Problem Set No 5 Solutions Nov 23 2009

Problem 4.22 of Hambley (Problem No 1) EECS 100/42

a) $i_2 = \frac{100}{10^3} = .1$ Amp (Inductor short, cap open)

$i_3 = 0$ (100 µF open) ; $i_4 = i_2 = .1$ Amp

$i_1 = .2$ Amps

b) Initial current (Inductor open, Capacitor Short)

Thus $i_1 = 100/500 + 100/1000 = .3$ Amps

c $100 \cos 10^2 t$ $\omega = 10^2$ rad/sec

For the steady-state

$$Z(\omega) = (100j + 1000) \,||\, ((500 - j100) \,||\, 1000)$$

$$= (100j + 1000) \,||\, \left(\frac{(500 - j100)1000}{1500 - j100}\right)$$

$$= \frac{(100j + 1000)(5\cancel{00} - j1\cancel{00})10\cancel{00}/(15\cancel{00} - j1\cancel{00})}{10\cancel{0}j + 100\cancel{0} + \frac{(5\cancel{00} - j1\cancel{00})(10\cancel{00})}{15\cancel{00} - j1\cancel{00}}}$$

$$= \frac{100(j + 10)(5 - j)\, 10/\cancel{(15 - j)}}{(15j + 1 + 150 - 10j + 50 - 10j)/\cancel{(15 - j)}}$$

$$= \frac{100(1 + 50 - 5j)\, 10}{-5j + 201}$$

$$= \frac{100(51 - 5j)(201 + 5j)\, 10}{(201)^2 + 25}$$

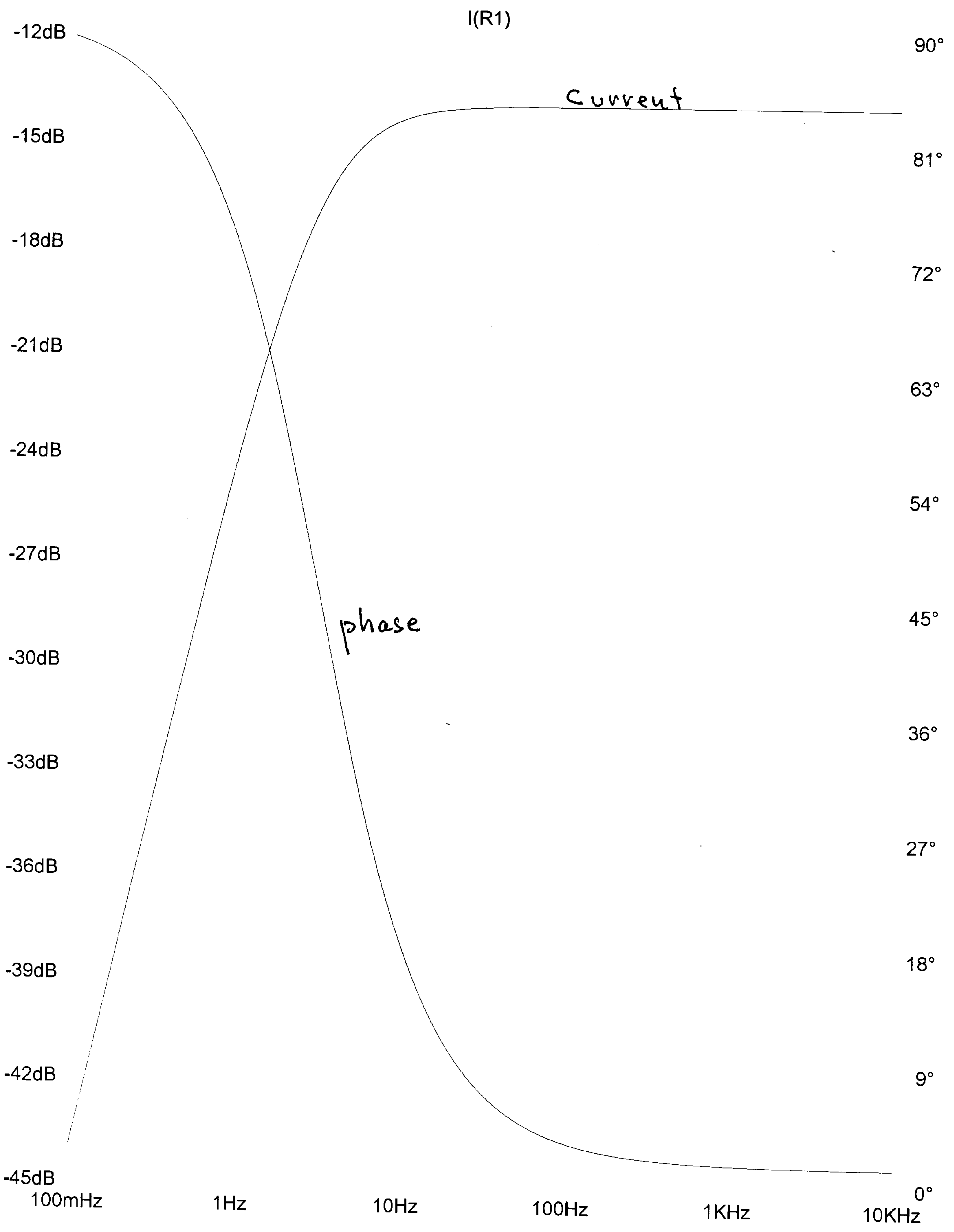
$$= 1000 \frac{(51 \times 201 + 25 - j(5)(201 - 51))}{(201)^2 + 25}$$

