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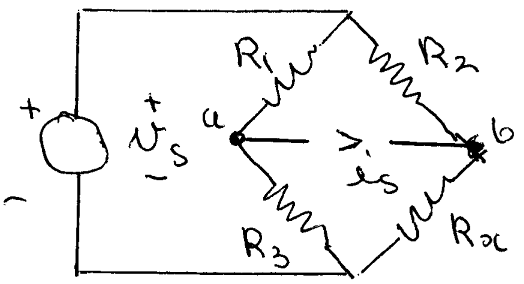
Solutions
Problem Set 2

Sept 23 2009

Problem 2.101 - Wheatstone Bridge



Balanced

$i_s = 0$ use voltage divider

$$\boxed{v_{ab} = v_s\left(\frac{R_3}{R_1+R_3} - \frac{R_2}{R_2+R_x}\right)}$$

$R_1 = 10k\Omega$, $R_3 = 3419\,\Omega$, $R_2 = 1k\Omega$

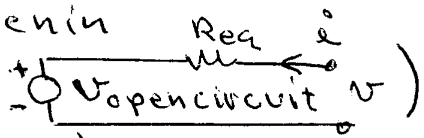
$$\frac{R_3}{R_1+R_3} = \frac{R_2}{R_2+R_x} \quad \text{solve for } R_x$$

Prob No. 1

Problem 2.102 Now the $i_s \neq 0$

Problem suggests using Thevenin

$(v = v_{open\,circuit} + i\,R_{eq}$



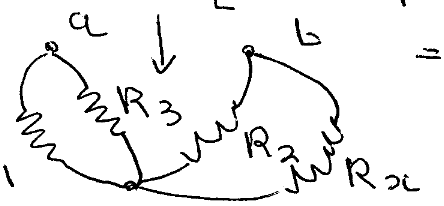
$$v_{open\,circuit} = v_s\left(\frac{R_3}{R_1+R_3} - \frac{R_2}{R_2+R_x}\right)$$

$$\boxed{v_{open\,circuit} = v_s\left(\frac{R_3R_x - R_2R_1}{(R_1+R_3)(R_2+R_x)}\right)} \quad \text{Eq (1)}$$

To find R_{eq} set $v_{open\,circuit} = 0$

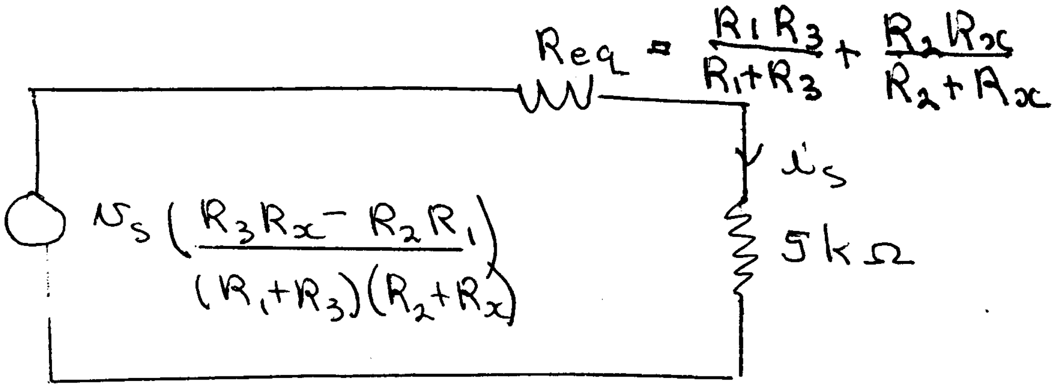
Equivalent is

$$R_{eq} = R_1 \| R_3 + R_2 \| R_x = \frac{R_1R_3}{R_1+R_3} + \frac{R_2R_x}{R_2+R_x}$$



