T. K. Gustafson Prob Set No. 1 Solutions

Sept 4, 2009

1) Hambley 1.14 $q(t) = (3 - 3e^{-2t})$ Coulombs

$$i(t) = \frac{dq(t)}{dt} = (-3)(-2)e^{-2t} = 6e^{-2t}$$

2) Hambley 1.24

$v(t) = 10\,V$

$i(t) = 2e^{-t}$

Note standard reference

$p = v(t)\,i(t)$ = power into element

$= 20e^{-t}$ watts

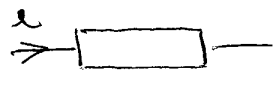
Energy transfered $= \int_0^\infty p(t)\,dt$

$$= \int_0^\infty 20e^{-t}\,dt = \frac{20}{(-1)}\left(e^{-\infty} - 1\right)$$

$= 20$ joules

3) Hambley 1.27

a) $i = +2$ Amps $v = 30$ Volts



vi = power = 60 watts into element

b) $i = -2$ Amps

$vi = -60$ watts i.e. 60 watts delivered by the element

Solution Problem 4 P.S. 1 Hambley 1.30

Battery consumes 50 Watts at 12.6 V

Thus current $= \frac{50}{12.6} = 3.968$ amps

Battery rated at 100 Amp Hrs. Thus it will "run" 1 hr at 100 Amps. A 3.968 amps it will run $\frac{100}{3.968} = \underline{25.2 \text{ hrs}} \rightarrow$

Initial Battery storage is

12.6×100 Volt Amp Hrs

$= 1.26 \times 10^3$ Watt Hrs

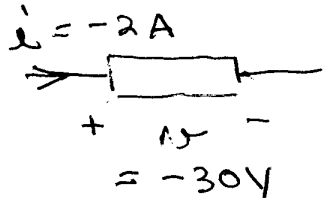
$= \underline{1.26 \text{ kw hrs}} \rightarrow$

Battery costs \$75 and can be charged 300 times. Thus total storage possible is 300×1.26 kw hrs

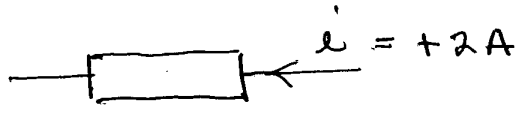
$$\text{Cost} = \frac{75}{300 \times 1.26} = \underline{19.84 \text{ cents/kw hr}}$$

c) $v = -30V$

$i = -2A$



same as



$v = +30V$

Still standard reference

so power = 60 Watts into the element.

5) a) Hambley Problem 1.68 Part c)

Fig (1)

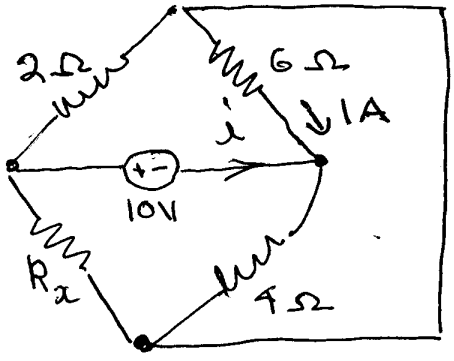


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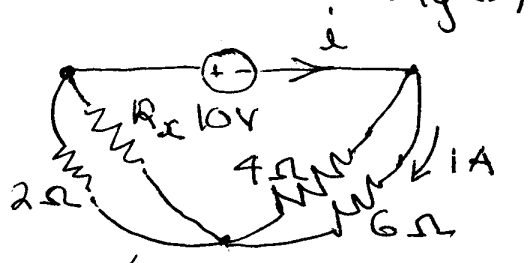
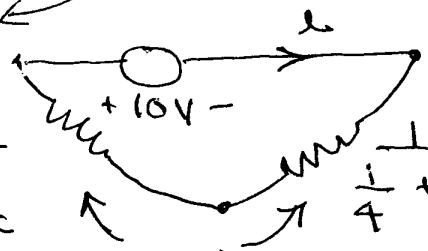
Fig (2)



Combine resistors

Fig (3)



$\frac{1}{\frac{1}{2}+\frac{1}{R_x}}$ $\qquad$ $\frac{1}{\frac{1}{4}+\frac{1}{6}} = \frac{24}{10}$

combine series just add

$$i = \frac{10V}{\frac{2R_x}{R_x+2} + \frac{24}{10}}$$

$$= \frac{10 \times 10 \times (R_x+2)}{20R_x + 24R_x + 48} = \frac{100(R_x+2)}{44R_x+48} \quad (1)$$

But by current division

$$1\,A = i\,\frac{(1/6)}{1/4 + 1/6} = i\,\frac{4}{10}$$

Thus from (1)

