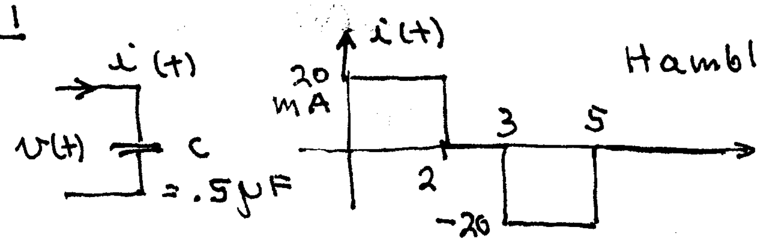
Solutions

Problem 1

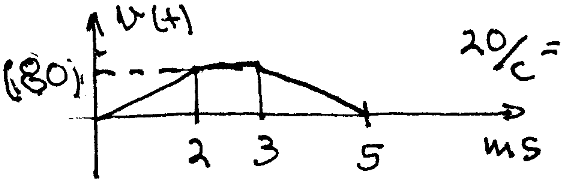
a)



Hambley 3.11

$i = C \frac{dv}{dt}$

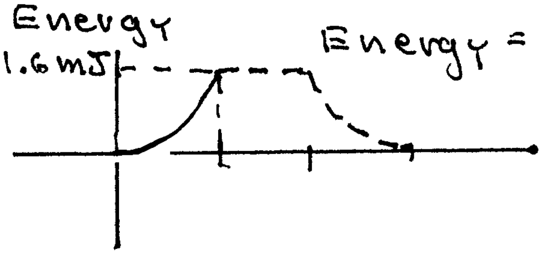
$v = \int \frac{i}{C}\, dt$



$20/C = \frac{20 \text{ mA}}{.5 \times 10^{-6} \text{ F}} = 40 \times 10^{3} \text{ V/s}$

At 2 msec

V = 80 V



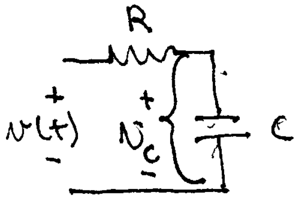
Energy $= \frac{1}{2} C v^{2}$

Peak $= \frac{1}{2} \times \frac{1}{2} \times 10^{-6} (80)^{2}$ J

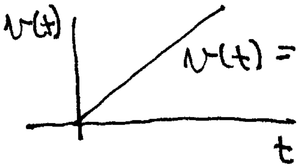
$= \frac{1}{4} \times 10^{-6} \times 64 \times 10^{2}$

$= 16 \times 10^{-4}$ J.

b)



Hambley 4.45



$$v_c(t) = \frac{1/pC}{1/pC + R}\, v(t) = \frac{1}{1 + pCR}\, t$$

$$= \left(\frac{1}{pCR}\right)\left(\frac{1}{1 + 1/pCR}\right) t$$

$$= \frac{1}{pCR}\left(1 - 1/pCR + (1/pCR)^{2} - (1/pCR)^{2} + \cdots\right) t$$

$$= \frac{1}{CR}\left(t^{2}/2 - \frac{1}{CR}\, t^{3}/3! + \frac{1}{(CR)^{2}}\, t^{4}/4! + \cdots\right)$$

$$= RC\left(-1 + t/RC + e^{-t/RC}\right)$$

Problem No. 1c

Hambley 3.71

$LC = 500\times10^{-6} \times 2\times10^{-3}$

$= 10^{-6} \quad \therefore \frac{1}{\sqrt{LC}} = \omega_0 = 1000$

$v_c(t) = 10\sin(1000t)$

$i = C\frac{dv_c}{dt} = \cancel{500\times10^{-6}} \times \cancel{10} \times \cancel{1000}\cos(1000t)$