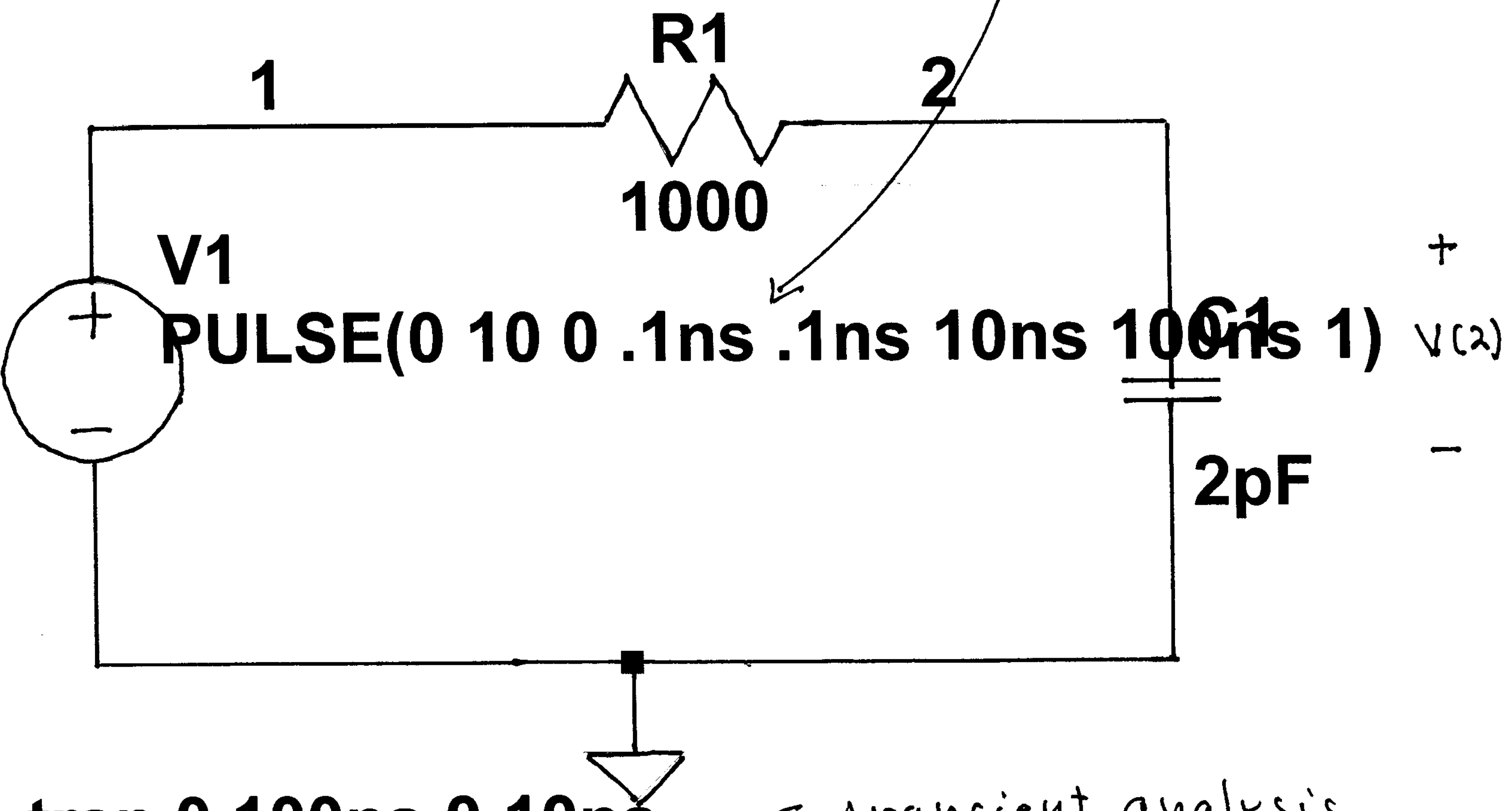
a) when bridge is balanced $i_s = 0$ $v_{ab} = 0$
So result above holds (Eq 1)

b)


$R_{eq} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_x}{R_2 + R_x}$
i_s
$5 k\Omega$
$v_s \left(\frac{R_3 R_x - R_2 R_1}{(R_1 + R_3)(R_2 + R_x)} \right)$

Fun With LT Spice. (Problem 2)

Pulse starting at 0 Volts ramping up to 10 Volts with a rise-time .1 nsec, fall time of .1 nsec, 10 nsec. in duration a period of 100 nsec. 1 cycle plotted. This is the spice netlist statement



.tran 0 100ns 0 10ns ← transient analysis from 0 to 100 nsec for a pulse from 0 to 10 nsec

result for V(2) next page

(Note your problem asked for a 4 nsec delay Here it is taken as 0. Just shift these results by 4 nsec to obtain the delay)