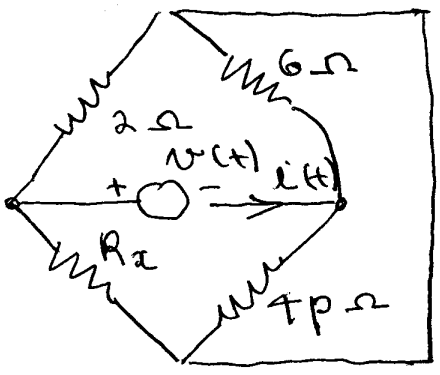
$$\frac{4}{10} + \frac{100\,(R_x + 2)}{44 R_x + 48} = 1$$

$$40 R_x + 80 = 44 R_x + 48$$

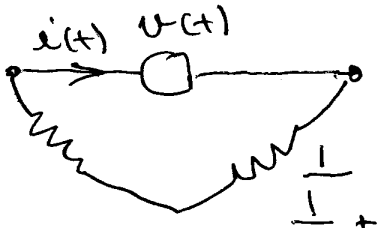
or $$32 = 4 R_x$$

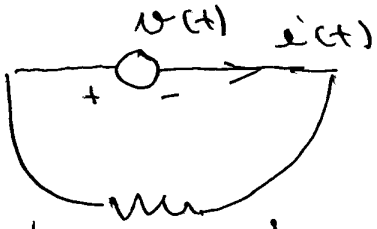
$$R_x = 8\ \Omega$$

b)



Follow the same procedure combining resistors





$$\frac{1}{\frac{1}{2} + \frac{1}{R_x}} + \frac{1}{\frac{1}{4p} + \frac{1}{6}} = \frac{2R_x}{R_x + 2} + \frac{24p}{6 + 4p}$$

$$R_{eq}(p) = \frac{2R_x(6 + 4p) + 24p(R_x + 2)}{(R_x + 2)(6 + 4p)}$$

$$i(t) = \frac{1}{R_{eq}(p)}\ v(t)$$

$$i(t) = \frac{(R_x+2)(6+4p)}{12R_x + p(8R_x + 24R_x + 48)} \quad v(t)$$

clear the denominator

$$[12R_x + (32R_x+48)p]\, i(t) = (6R_x+12+[4R_x + 8]p)\, v(t)$$

To obtain D.E replace p by $\frac{d}{dt}$

$$[12R_x i(t) + (32R_x+48)\frac{di(t)}{dt}] = (6R_x+12)v(t) + (4R_x+8)\frac{dv(t)}{dt}$$

Divide through by 4

$$[3R_x i(t) + (8R_x+12)\frac{di(t)}{dt} = (\tfrac{3}{2}R_x + 3)v(t) + (R_x+2)\frac{dv(t)}{dt}$$

(known)

This is the DE for $i(t)$

6) Hambley 2.22 Part a)

Fig 1

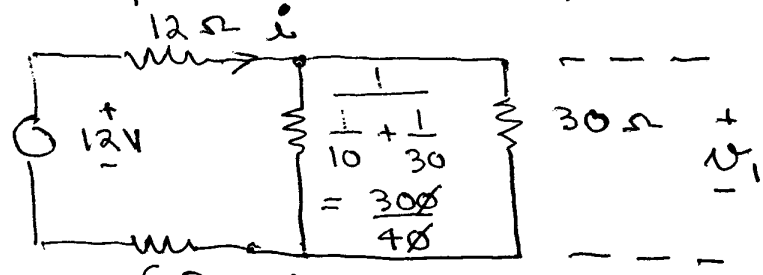


Fig 2

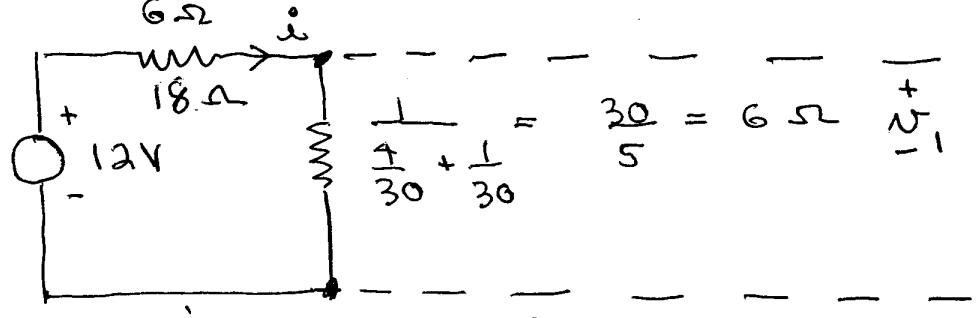
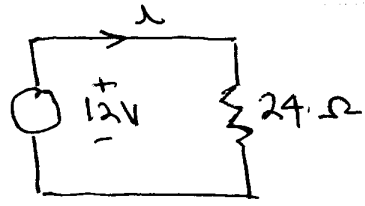