$= 5\cos(\omega_0 t)$

$v_L(t) = L\frac{di}{dt} = -2\times10^{-3}\times 5\,\omega_0 \sin(\omega_0 t)$

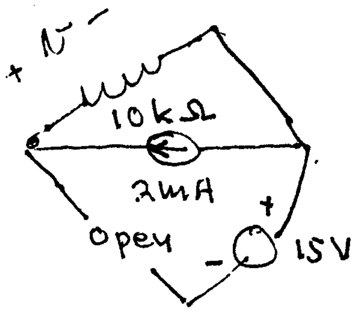
$= -10\sin\omega_0 t$

$v(t) = 0$

<u>Problem No. 2</u>

a) Hambley 4.27 Inductor is short

Capacitor is open



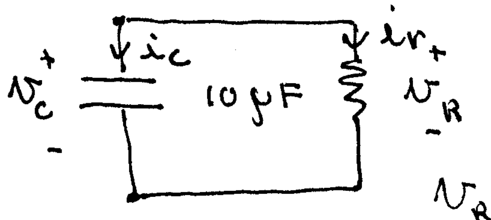
$v = 2mA \times 10k\Omega$

$= 20V$

$i_R = \underline{2mA}$

$v_c = 20 + 15 = \underline{35V}$

b) Hambley 4.11



$i_c + i_R = 0$

$\left(\frac{1}{pC} + R\right)v = 0$

$v_R = v_c = v$

$\frac{1}{C}v(t) + pRv = 0$

Solution to Differential Equation $v(t) = Ae^{st}$ $\quad v(0) = A$

$\frac{1}{C}A\,(e^{st}) + sRe^{st}A = 0$

$\therefore s = -RC \qquad A = 50V,$

$t = 30$ sec $\quad v(t) = 50e^{-30/RC}$

$= 25V$

$\therefore -30/RC = \ln\frac{25}{50} = -\ln 2 = .69$

$R = 30/.69C = 3/.69\ M\Omega = 4.34\ M\Omega$

Note: if you use integrals note that $\frac{1}{p}$ can involve a constant

$\therefore \frac{v}{C} + R(p)v) = 0$

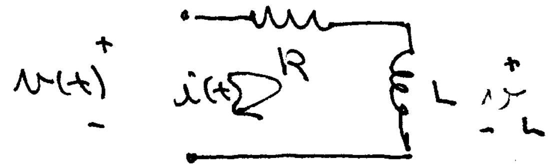
$\int_{0^+}^{t} \frac{v(t)}{C}dt + R\int_{0^+}^{t}\frac{dv}{dt}dt = 0$

$-\int_{0^+}^{t} v(t)dt + R(v(t) - \overset{\overset{A}{\|}}{v}(0^+)) = 0$

↳ 1/p

Prob. 2c)

Problem 4, 49



$$v(t) = (R + Lp)\, i(t)$$

$$i(t) = \left(\frac{1}{R+Lp}\right) v(t)$$

$$= \frac{1}{Lp}\left(\frac{1}{1 + R/Lp}\right) i(t)$$

$$= \frac{1}{Lp}\left(1 - \frac{R}{L}\frac{1}{p} + \left(\frac{R}{L}\frac{1}{p}\right)^2 + \cdots\right) v(t)$$

$$= \frac{1}{R}\left(\frac{R}{Lp} - \left(\frac{R}{Lp}\right)^2 + \left(\frac{R}{Lp}\right)^3 + \cdots\right) v(t)$$

$$= \frac{1}{R}\left(\frac{1}{2}\frac{R}{L}t^2 - \left(\frac{R}{L}\right)^2\frac{t^3}{3!} + \left(\frac{R}{L}\right)^3\frac{t^4}{4!} + \cdots\right)$$

$$= \frac{L}{R^2}\left(\frac{1}{2}\left(\frac{Rt}{L}\right)^2 - \frac{1}{3!}\left(\frac{Rt}{L}\right)^3 + \cdots\right)$$

$$= -\frac{L}{R^2} + \frac{1}{R}t + \frac{L}{R^2}\left(1 - \frac{Rt}{L} + \frac{1}{2}\left(\frac{Rt}{L}\right)^2 \cdots\right)$$

$$= -\frac{L}{R^2} + \frac{t}{R} + \underbrace{\frac{L}{R^2}e^{-Rt/L}}_{\text{transient solution}}$$

Note

$v_L = L\frac{di}{dt}$

∴ i can't change instantaneously unless $v_L \to \infty$

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{1}{R} - \frac{R}{L}\frac{L}{R^2} = 0$$

No. 3

$t = 0 \qquad 10\sin(300t)$

$$\boxed{v(t) = Ri + L\frac{di}{dt}}$$

Multiply both sides by $e^{R/L\,t}$

$$\left(10\sin(300t)\,e^{R/L\,t}\right) = L\left(\frac{d}{dt}\left[e^{R/L\,t}\,i(t)\right]\right)$$

① $$\therefore \int_{0^+}^{t} \frac{10}{L}\sin(300t)\,e^{R/L\,t}\,dt = e^{R/L\,t}\,i(t) - i(0^+)$$

We can do this this way

$$2\,\mathrm{Re}\int_{0^+}^{t} \frac{10}{L}\,\frac{e^{300jt}}{2j}\,e^{R/L\,t}\,dt$$

$$= \mathrm{Re}\,\frac{10}{Lj}\cdot\left[\frac{1}{300j + R/L}\left(e^{300jt + R/L\,t} - 1\right)\right]$$

$$= \frac{10}{L}\,\mathrm{Re}\left[\frac{1}{-300 + R/L\,j}\cdot e^{300jt + R/L\,t} - \frac{1}{-300 + R/L\,j}\right]$$

$$= \frac{10}{L}\,\mathrm{Re}\left[\frac{-300 - R/L\,j}{(300)^2 + (R/L)^2}\,e^{300jt + R/L\,t} + \frac{300}{(300)^2 + (R/L)^2}\right]$$

$$= \frac{10}{L}\,\mathrm{Re}\left[\frac{(-1)e^{300jt + R/L\,t + j\tan^{-1}(\frac{R}{300L})}}{\sqrt{(300)^2 + (R/L)^2}}\right] + \frac{3000}{(300)^2 + (R/L)^2}$$