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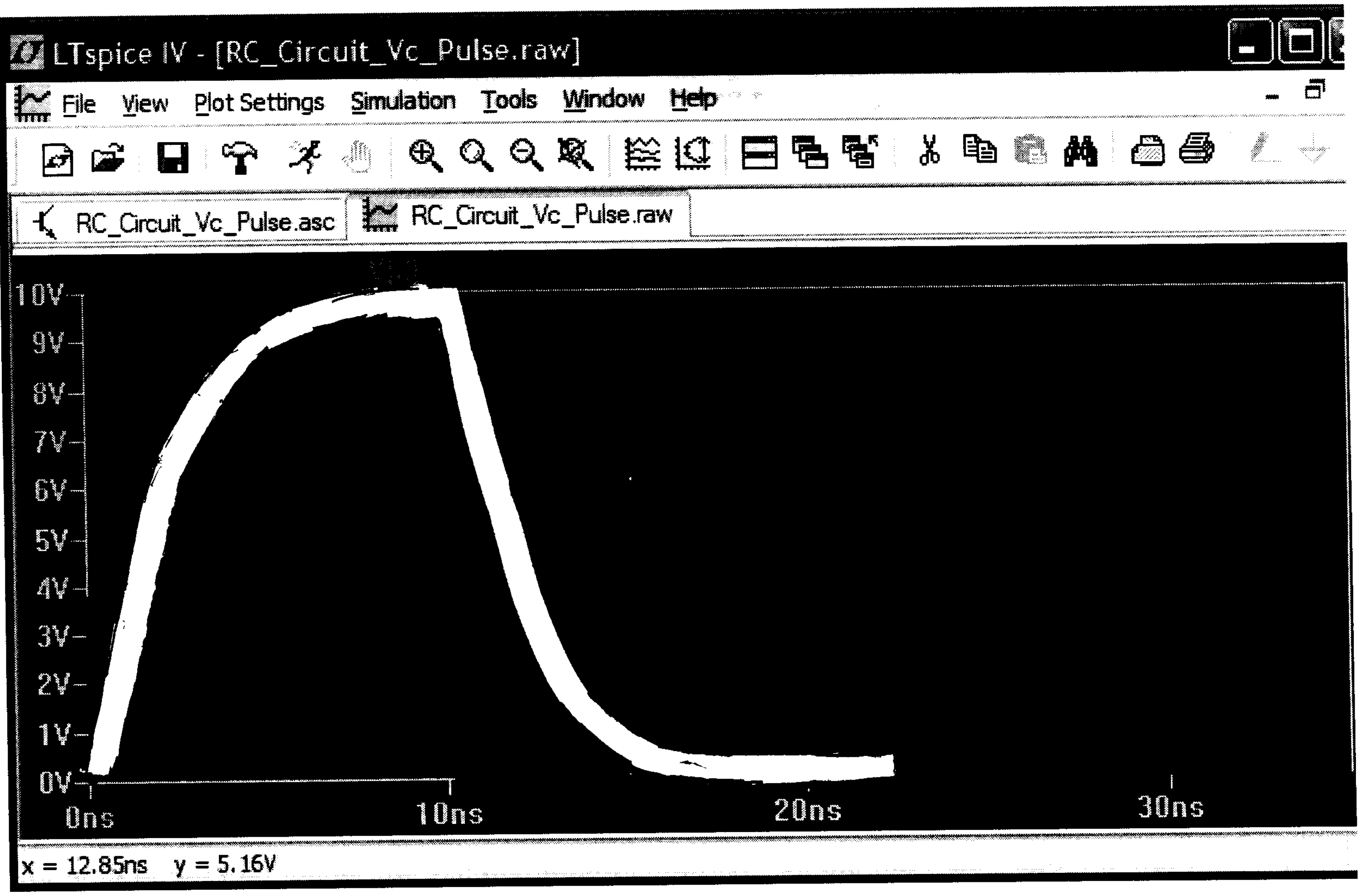


