UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering and Computer Scences
EECS 100/42
Midterm Examination Fall 2009

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PRINT YOUR NAME: Solutions

S.I.D. :

SIGNATURE:

Do your work on the exam. If you do need to use extra sheets, attach these to the exam so that these are considered as well. Make your methods clear so that partial credit is possible. There are three problems. THE PROBLEMS ARE EACH WORTH 12.

Useful Equations

Complex Exponentials

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad \sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad e^{jx} = \cos x + j \sin x$$

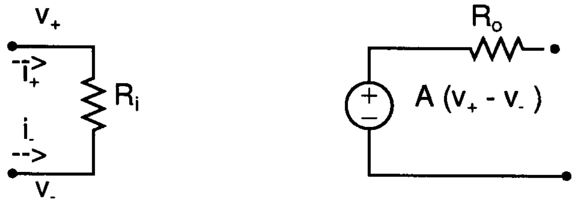
Natural Solution Forms for First and Second Order Differential Equations

Differential Equation Form	Natural Solution Form
$\frac{df(t)}{dt} + af(t) = x(t)$	$f_n(t) = ke^{-at}$
$\frac{d^2f(t)}{dt^2} + a\frac{df(t)}{dt} + bf(t) = x(t)$, overdamped	$f_n(t) = k_1e^{s_1t} + k_2e^{s_2t}$
$\frac{d^2f(t)}{dt^2} + a\frac{df(t)}{dt} + bf(t) = x(t)$, critically damped	$f_n(t) = k_1e^{s_1t} + k_2te^{s_1t}$
$\frac{d^2f(t)}{dt^2} + a\frac{df(t)}{dt} + bf(t) = x(t)$, underdamped	$f_n(t) = e^{-\alpha t}(k_1 \cos \omega_d t + k_2 \sin \omega_d t)$, where $s_{1,2} = -\alpha \pm j\omega_d$

For second order differential equations, s_1 and s_2 are the roots of the characteristic equation.

Problem Number One) (Total 12)

3/12 a) For the operational amplifier model shown, give the conditions that apply when it is to be treated as ideal.



$R_i =$ ∞

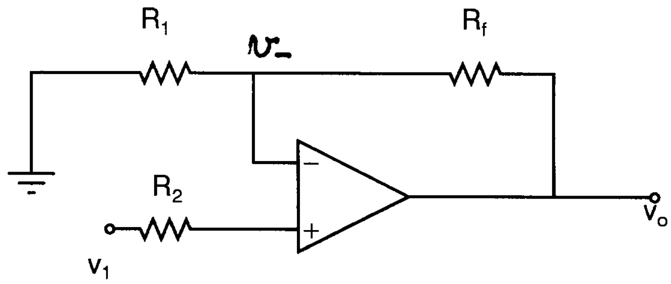
$R_o =$ 0

i_+ and $i_- =$ 0

$A =$ ∞

Relation between v_+ and v_- $v_+ = v_-$

For the circuit shown below assume ideal operational amplifier conditions



3/ 12 b) Determine $\frac{v_o}{v_1}$

KCL $\dfrac{v_o - v_-}{R_f} = \dfrac{v_-}{R_1}$

So $v_o = v_- \left(\dfrac{1}{R_f} + \dfrac{1}{R_1}\right) R_f$

$v_- = v_1$ (ideal Op Amp)

$\frac{v_o}{v_1} =$ $R_f(R_1 + R_f)/(R_1 R_f)$

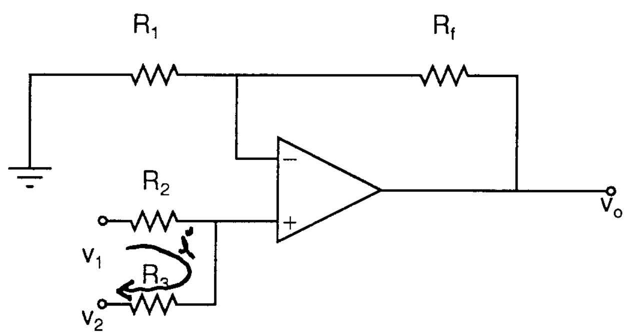
2/12 c) What is the input resistance, R_{in} for v_o open (not connected to any load) ?

