denominator $= 0$

Thus $\quad i(t) = B e^{-R/L\, t} = B e^{-500t}$

$i(0) = B = -\dfrac{1}{10\sqrt{2}} \cos(\pi/4) = -\dfrac{1}{20}$ from the previous page. Thus complete solution is

$$\boxed{i(t) = \frac{1}{10\sqrt{2}} \cos(500t - \pi/4) - \frac{1}{20} e^{-500t}}$$

Integration Factor Approach

obtain DE. From ① above

$$\left(R + L\frac{d}{dt}\right) i(t) = v(t)$$

or $\quad L\left(\dfrac{d}{dt}\left(i(t)\, e^{R/L\, t}\right)\right) = v(t)\, e^{R/L\, t}$ integration faltor

integrate (definite integral) both sides

$$L\,[\,i(t)\, e^{R/L\, t} - \underset{\substack{\parallel \\ 0}}{i(0)}\,] = \int_0^t \underset{\substack{\uparrow \\ 10\cos 500t}}{v(t')}\, e^{R/L\, t'}\, dt'$$

$$\int_0^t 10\cos 500t'\, e^{R/L\, t'}\, dt' = 2\,\mathrm{Re}\left[\int_0^t \frac{10}{2} e^{j500t' + R/L\, t'}\, dt'\right]$$

$$= \mathrm{Re}\left[10\,\frac{\left(e^{j500t + R/L\, t} - 1\right)}{j500 + R/L}\right], \quad R/L = 500$$

rect to polar $\;\rightarrow\; 500\sqrt{2}\, e^{j\tan^{-1} 1}$

$$\therefore\; L\, i(t)\, e^{500t} = \mathrm{Re}\left[\frac{10}{500\sqrt{2}}\, e^{-j\pi/4}\left(e^{j500t + 500t} - 1\right)\right]$$

or $\quad \boxed{i(t) = \dfrac{1}{10\sqrt{2}} \left(\cos(500t - \pi/4) - \cos\pi/4\; e^{-500t}\right)}$

which is the same as above