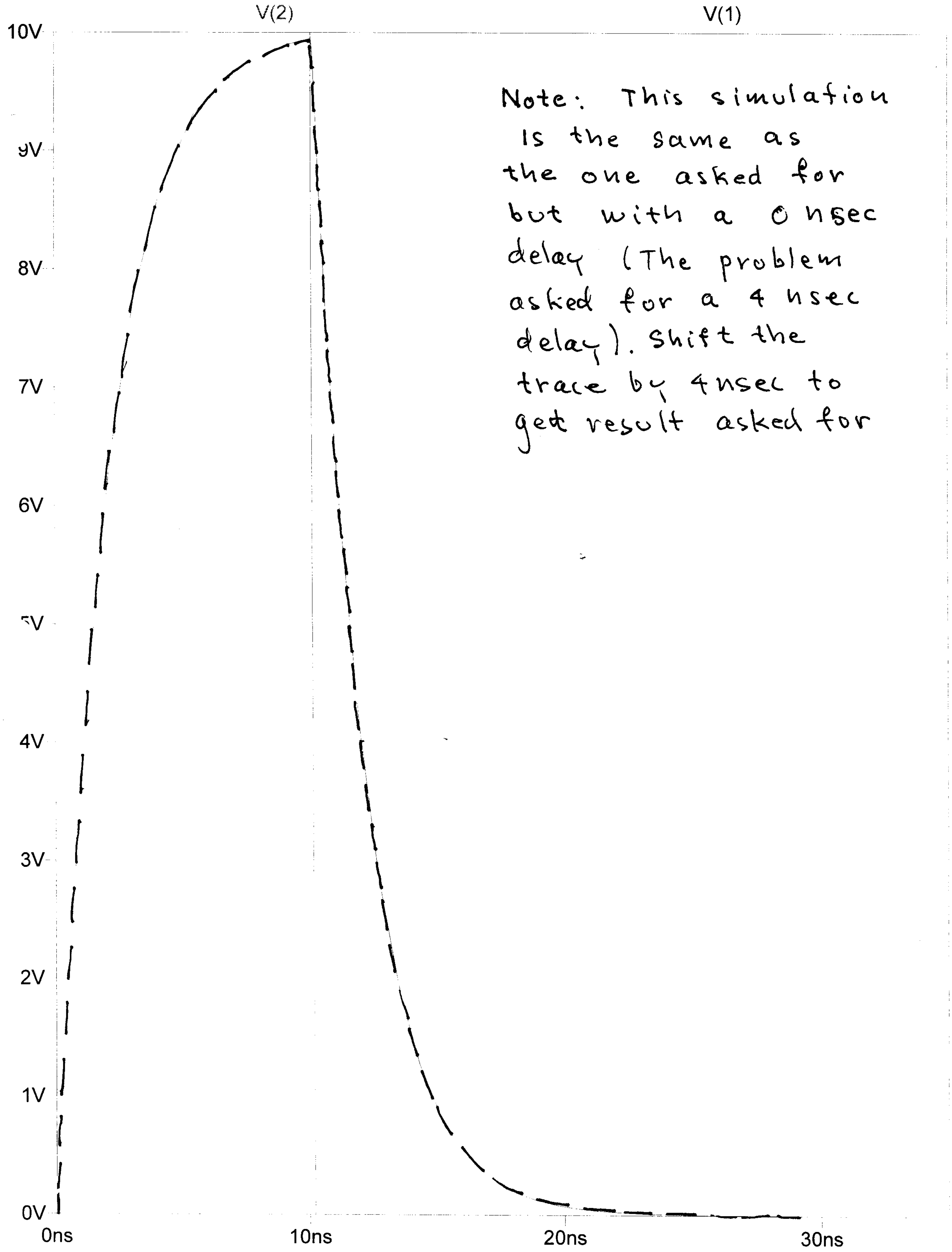
Can download LTspice from

www.linear.com

Getting started guide

www.linear.com/designtools/software/

LTSpice Getting Started Guide.pdf



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Ans ? $v_o \simeq - R_f i_s$

The sub-circuit disappears

Node ①

① $$v_i\left(\frac{1}{R_i}+\frac{1}{R_s}\right)+\frac{v_i-v_o}{R_f}=+i_s$$

Node ②

② $$\frac{v_o+Av_i}{R_o}+\frac{v_o-v_i}{R_f}=0$$

① $$v_i\left(\frac{1}{R_i}+\frac{1}{R_s}+\frac{1}{R_f}\right)=+i_s+\frac{v_o}{R_f}$$

sub in ②

$$v_o\left(\frac{1}{R_o}+\frac{1}{R_f}\right)+\left(\frac{A}{R_o}-\frac{1}{R_f}\right)\frac{\left(+i_s+\frac{v_o}{R_f}\right)}{\frac{1}{R_i}+\frac{1}{R_s}+\frac{1}{R_f}}=0$$

$$v_o\left(\frac{1}{R_o}+\frac{1}{R_f}+\frac{1}{R_f}\left(\frac{A}{R_o}-\frac{1}{R_f}\right)\frac{1}{\frac{1}{R_i}+\frac{1}{R_s}+\frac{1}{R_f}}\right)+\left(\frac{A}{R_o}-\frac{1}{R_f}\right)\frac{i_s}{\left(\frac{1}{R_i}+\frac{1}{R_s}+\frac{1}{R_f}\right)}=0$$