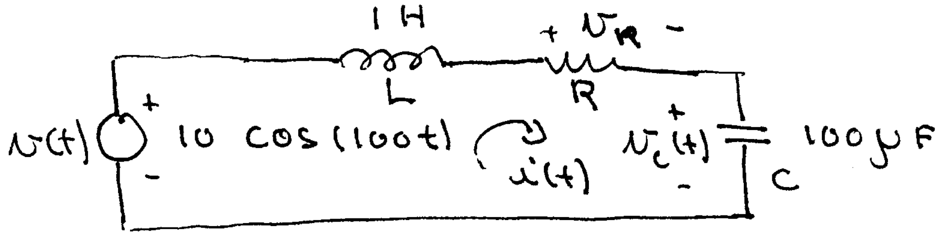
$$= -\frac{10}{L}\,\frac{\cos(300t + \tan^{-1}(1))}{300\sqrt{2}}\,e^{R/L\,t} + \frac{10}{300\,(2)}$$

$i(0^+) = 0$ so (1) results in

$$i(t) = \underbrace{-\frac{10}{300\sqrt{2}}\cos(300t + \pi/4)}_{\text{Steady-State Response}} + \underbrace{\frac{10}{300\times 2}\,e^{-R/L\,t}}_{\text{Natural Respons}}$$

Check $i(t=0) = -\dfrac{10}{300\sqrt{2}}\underbrace{\cos\pi/4}_{=\frac{1}{\sqrt{2}}} + \dfrac{10}{600} = 0$

Problem No. 4 Resonant Circuit



Steady state

$$i(t) = \frac{1}{R + pL + \frac{1}{pC}} v(t)$$

$$\therefore\ i(t) = \text{Re}\left[\frac{1}{R + j\omega L + \frac{1}{j\omega C}} 10 e^{j100t}\right]$$

$$= \text{Re}\left[\frac{1}{R + j(\omega L - \frac{1}{\omega C})} 10 e^{j100t}\right]$$

$$= \text{Re}\left[\frac{10}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} e^{-j\tan^{-1}\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right] + j100t}\right]$$

$$= \frac{10}{(R^2 + (\omega L - \frac{1}{\omega C})^2)^{1/2}} \cos\left(100t - \tan^{-1}\left(\frac{(\omega L - 1/\omega C)}{R}\right)\right)$$

$\omega = 100$ rad/sec

Transient Solution $i(t) \neq 0$ when $v(t) = 0$ for

$$R + pL + \frac{1}{pC} = 0 \qquad RpC + p^2LC + 1 = 0$$

$$p^2 + p\frac{R}{L} + \frac{1}{LC} = 0 \qquad \boxed{p = -\frac{1}{2}\frac{R}{L}} \pm j\underbrace{\sqrt{\frac{1}{LC} - \left(\frac{1}{2}\frac{R}{L}\right)^2}}_{\omega_1}$$

$$\therefore\ i_t(t) = e^{-R/L\, t}\left[A e^{j\omega_1 t} + B e^{-j\omega_1 t}\right]$$

$$i_{Total} = i_t(t) + i(t) = 0 \text{ at } t = 0$$

$$(1) \quad -- \therefore A + B = -\left[\frac{10}{(R^2 + (\omega L - 1/\omega C)^2)^{1/2}}\right] \cos\left(\tan^{-1}\left[\frac{(\omega L - 1/\omega C)}{R}\right]\right)$$

Also $i = C\frac{dv_c}{dt}$ $\quad v_L = L\frac{di}{dt}$ $\quad v_c$ cant change instantaneously
i cant change

so $i = 0$ at $t = 0^+$ $\quad v_c = 0$ at 0^+ $\quad v_R = 0$ at 0^+

so by KVL $v_L = 0$ at 0^+ implying $\frac{di}{dt} = 0$ at 0^+

Thus

$$(2) - - \quad j\omega_1 [A - B] = \frac{1000}{(R^2 + (\omega L - \frac{1}{\omega C})^2)^{1/2}} \sin(\tan^{-1}(\frac{\omega L - 1/\omega C}{R}))$$

parameters

$$\omega L - \frac{1}{\omega C} = 100 - \frac{1}{10^4 \times 10^{-6}} = 0$$

so the circuit is resonant. It just acts as a resistor for the St. St.