$i_+ = 0$ (ideal Op Amp)

Thus $\frac{v_1}{i_+} = \infty$

$R_{in} =$ ∞

A second input is added as shown:



2/12 d)Obtain the output v_o in terms of the two inputs v_1 and v_2.

ideal Op Amp $i_+ = 0$

KCL around input loop $v_1 - v_2 = i(R_2 + R_3)$

$v_- = v_+ = v_1 - iR_2 = v_1 - \frac{(v_1 - v_2)R_2}{R_2 + R_3}$

$= \frac{v_1 R_3 + v_2 R_2}{R_2 + R_3}$

KCL $\frac{v_o - v_-}{R_f} = \frac{v_-}{R_1}$

$v_o = R_f\left(\frac{1}{R_f} + \frac{1}{R_1}\right)v_- = \left(\frac{R_f + R_1}{R_1}\right)\left(\frac{v_1 R_3 + v_2 R_2}{R_2 + R_3}\right)$

2/12 e)Now what is the input resistance,R_{in1} seen by v_1 with v_2 set to zero, and v_o open?

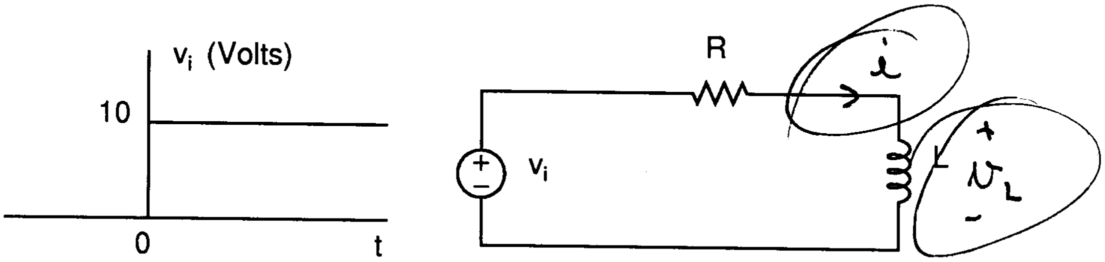
KCL around input loop

$v_1 - v_2 = i(R_2 + R_3)$

$v_2 = 0$; $\therefore \frac{v_1}{i} = (R_2 + R_3)$

$R_{in1} =$ $R_2 + R_3$

Problem Number Two) (Total 12)
For the circuit shown below $V_i(t)$ is the step waveform shown:



3/12 a) The initial current through the inductor ~~is 1 mA.~~ What is the voltage across the inductor at $t = 0^+$ in terms of R (and possibly L) (t^+ is Just after the $V_i(t)$ has jumped from 0 to 10 Volts)?

$t = 0^+$

i in the inductor can't change instaneously unless $v_L \to \infty$ ($v_L = L\frac{di}{dt}$ thus $\Delta i = [i(0^+) - i(0^-)] = \int_{0^-}^{0^+} \frac{v_L}{L} dt \to 0$ if v_L finite)

$\therefore i(0^+) = 1mA \quad \therefore v_L = 10 - iR = 10 - 10^{-3}R$

$v_L(t = 0^+) =$ $10 - 10^{-3} R$

3/12 b) What is the voltage across the inductor as $t --> \infty$?

$v_L(t = \infty) =$ 0 Volts

L becomes short ($p \to 0$)

3/12 c) Determine the inductor current as a function of time ($i_L(t=0^-) = 1mA$):

$i_L(0^+) = 10^{-3}$ Solution must be $e^{-t/T}$ + Const

$i_L(\infty) = 10/R$ $T\,[\sec] = (L/R)\,\frac{[H]}{[\Omega]}$

$\therefore\ i_L(t) = 10^{-3} e^{-Rt/L} + \frac{10}{R}\left(1 - e^{-tR/L}\right)$

Dimension of L/R is $H/\Omega = \frac{\text{Volt}}{\Omega\ \text{Amp/sec}} = \cancel{\frac{\Omega}{\Omega}} \times \sec$

$i_L(t) = \underline{10^{-3}\left(e^{-Rt/L}\right) + \frac{10}{R}\left(1 - e^{-tR/L}\right)}$

3/12 d) Find the steady-state inductor voltage if $v_i(t) = A\sin(\omega t)$ and $\omega L = R$.