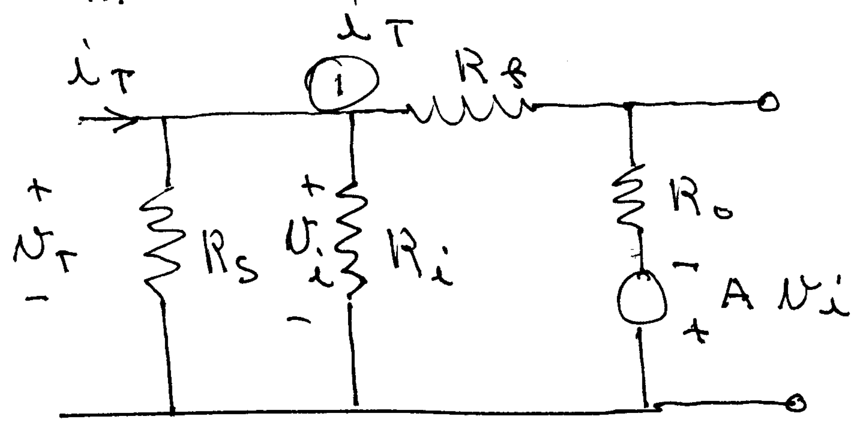
a) so $$\boxed{v_o/i_s=\frac{-\left(A/R_o-1/R_f\right)}{\left(\frac{1}{R_i}+\frac{1}{R_s}+\frac{1}{R_f}\right)\left(\frac{1}{R_o}+\frac{1}{R_f}+\frac{1}{R_f}\left(\frac{A}{R_o}-\frac{1}{R_f}\right)\frac{1}{\left(1/R_i+1/R_s+1/R_f\right)}\right)}}$$

b) Input Resistance - Resistance measured (8) when looking into terminals a) and b)

Note: <u>Should specify</u> condition of the output terminals: We shall assume them to be open

Approach: Excite with a test voltage v_T and calculate current i_T then

$$R_{in} = v_T / i_T$$



observe $v_i = v_T$ $\quad A v_i = A v_T$

Use KCL at 1

$$i_T = \frac{v_T}{R_S} + \frac{v_T}{R_i} + \frac{v_T - (-A v_T)}{R_o}$$

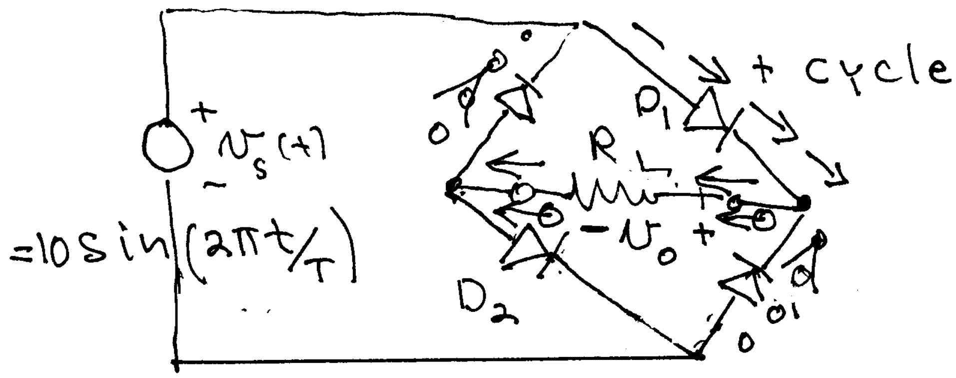
Thus $$\boxed{\frac{v_T}{i_T} = R_{in} = \frac{1}{\frac{1}{R_S} + \frac{1}{R_i} + \frac{(1+A)}{R_o}}}$$

c) When $A \to \infty$

a) $\to$ $v_o / i_s = -R_f$ (as expected)

b) $\to$ $R_{in} \to 0$

Problem No 4 : The basic bridge rectifier (9)