$$i(t) = \frac{10}{R} \cos(100t)$$

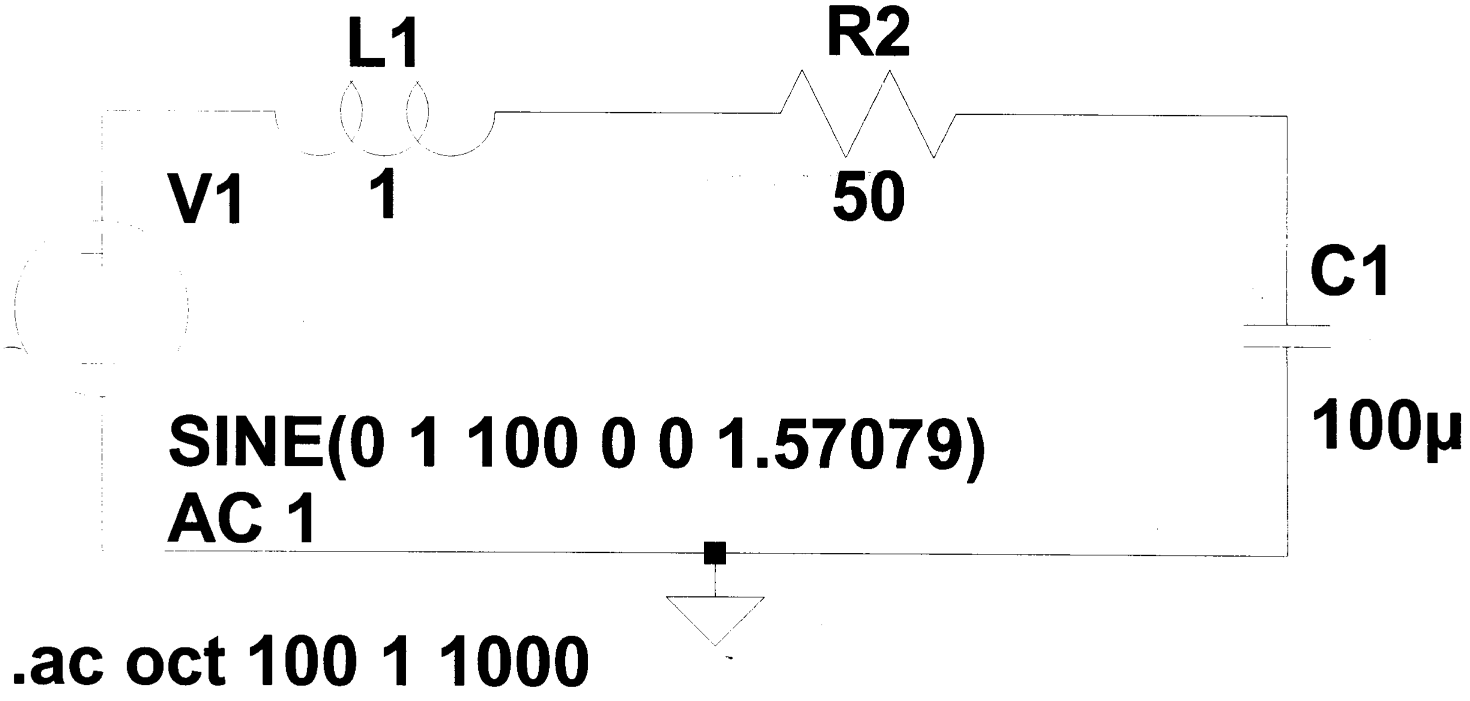
Transient

$$\left.\begin{array}{l} A + B = -\frac{10}{R} \\ A - B = 0 \end{array}\right\} \rightarrow \quad B = A = -5/R$$

$$i_{Total}(t) = \frac{10}{R} \cos(100t) - 10/R \, e^{-R/L \, t}$$

$$= 200 \, (\cos(100t) - e^{-R/L \, t})$$

$$R/L = \frac{50}{1} = 50 \sec^{-1} \; ; \; L/R = 20 \mathrm{msec}$$



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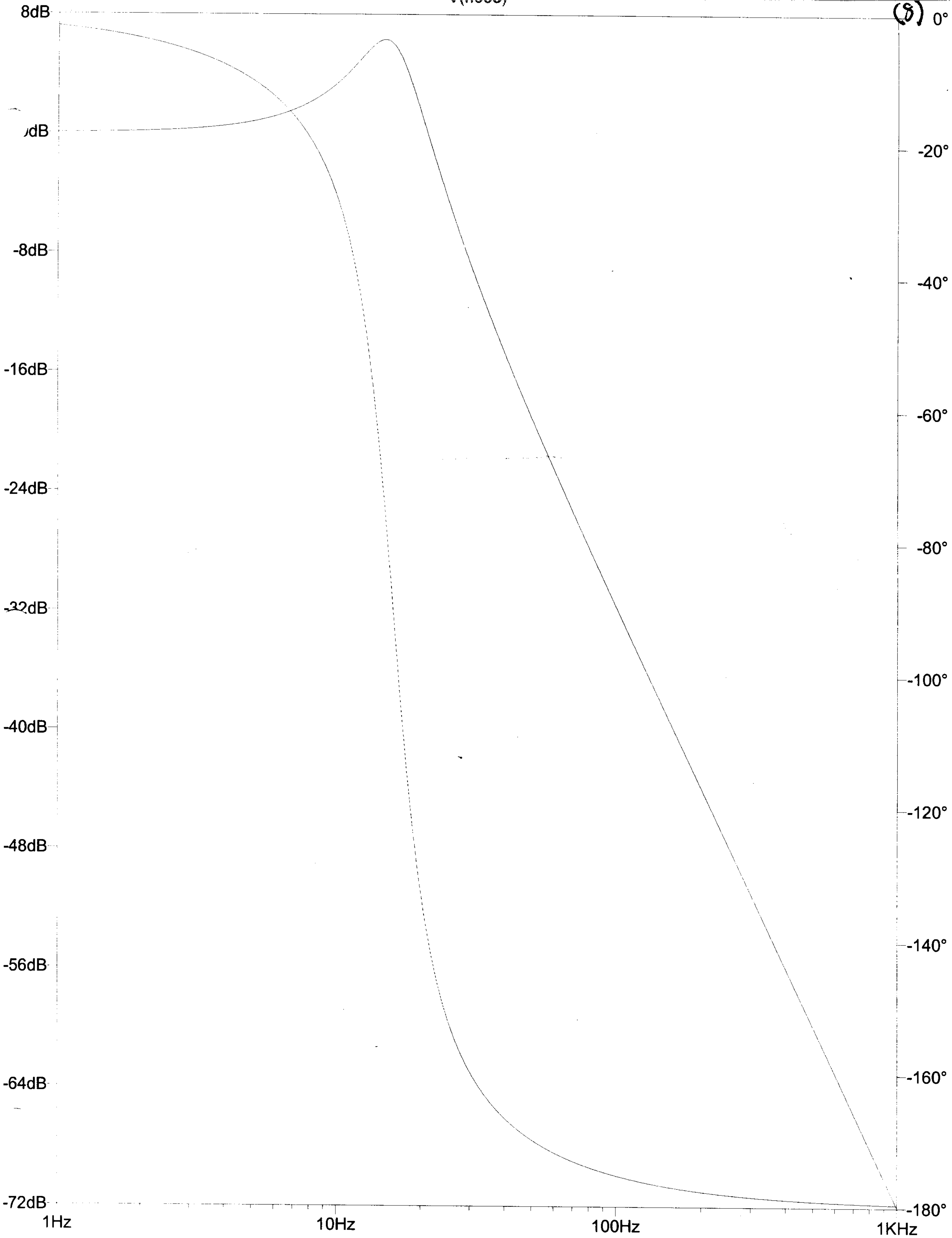