Fig 3



Thus $i = \frac{12}{24}$ Amps

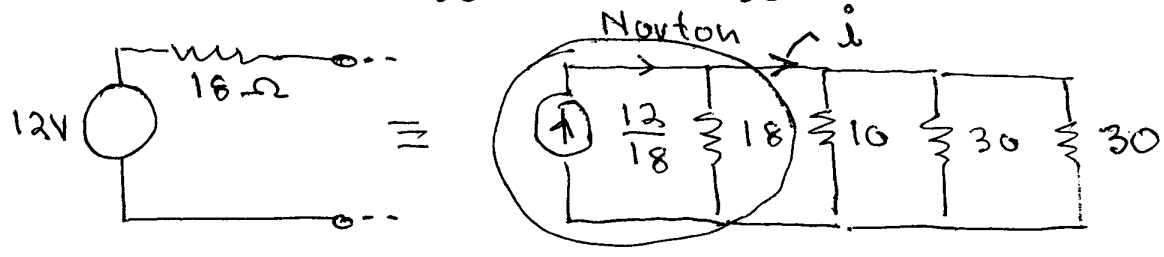
$= .5$ A

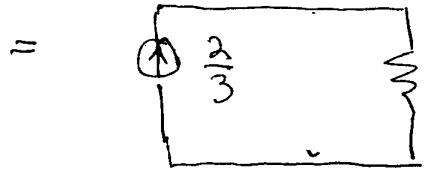
From Fig 2 using voltage division

$$v_1 = \frac{6}{24} \times 12 = 3 \text{ V}$$

From book Fig using voltage division

$$v_2 = \frac{5}{30} \times v_1 = \frac{15}{30} = \frac{1}{2} \text{ V}$$

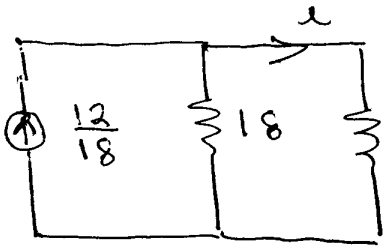




$$\frac{1}{\frac{1}{18} + \frac{1}{10} + \frac{1}{30} + \frac{1}{30}} = \frac{1}{\frac{10}{180} + \frac{18}{180} + \frac{12}{180}}$$

$$= \frac{180}{40} = \frac{18}{4}\ \Omega$$

To unwind this find i by current division



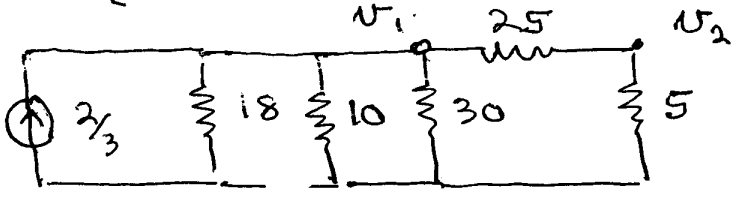
$$\frac{1}{\frac{1}{10}+\frac{2}{30}} = \frac{30}{3+2} = \frac{30}{5} = 6\ \Omega$$

$\therefore\ i = \frac{12}{18} \times \frac{18}{24} = \frac{1}{2}$ A as before

$\therefore\ v_1 = \frac{1}{2} \times 6\ \Omega = 3$ V as before

and v_2 is found as previously

Node Equation Approach



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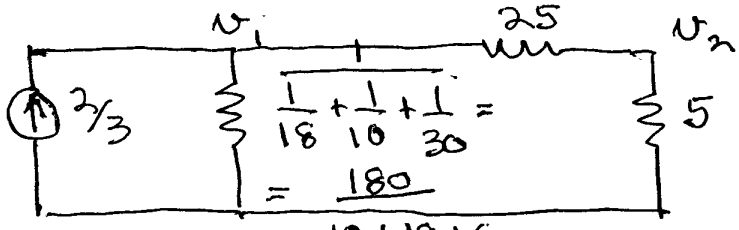

$$= \frac{180}{10+18+6}$$

$$= \left(\frac{180}{34}\right)$$

Node equations

$$\frac{v_1}{\left(\frac{180}{34}\right)} + \frac{(v_1 - v_2)}{25} = 2/3$$

$$\frac{v_2}{5} + \frac{v_2 - v_1}{25} = 0 \rightarrow \text{solve for } v_1$$

$$v_2\left(\frac{6}{25}\right) = \frac{v_1}{25}$$

Substitute for V_1

$$\frac{\cancel{34.6}}{\cancel{180}\ 30} V_2 + \frac{\cancel{8} V_2}{\cancel{25}\ 5} = 2/3$$

$$\frac{40}{30} V_2 = 2/3$$

$$V_2 = \frac{10}{20} = \frac{1}{2} \text{ volt}$$

$$V_1 = 6 V_2 = 3 \text{ Volts}$$

Prob No 7 – The difference internal resistance of the two and over time the storage capacity of the car batt. as discussed in recitations