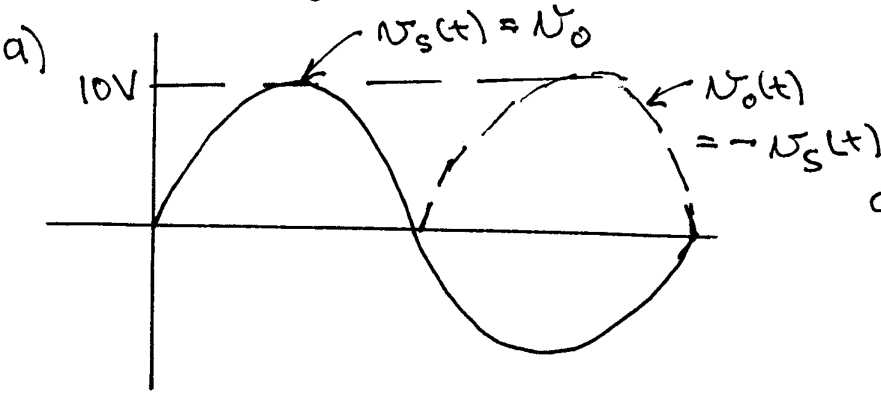
Picture is clearer if we include a finite load resistance R_L.

During the positive cycle of $\sin(2\pi t/T)$ the current follows the →'s, since D_1 and D_2 are forward biased and are thus shorts.

During the negative cycle D_1 and D_2 are open (reverse biased) and the other two are forward biased so shorted. The current then follows the o→'s.

The current goes through the load resistor in the same direction regardless of the sign of $v_s(t)$. Thus

a)

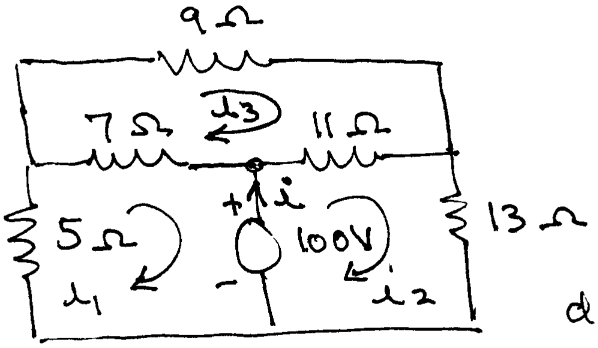


b) $\int_0^T v_s(t)\,\frac{dt}{T} = 0$

c) $\int_0^T v_o(t)\,\frac{dt}{T} = \int_0^{T/2} v_o(t)\,dt\,\frac{2}{T}$

$= 20/\pi$

Problem No. 3)



Power delivered

$= 100\,V \times i$

= sum of powers dissipated in the resistors

KVL for Meshes ① ② ③ in matrix for

i_2 & i_3 in opposite directions

$$\begin{pmatrix} 5+7, & 0, & -7 \\ 0, & 11+13, & -11 \\ -7, & -11, & 11+7+9 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -100 \\ 100 \\ 0 \end{pmatrix}$$

Symmetrical for this circuit (all resistors are reciprocal)

want to find i_1 & i_2

$$i_1 = \frac{\begin{pmatrix} -100 & 0 & -7 \\ 100 & 24 & -11 \\ 0 & -11 & 27 \end{pmatrix}}{\Delta} = [-100(24\times 27 - 11^2) - 7(100)(-11)]/\Delta$$

$$\Delta = 12(24 \times 27 - 11^2) - 7(7 \times 24)$$

$$i_2 = \begin{pmatrix} 12 & -100 & -7 \\ 0 & 100 & -11 \\ -7 & -11 & 27 \end{pmatrix} = [12(100\times 27 - 11^2) - 7(100\times 11 + 7\times 100)]/\Delta$$