$$= 1000 \frac{(10276 - j750)}{40426}$$

$$= 1000(.2542 - j.0186)$$

$$= 255 e^{-j.073}$$

Thus $i_1(t) = \frac{100}{255} \cos(100t + .073)$ (Almost Resonant)

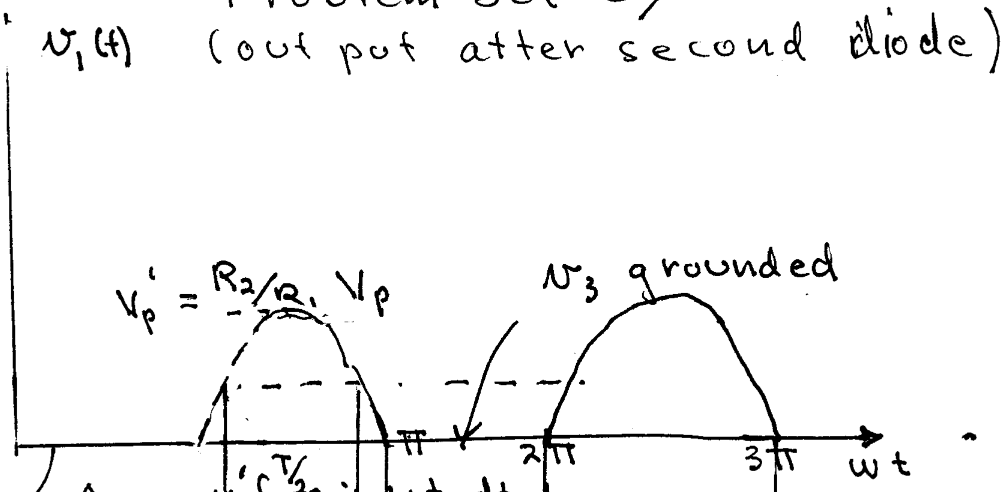


I(R1)
-12dB
-15dB
-18dB
-21dB
-24dB
-27dB
-30dB
-33dB
-36dB
-39dB
-42dB
-45dB
90°
81°
72°
63°
54°
45°
36°
27°
18°
9°
0°
100mHz
1Hz
10Hz
100Hz
1KHz
10KHz
Current
phase

Problem No 2: (Reference Sedra/Smith page 1020

Problem Set 5/

$v_1(t)$ (out put after second diode) (before filter)

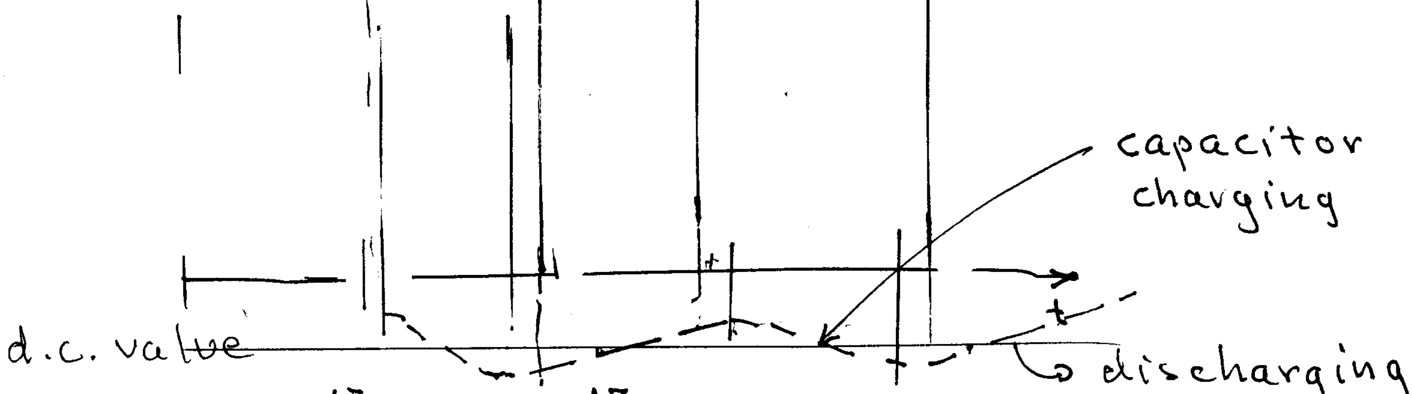


Diodes on so v_3 is grounded

$$\text{Avg} = V_p' \int_0^{T/2} \sin \omega t\, dt\ \frac{1}{T}$$

$$= \frac{V_p'}{\omega T} \int_0^{\pi} \sin \alpha\, d\alpha = V_p \frac{2}{\omega T} = \frac{V_p'}{\pi}$$