$v_L(t) = \left(\frac{pL}{pL+R}\right) v_i(t)$

use $A \sin \omega t = \frac{A}{2i} e^{i\omega t} - \frac{A}{2i} e^{-i\omega t}$

use Superposition

$v_{L_1}(t) = \left(\frac{pL}{pL+R}\right)\frac{A}{2i} e^{i\omega t}$ for first term

$v_L(t) = \underline{\frac{A}{\sqrt{2}} \cos(\omega t - \pi/4)}$

$= \frac{i\omega L}{i\omega L + R} \frac{A}{2i} e^{i\omega t}$

$= \frac{\cancel{i}}{i+1} \frac{A}{2\cancel{i}} e^{i\omega t}$

for second term $v_{L_2}(t) = \frac{1}{-i+1} A e^{-i\omega t}$

$v_L(t) = v_{L_1}(t) + v_{L_2}(t)$

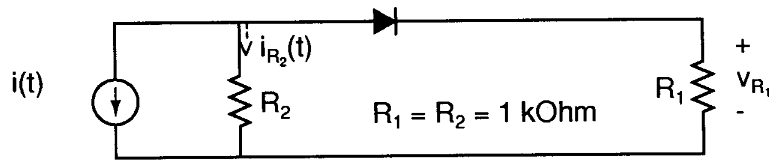
$= \cancel{2} \operatorname{Re}\left(\frac{1}{1+i} \frac{A}{\cancel{2}} e^{i\omega t}\right)$

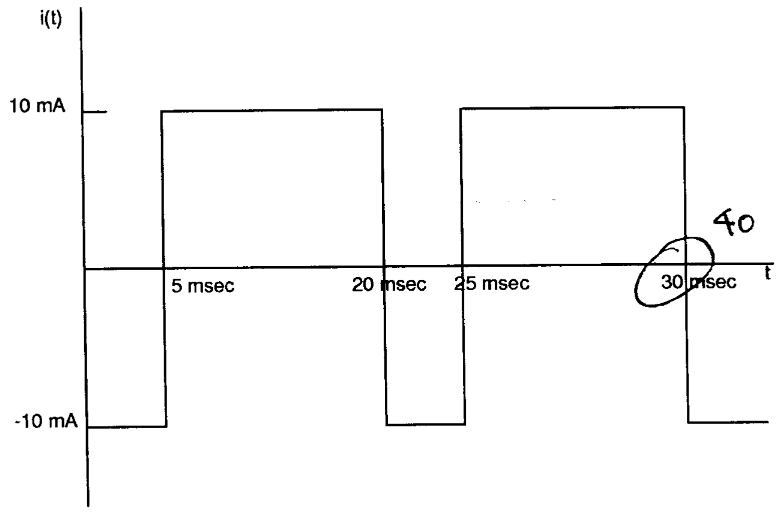
$= \operatorname{Re}\left(\frac{1}{\sqrt{2}\, e^{i\pi/4}} A e^{i\omega t}\right)$

$= A \operatorname{Re} \frac{1}{\sqrt{2}} e^{i(\omega t - \pi/4)} = \frac{A}{\sqrt{2}} \cos(\omega t - \pi/4)$

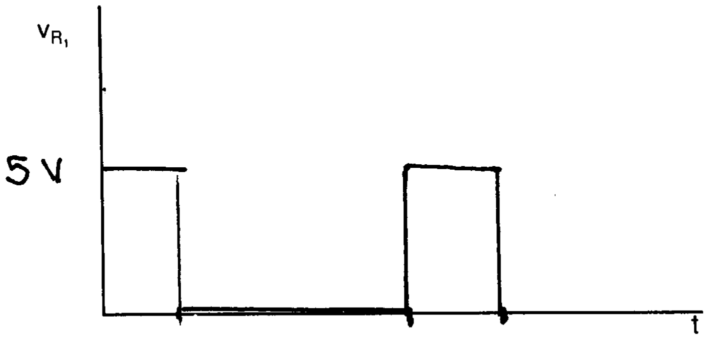
Problem Number Three) (Total 12)

For the following circuit, assume the diode to be ideal.