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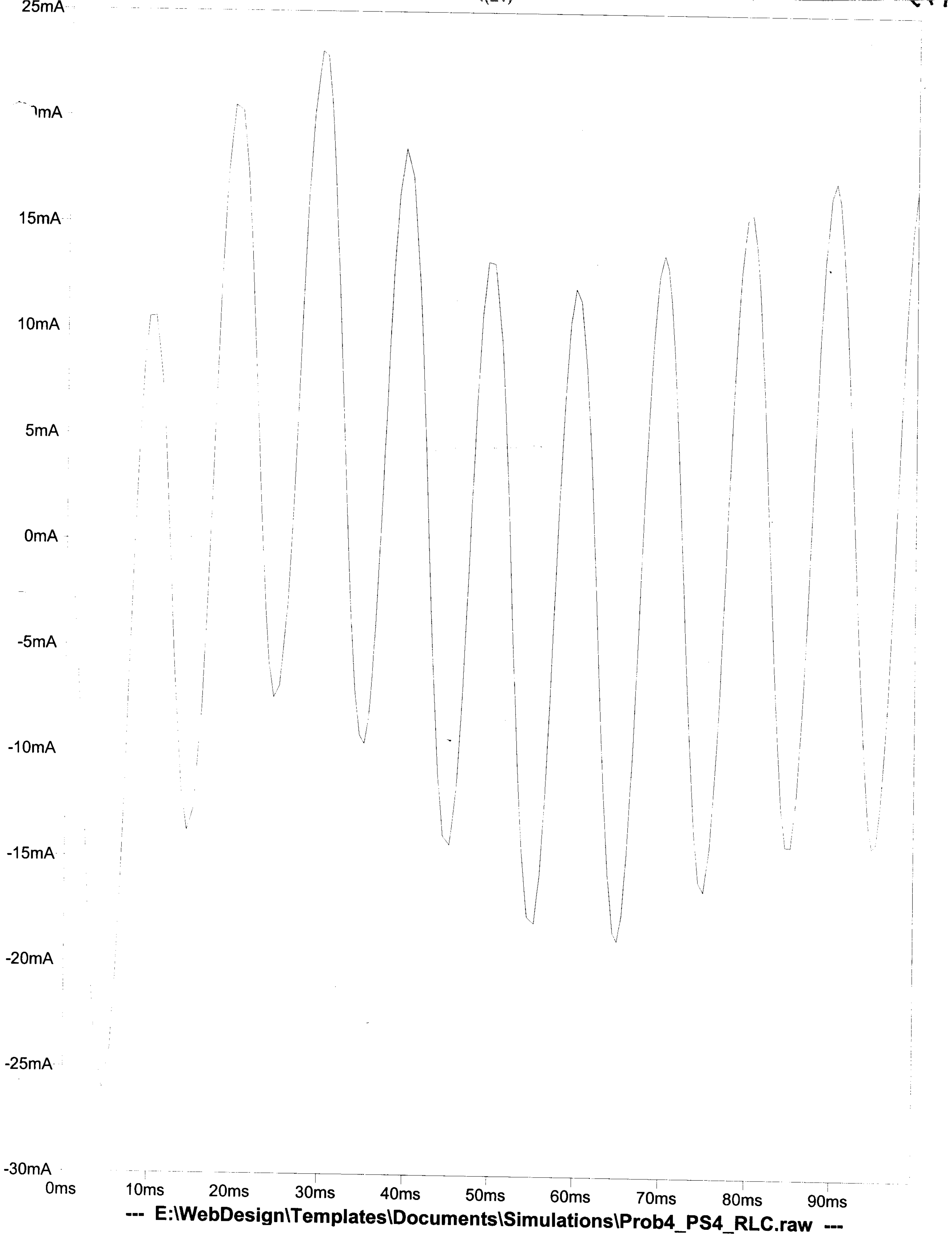
25mA
15mA
10mA
5mA
0mA
-5mA
-10mA
-15mA
-20mA
-25mA