$v_2(t)$ (out put after RC filtering)



$$\frac{v_2}{\frac{1}{pC} \parallel R_4} = -\frac{v_1}{R_3}$$

$$\therefore v_2 = -\frac{R_4}{1 + R_4 pC} \frac{v_1}{R_3}$$

R_4C is large

Governing D.E.

$$(1 + R_4 C p)\, v_2 = -\frac{R_4}{R_3} v_1$$

$$\therefore \left(\frac{1}{R_4 C} + \frac{d}{dt}\right) v_2 = -\frac{1}{R_3 C} v_1$$

$$\frac{d}{dt}\left(v_2 e^{+t/R_4 C}\right) = -\frac{1}{R_3 C} e^{t/R_4 C}$$

Consider signal $v_1(t)$ above

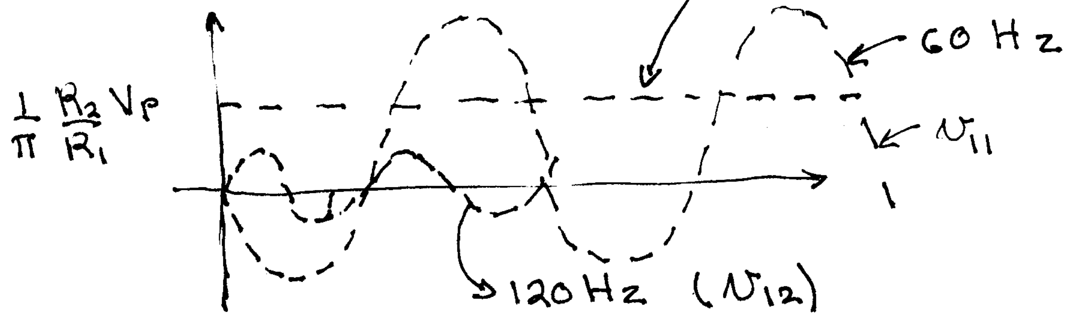
The average of the signal $v_1(t)$

$$= V_p \int_0^{T/2} \sin \omega t \; dt\omega \times \frac{1}{\omega T} \frac{R_2}{R_1} = V_p \frac{R_2}{R_1} \times \frac{1}{\underset{\uparrow}{\omega T}} 2 = V_p \frac{R_2}{R_1} \times \frac{1}{\pi}$$

$2\pi/\omega$

This is the d.c. component.
which gets filtered out

model $v_1(t)$ as $v_{10} + v_{11} + v_{12} + \cdots$



Now look at v_2 for <u>each of these</u>

$p = 0$ for d.c.

$$v_{20} = -\frac{R_4}{R_3} v_{10} = -\frac{R_4}{R_3} \frac{R_2}{R_1} \times \frac{1}{\pi}$$

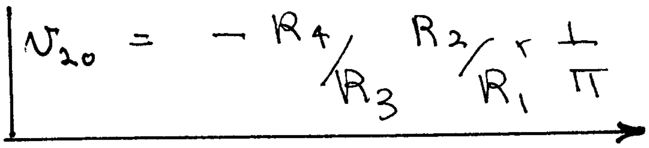
$$v_{21} = -\frac{R_4}{1 + R_4 j\omega C} \frac{v_{11}}{R_3} \quad \text{for} \quad 60\text{ Hz} \quad \omega = 2\pi 60$$

$$\approx -\frac{1}{j\omega C} \frac{v_{12}}{R_3} = -\frac{R_4}{R_3}\frac{R_2}{R_1} \frac{1}{j(\omega C R_4)}$$

