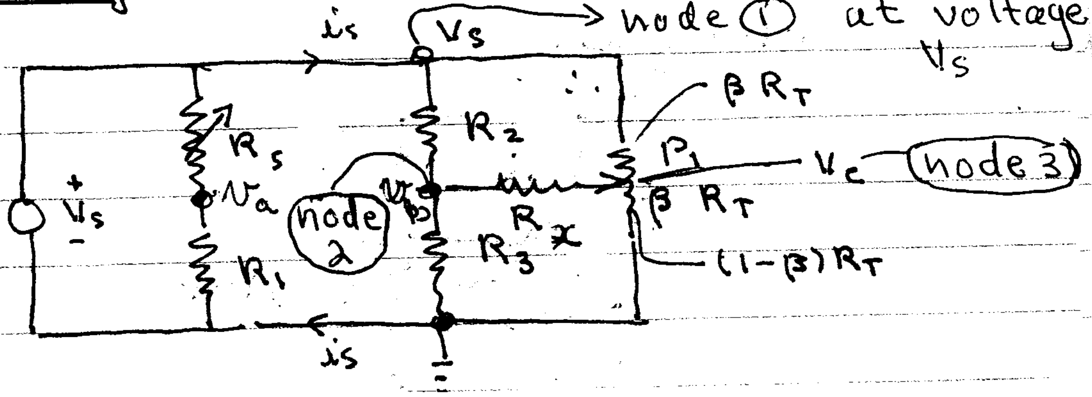
Note on Lab 2 Equation page 3

Full Bridge Alternative to Δ-Y Transform



node equations

node 1 KCL: $i_s - \frac{V_s - v_b}{R_2} - \frac{V_s - v_c}{\beta R_T} = 0$ — gives i_s in terms of V_s, v_b & v_c

node 2 KCL: $\frac{v_b - V_s}{R_2} + \frac{v_b - v_c}{R_x} + \frac{v_b}{R_3} = 0$

node 3 KCL: $\frac{v_c - V_s}{\beta R_T} + \frac{v_c - v_b}{R_x} + \frac{v_c}{(1-\beta)R_T} = 0$

→ give v_b and v_c

3 gives $v_c = \left(\frac{V_s}{\beta R_T} + \frac{v_b}{R_x}\right) \Big/ \left(\frac{1}{\beta R_T} + \frac{1}{R_x} \frac{1}{(1-\beta)R_T}\right)$

$= \left(\frac{V_s}{\beta R_T} + \frac{v_b}{R_x}\right) \Big/ \left(\frac{R_x + \beta R_T(1-\beta)}{R_x \beta R_T(1-\beta)}\right)$

sub in 2 to obtain v_b

$$\left(\frac{1}{R_2}+\frac{1}{R_x}+\frac{1}{R_3}\right)v_b = \left(\frac{V_s}{R_2} + \frac{v_c}{R_x}\right)$$

Let $R = R_2 = R_3 = R_{ref} = R$

Then

$$\left(\frac{2}{R}+\frac{1}{R_x}\right)v_b = \left(\frac{V_s}{R} + \frac{1}{R_x}\left[\frac{V_s}{\beta R_T} + \frac{v_b}{R_x}\right] \Big/ \left(\frac{R_x+\beta(1-\beta)R_T}{R_T R_x \beta(1-\beta)}\right)\right)$$

$$v_b\left(\frac{2}{R}+\frac{1}{R_x} - \frac{1}{R_x^2}\frac{R_x\beta(1-\beta)R_T}{R_x+\beta(1-\beta)R_T}\right) = V_s\left(\frac{1}{R} + \frac{(1-\beta)}{R_x+\beta(1-\beta)R_T}\right)$$

cancel

or $$\frac{v_b}{V_s}\left(\frac{(R_x+\beta(1-\beta)R_T)(2R_x+R) - R_T\beta(1-\beta)R}{R R_x(R_x+\beta(1-\beta)R_T)}\right) = \left(\frac{1}{R}+\frac{(1-\beta)}{R_x+\beta(1-\beta)R_T}\right)$$

Thus

$$\frac{v_b}{V_s}\left(\frac{2R_x^{\cancel{2}} + \cancel{R_x}R + 2\cancel{R_x}\beta(1-\beta)R_T}{R\cancel{R_x}\cancel{(R_x+\beta(1-\beta)R_T)}}\right)$$

$$= \frac{\cancel{R_x + \beta(1-\beta)R_T} + R(1-\beta)}{R\,\cancel{(R_x+\beta(1-\beta)R_T)}}$$

Thus

$$\frac{v_b}{V_s} = \frac{(1-\beta)\overset{R_{ref}}{R} + \beta(1-\beta)R_T + R_x}{R + 2R_x + 2\beta(1-\beta)R_T}$$

v_a found simply by a voltage divider relation

$$v_a = \frac{R_1}{R_1 + R_S} \times V_S$$

Note: have used

R_1 for $R_{1\,ref}$, etc

and then assumed

$\ldots\ R_2 = R_3$