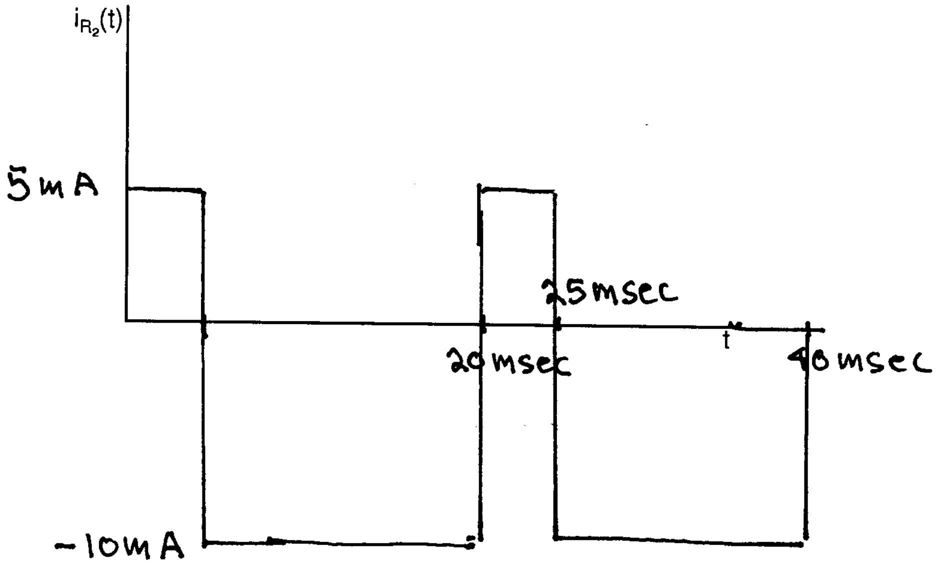




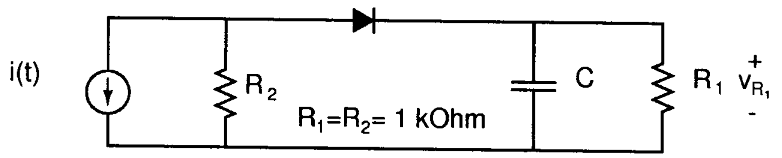
3/12 a) For current excitation shown plot the voltage $v_{R_1}(t)$ across R_1 as a function of time.



2/12 b) Similarly plot the current $i_{R_2}(t)$ through R_2 as a function of time.



2/12 c) A capacitor is added as shown:



If its value is .0001μ F what is the value of the output voltage across R_1 after 5 ms? (The capacitor is initially uncharged).

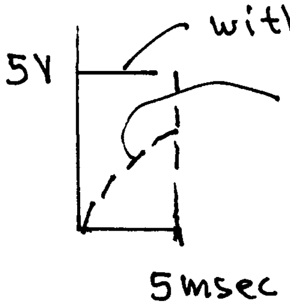
$RC = (500\,\Omega) \times 10^{-10}\,F = 5 \times 10^{-7}$ sec - very short compared with 5 msec so result does not change (very small change)

$v_{R_1}(t) =$ 5 Volts

3/12 d) The value of the capacitance is increased to $10\mu F$. What is the output voltage after 5ms? (The capacitor is initially uncharged.)

RC is now 506×10^{-5} sec = 5 msec so
result now changes

without C



with c exponentially approaches 5V

$$v_c(t) = 5\,(1 - e^{-t/5})$$

$$= 5\left(1 - \tfrac{1}{e}\right) = 5\,\frac{(1.718)}{(2.718)}$$

$v_{R_1}(t) =$ 3.16 Volts

2/12 e) What is the output voltage after 20ms for $C = 10\mu F$?

Voltage exponentially decays with
time constant 1000×10^{-5} sec = 10 msec

After 20 msec

$$\frac{5\,(1.718)}{2.718}\, e^{-15/10} = \frac{5 \times 1.718}{2.718} \times \frac{1}{2.718\sqrt{2.718}}$$

$$= .705\ V$$

$v_{R_1}(t) =$ .705 V