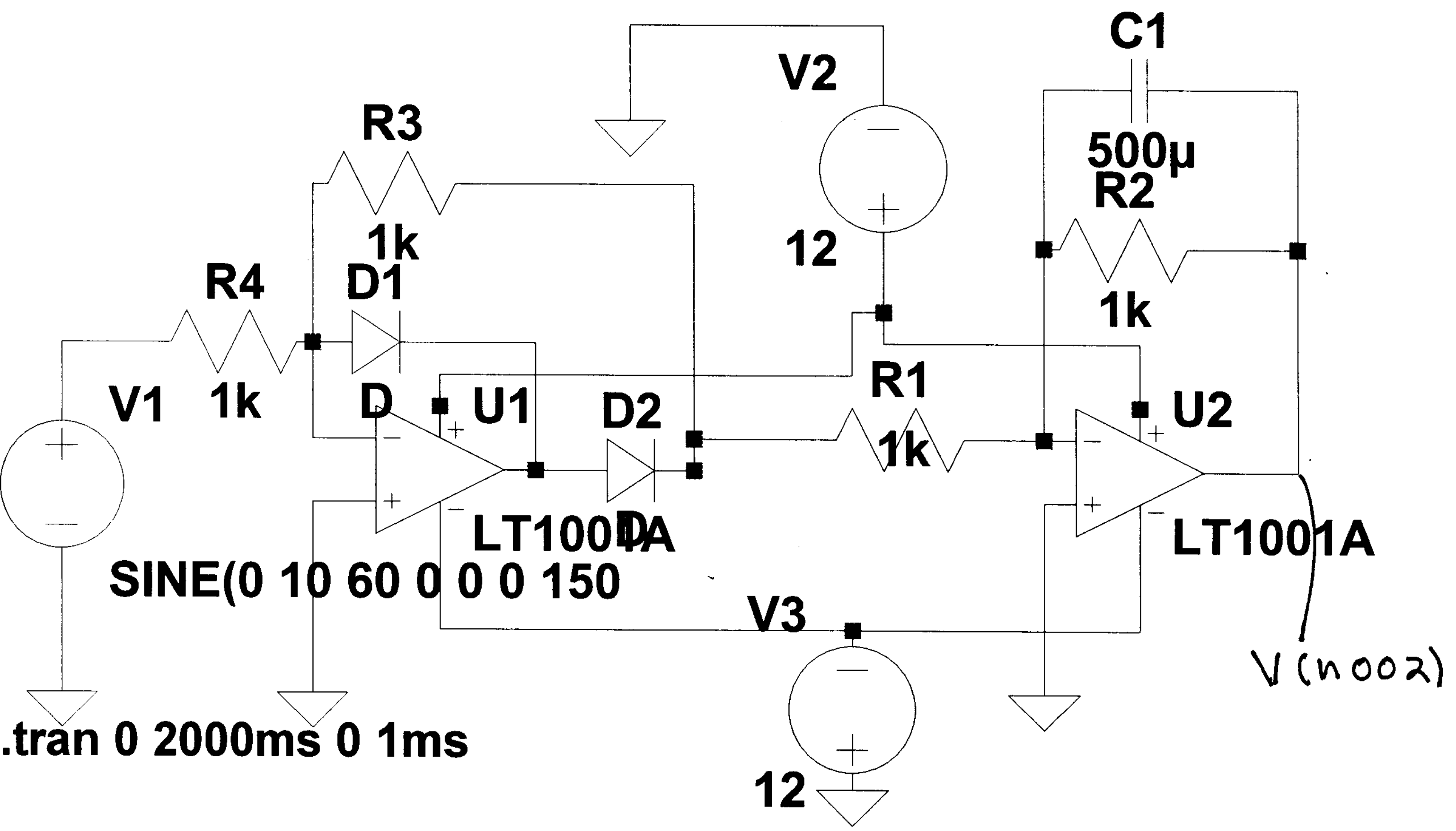
$\omega C R_4 >> \pi$

Thus v_{21} is small. (example $1k\Omega \times 50\mu F \times 60$ Hz) $= 50 \times 10^{-3} \times 2\pi \times 60 \approx 6\pi$