-30mA
0ms
10ms
20ms
30ms
40ms
50ms
60ms
70ms
80ms
90ms
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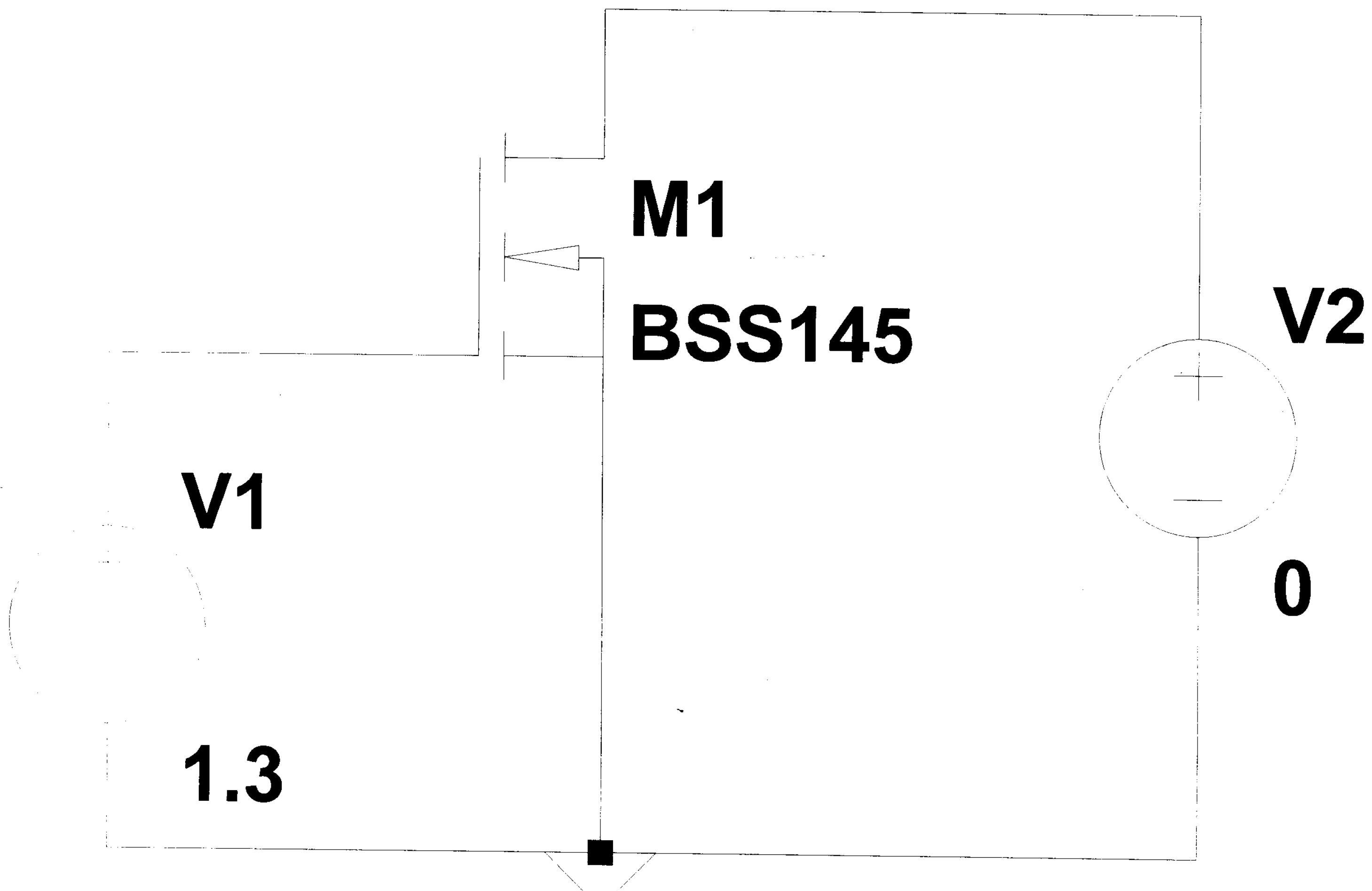