i_1 and i_2 are -8.741 and 3.846 amps respectively

$i = (i_2 - i_1) = 12.597$

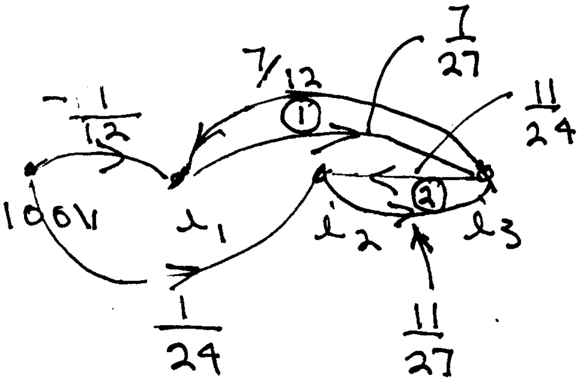
∴ power consumed = 1259. Watts

Flow graph sol$^{\underline{n}}$

$$i_1 = \frac{7}{12} i_3 - \frac{100}{12}$$

$$i_2 = \frac{11}{24} i_3 + \frac{100}{24}$$

$$i_3 = \frac{11}{27} i_2 + \frac{7}{27} i_1$$



two loops ① $\frac{7}{27} \times \frac{7}{12}$ ② $\frac{(11)^2}{27 \times 24}$

$$\therefore \det = 1 - \frac{49}{27(12)} - \frac{(11)^2}{27 \times 24}$$

$$= 1 - \frac{1}{27(24)}\Big(\underbrace{49 \times 2}_{98} + 121\Big)$$

$$= 1 - \frac{1}{27(24)}(219) = \frac{27 \times 24 - 219}{27 \times 24}$$

$$= 429/648 = \frac{143}{216}$$

$$i_1 = 100\left(-\frac{1}{12}\underbrace{\left(1 - \frac{11 \times 11}{27 \times 24}\right)}_{\substack{\text{(1 − sum of Loops that}\\ \text{do not touch this path)}}} + \frac{1}{24} \times \frac{11}{27} \times \frac{7}{12}\right) \underbrace{\frac{216}{143}}_{\text{1 − Sum of Loops}}$$

New term

$$= 100\left(-\frac{2}{\cancel{24}} + \frac{1}{12 \times 27 \times \cancel{24}}(11) \times 18\right) \frac{\overset{9}{\cancel{216}}}{143}$$

$$= \underline{100}\left(-2 + \frac{\cancel{2}}{\underset{18}{\cancel{36}}} \times 11\right)\left(\frac{9}{143}\right)$$

$$i_1 = -100\left(\frac{25}{18}\right)\frac{9}{143} = -8.741$$

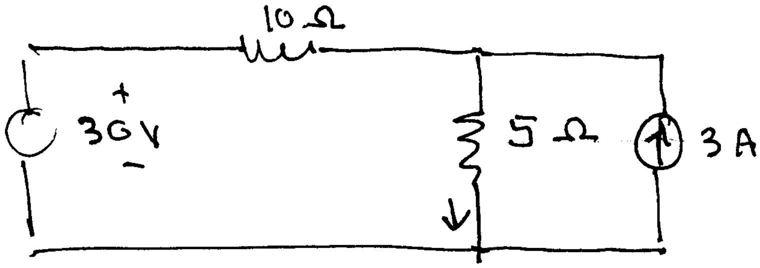
$$i_2 = 100\left(\frac{1}{\cancel{24}}\overbrace{\left(1 - \frac{7}{27} \times \frac{7}{12}\right)}^{\text{(1 − sum of Non Touching Loops)}} - \frac{1}{12} \times \frac{7}{27} \times \frac{11}{\cancel{24}}\right) \times \frac{\overset{9}{\cancel{216}}}{143}$$

$$= 100\left(1 - \frac{7\times\cancel{18}}{\underset{2}{\cancel{12}}\times\underset{9}{\cancel{27}}}\right)\times\frac{9}{143}$$

$$= 100\left(\frac{18-7}{18}\right)\times\frac{9}{143} = \frac{100\times 11}{2}\times\frac{1}{143}$$

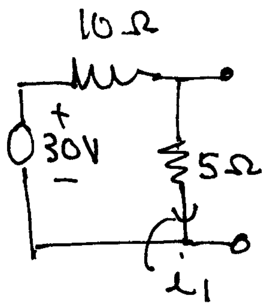
$$= 3.846 \text{ amps.}$$

Problem No. 6 2-89 Hambley



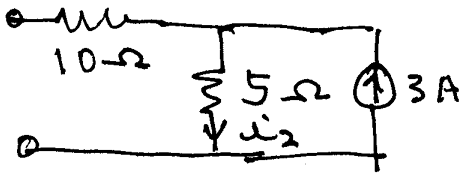
$i = i_1 + i_2$ → due to 3A source with 30V set to 0 (short it)

i_1: due to 30V Source with 3 A set to 0 (open)



$$\therefore\ i_1 = 30V/(5+10)\,\Omega = 30/15 = 2\text{ A}$$

$i_2 = 3$ A →



$\therefore\ \boxed{i = 5\text{ A}}$