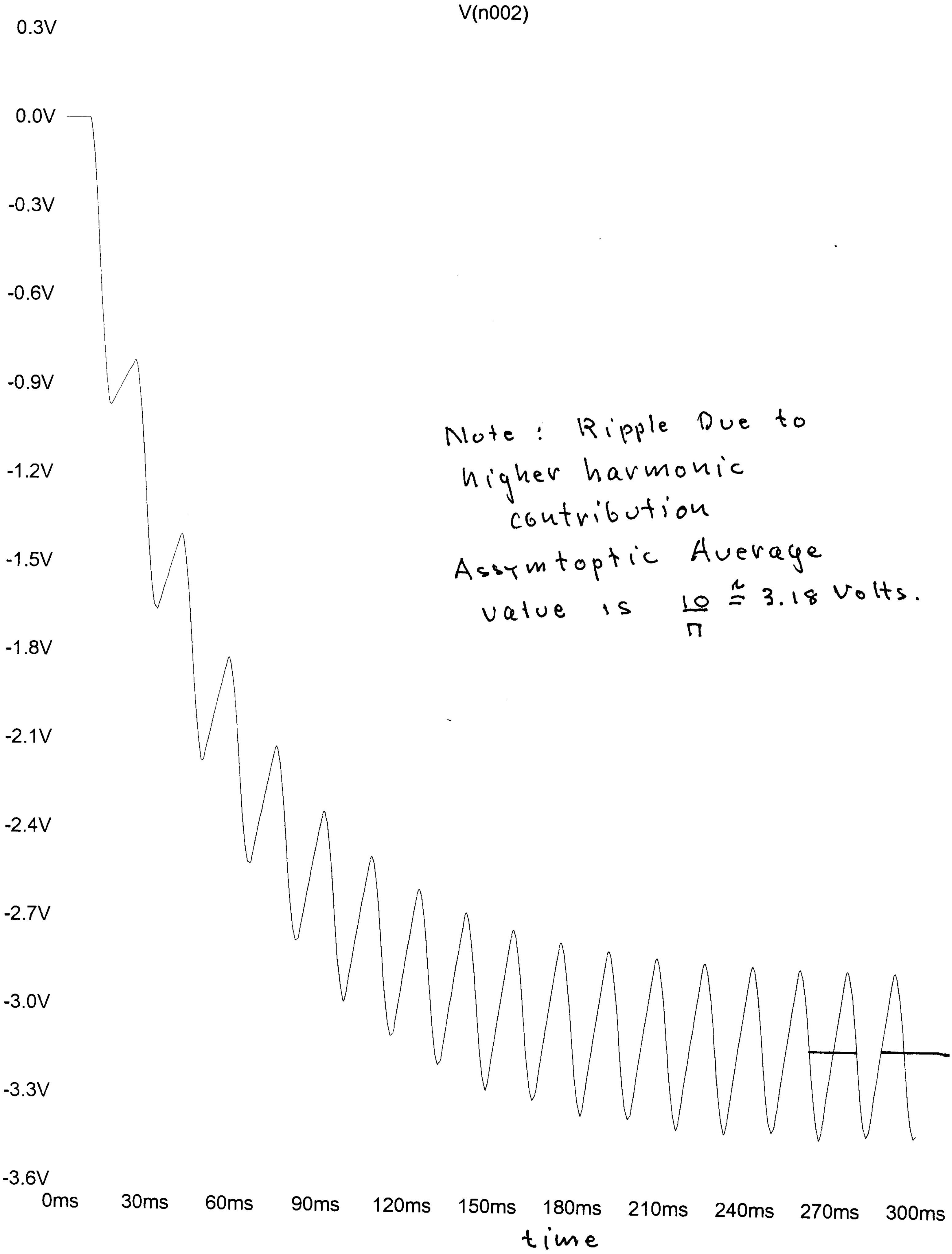
And output is close to a constant v_{20}



LT Spice Problem 2 a.c. voltmeter



- E:\WebDesign\Templates\Documents\Assignments\Problem_Sets\Problem_Set_5\VoltMeter.asc --

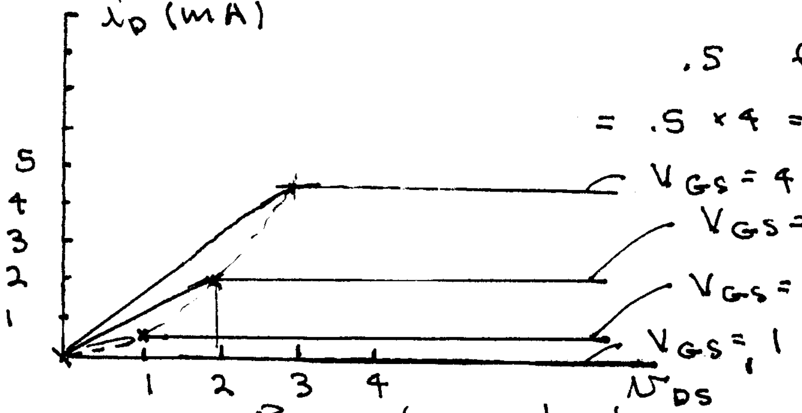

V(n002)
0.3V
0.0V
-0.3V
-0.6V
-0.9V
-1.2V
-1.5V
-1.8V
-2.1V
-2.4V
-2.7V
-3.0V
-3.3V
-3.6V
0ms
30ms
60ms
90ms
120ms
150ms
180ms
210ms
240ms
270ms
300ms
time
Note : Ripple Due to higher harmonic contribution
Assymtoptic Average value is $\frac{10}{\pi} \approx 3.18$ Volts.

Problem No. 3 Hambley 12.20

FET $V_{to} = 1V$ $K = .5\ mA/V^2$

$$i_D = K(V_{GS} - V_{TO})^2 = .5\,(V_{GS} - V_{TO})^2$$
$$= .5\,(V_{GS} - 1)^2$$

b)