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.dc V2 0 5 .01 V1 2. 5 .5
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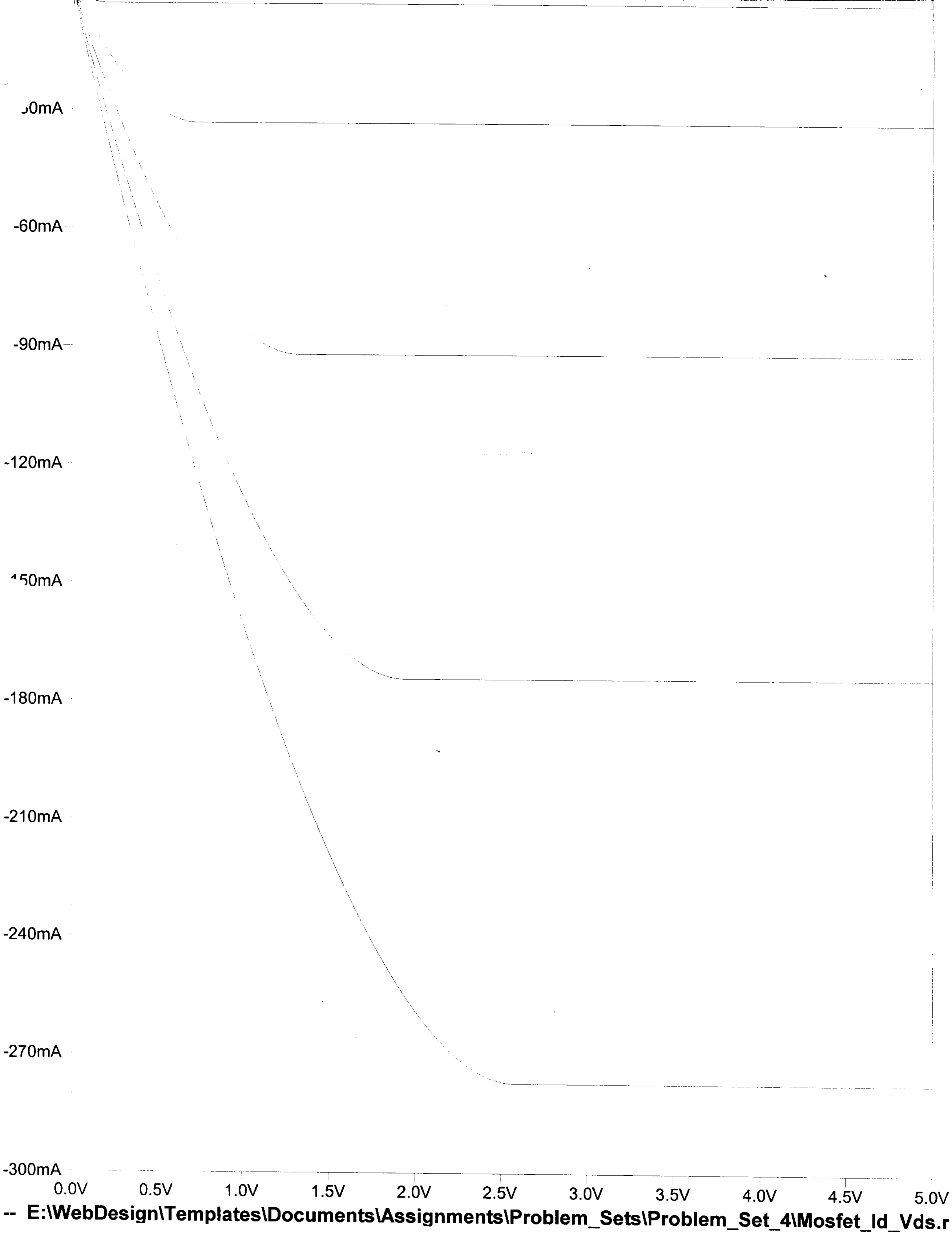
0mA
-60mA
-90mA
-120mA
150mA
-180mA
-210mA
-240mA
-270mA
-300mA
0.0V
0.5V
1.0V
1.5V
2.0V
2.5V
3.0V
3.5V
4.0V
4.5V
5.0V
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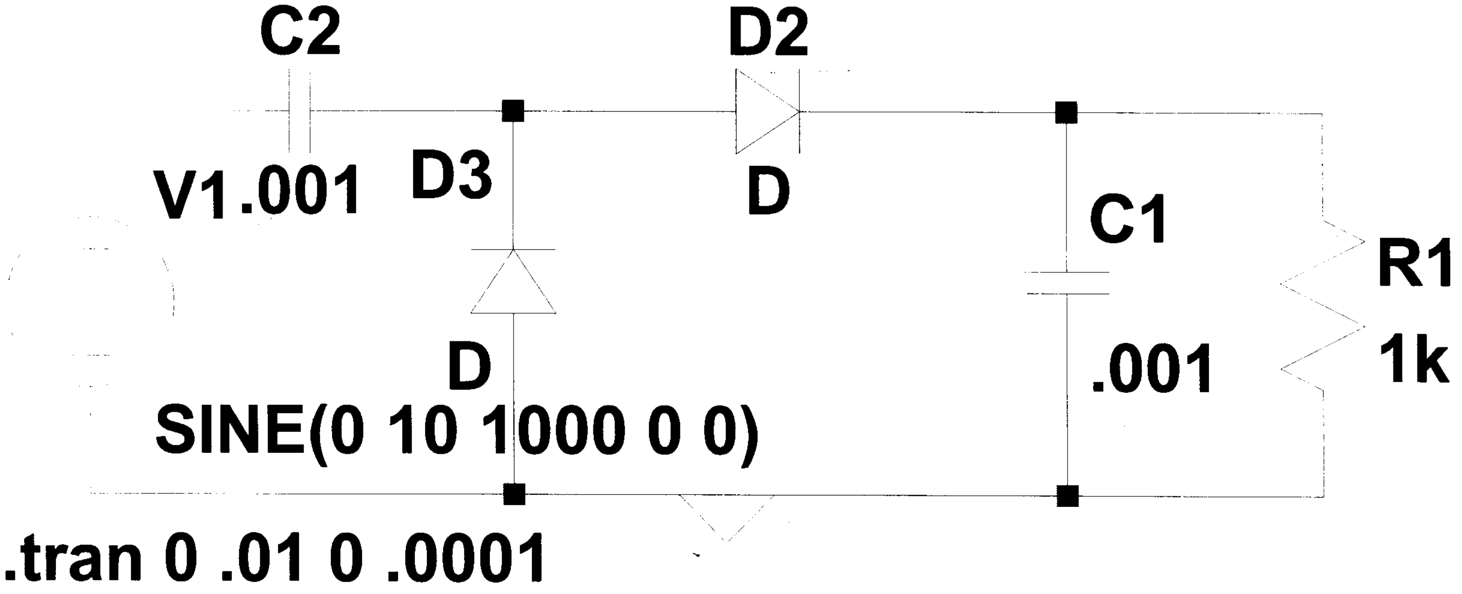