$.5$ for $V_{GS} = 2V$

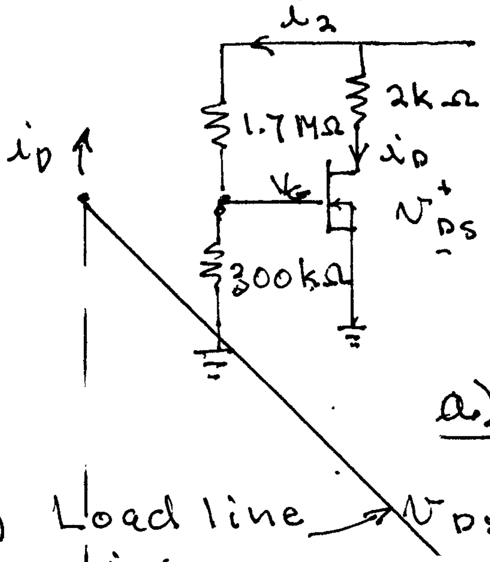
$= .5 \times 4 = 2$ for $V_{GS} = 3V$

$= .5 \times 9 = 4.5\,mA$ for $V_{GS} = 4V$

Boundary between Sat + Triode

$$V_{DS} = V_{GS} - V_T$$
$$= V_{GS} - 1$$

D.C. Load Line



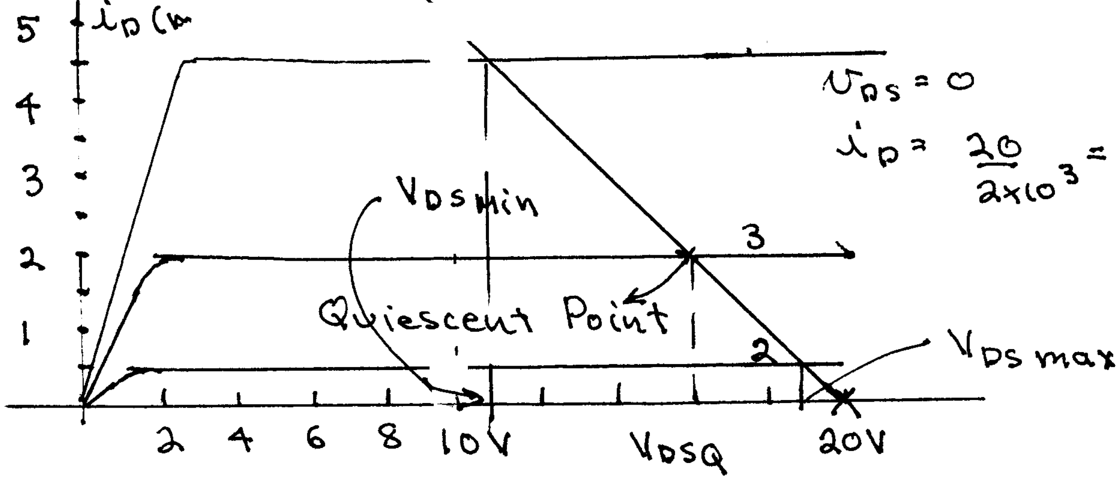
Volt divider for D.C.

$$V_G = 20\,\frac{.3}{1.7 + .3} \approx 20 \times \frac{3}{20} = 3V$$

Thus

a) $V_{GS}(t) = 3 + \sin(2000\pi t)$

c) Load line $v_{DS} = -2k\Omega\, i_D + 20$



$v_{DS} = 0$

$$i_D = \frac{20}{2 \times 10^3} = 10\,mA$$

$$v_{DS} = -2\times10^3 (K)(V_{GS}-V_{TO})^2 + 20$$

$$= -2\times . \times .5 (V_{GS}-1)^2 + 20$$

$$V_{GS\,min} = 2\text{ V}$$

$$\therefore v_{DS\,max} = -1 + 20 = \underline{19\text{V}}$$

$v_{DS\,min}$ for $V_{GS\,max} = 4\text{ V}$

$$= -(3)^2 + 20$$

$$= \underline{11\text{ V}}$$

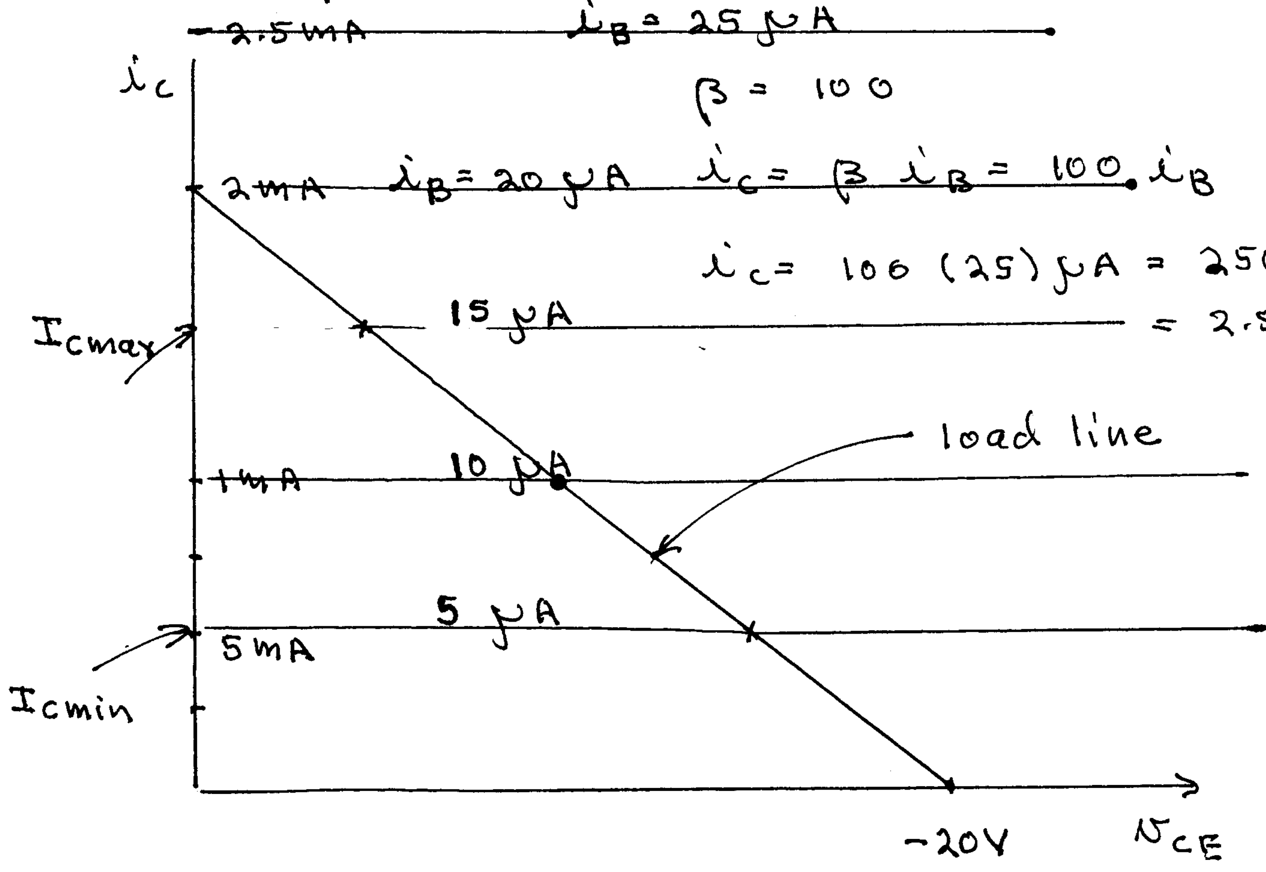
b Hambley 13.29

$\beta = 100$

$i_C = \beta\, i_B = 100\, i_B$

$i_C = 100\,(25)\,\mu A = 2500\,\mu A$

$= 2.5\text{ mA}$



For case e) During the positive cycle the transistor is saturated so $i_c = 2\text{mA}$

i_{cmin} occurs for $i_B = 15\,\mu A$ $(i_c = 1.5\text{ mA})$

Hambley 13.19

Note $i_C = 10$ mA for $i_B = 25\,\mu A$

$\therefore\ \beta = i_C / i_B = \frac{10}{.025} = 40$

$\beta = \frac{\alpha}{1-\alpha}$ or $\alpha = \frac{\beta}{1+\beta} = \frac{40}{41}$

Problem No. 4 Hambley 7.8

Binary $2^6\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

| 64 | 32 | 16 | 8 | 4 | 2 | 1 → sum is 63

sum is 127

So 7 bits needed for 100 →

128 8'th — sum is 255

256 9'th — sum is 511

So 10 bits needed for 1000 →

512 10'th — 1023

1024 11'th — 2047

2048 12'th — 4095

4096 13'th → 8191

8192 14'th → 16383 — 14 bits

16384 15'th → 32777 — 15 bits

32768 16'th → 65545 → 16 bits

65536 17'th → 131181 → 17 bits

131072 18'th — add — yields → 262253 → 18 bits

262154 19'th — add — yields 524417 → 19 bits

So 20 bits are needed → $2^{20} = 2(262154) = 524308$

Large Number Law

$2^n = 100$ $n = 6.64 \rightarrow 7$

Using large Number law

$2^n = 1000$

$n = 9.965 \rightarrow 10$

12 bits

Note Short Way $\sum_{i=0}^{n-1} 2^i \simeq 2^n$ Law of large numbers

we want this to be 10^6 so $2^n \simeq 10^6 \Rightarrow n = \log_2 10^6 = 19.931 \rightarrow 20$