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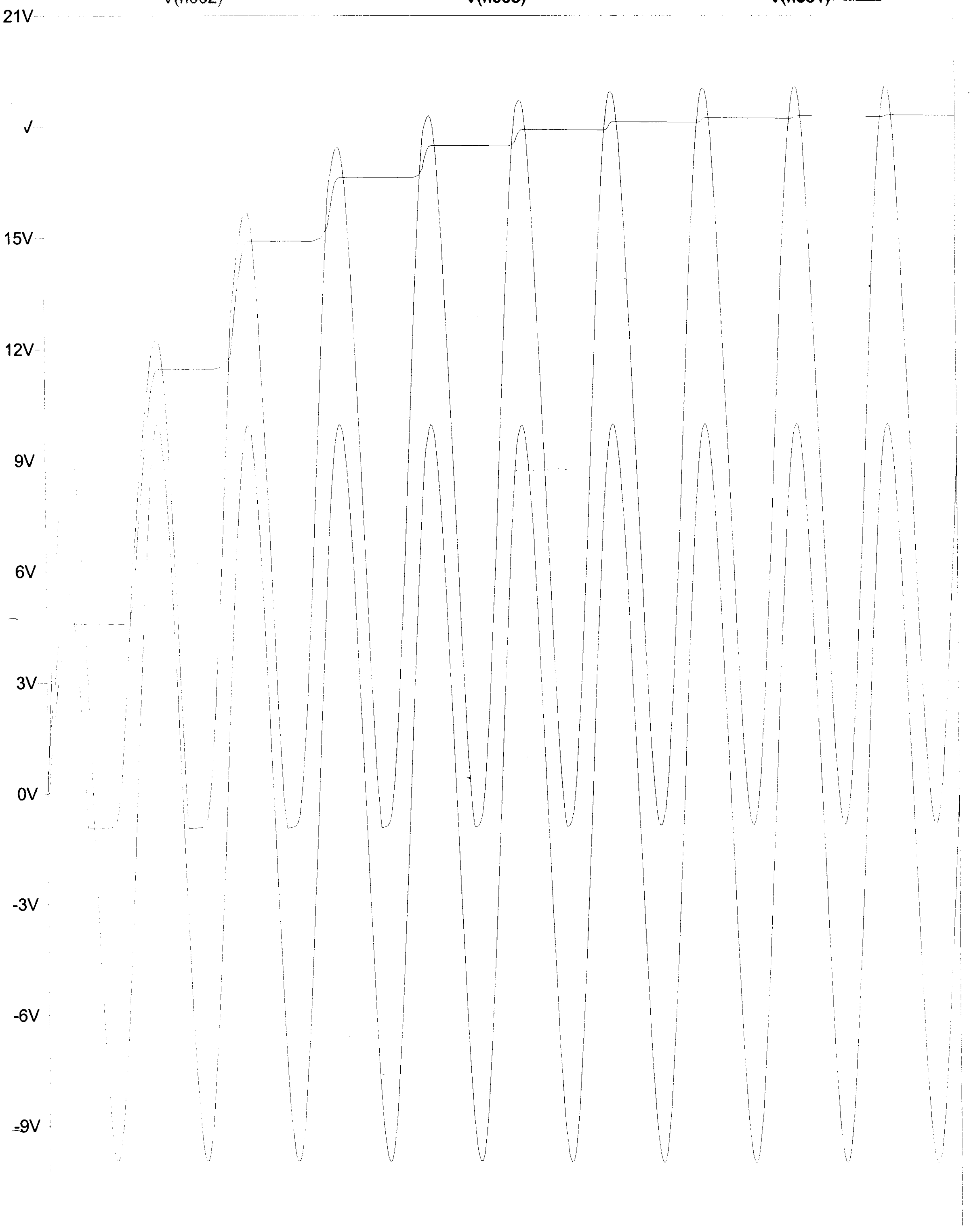

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