b) Hambley 7.37

a) $F = (A + B)\overline{C}$

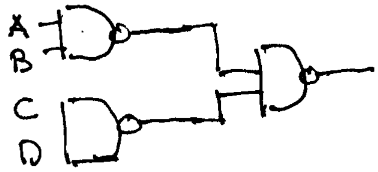
b) $F = (A + B) + \overline{BC}$

c) $F = AB + \overline{BC} + D$

c) Hambley 7.46

$F = AB + CD$

$= \overline{\overline{AB}\;\overline{CD}}$

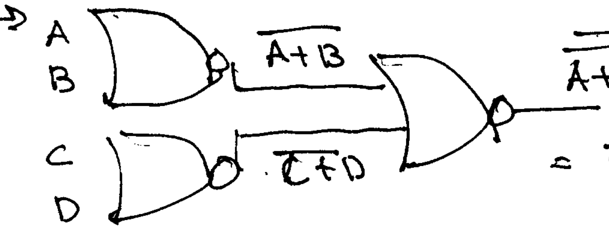
$=$ 

Hambley 7.47

$$= \overline{(A+B)}\,(C+D)$$

$$\overline{\overline{(A+B)} + \overline{C+D}}$$

since $\overline{A+B} + \overline{C+D} = \overline{(A+B)(C+D)}$



$\overline{\overline{A+B} + \overline{C+D}}$

$= \overline{\overline{(A+B)(C+D)}}$

Problem No. 5 Hambley 7.54

Inputs A, B, C

$D = \sum m(0, 3, 4)$

↑ these are the terms in truth table which give 1 for output

A	B	C		F	
0	0	0	→	1	$\bar{A}\bar{B}\bar{C}$ - 0
0	0	1		0	
0	1	0	→	0	$\bar{A}B\bar{C}$
0	1	1	→	1	$\bar{A}BC$
1	0	0		1	$A\bar{B}\bar{C}$
1	0	1		0	
1	1	0		0	
1	1	1		0	

C \ AB	00	10	11	01
0	1	1		1
1				

b) $F = \bar{A}\bar{C} + \bar{B}\bar{C}$

minimum <u>SOP</u>

c) POS

$D = \prod(1, 2, 5, 6, 7)$

$F = (A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + \bar{C})$

$(\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})$

or apply de Morgan's Theorem to

$F = \bar{A}\bar{C} + \bar{B}\bar{C}$

$= \overline{\overline{\bar{A}\bar{C}}\;\overline{\bar{B}\bar{C}}}$

$= \overline{(A + C)(B + C)}$

or use karnaugh map for $\bar{F}$

C \ AB	00	10	11	01
0	0	0	1	0
1	1	1	1	1

$$\bar{F} = C + AB$$

$$\therefore F = \overline{C + AB}$$

$$= \bar{C}\ \overline{AB}$$

$$= \bar{C}\,(\bar{A} + \bar{B})$$

Problem No. 6

Hambley 14.25

a) $\frac{v_o - v_A}{R} - \frac{v_A}{R} + i_{in} = 0$

v_- is grounded so

$v_A = -i_{in} R$

$\therefore \frac{v_o}{R} + 2 i_{in} + i_{in} = 0$

$\therefore \boxed{v_o = 3 R i_{in}}$

b) Looking in from v_o ($i_{in} = 0$) (input open)

$i_{R_f} = \frac{v_o - v_A}{R} = \frac{v_o}{R}$

$\therefore$ output Resistance $= R$

c) Input resistance

$v_{in} = 0$ $\frac{v_{in}}{i_{in}} = 0 = R_{in}$

d) Transimpedance Amplifier

Problem No. 7 - 6.24

$v_{in} = 5\cos(\omega_0 t) + 5\cos\omega_2 t + 5\cos\omega_4 t$

$\omega_0 = 500\pi$
$\left.\begin{array}{l}\omega_2 = 2\omega_0 \\ \omega_4 = 4\omega_0\end{array}\right\}$ even order harmonics of the fundamental at ω_0

$$v_{out} = \frac{1/pc}{1/pc + R} v_{in}(t)$$

For $e^{+j\omega t}$ terms $\omega = \omega_0, 2\omega_0, 4\omega_0$

$$V_{out} = \frac{5}{2}\frac{1}{1 + j\omega CR}$$

$$v_{out}(t) = 2\,\mathrm{Re}\Big(\sum_{m=1,2,4} \frac{5}{2} \cdot \frac{1}{1 + j\omega_0 m CR} e^{j\omega_0 m t}\Big)$$

$\omega_0 RC = 318.3 \times 10^{-6}\ 500\pi$
$= .5$

$$= 5\,\mathrm{Re}\Big[\frac{1}{1+.5j} e^{j500\pi t} + \frac{1}{1+j} e^{j1000\pi t} + \frac{1}{1+2j} e^{j2000\pi t}\Big]$$

convert to polar (with calculator!)

$$= 5\,\mathrm{Re}\Big[\Big(\frac{1}{1.12\, e^{j.46}}\Big) e^{j500\pi t} + \frac{1}{\sqrt{2}\, e^{j\pi/4}} e^{j1000\pi t} + \frac{1}{2.236\, e^{j1.11}} e^{j2000\pi t}\Big]$$

ote how it is
creasing the
plitude of higher
rmonics. This is
ow
s filter

$$= 5\,[.89\cos(500\pi t - .46) + .707\cos(1000\pi t - \pi/4) + .447\cos(2000\pi t - 1.11)]$$