EECS 219C: Computer-Aided Verification

# Abstraction & Symbolic Model Checking without BDDs

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## Key Optimizations in (Symbolic) Model Checking

- Abstraction
  - Compute a smaller state graph by "merging states" s.t. if the property holds on the smaller system model, it holds on the larger one
- Symmetry Reduction
  - Group states into equivalence classes by exploiting symmetries in the model
- Compositional Reasoning
  - Compose proofs of correctness of modules to prove the overall system correct

# Today's Lecture

- Abstraction
  - Counter-example guided abstraction refinement (CEGAR)
- Symbolic Model Checking without BDDs
  - Uses SAT instead of BDDs
  - Started with Bounded Model Checking
  - Extended to Unbounded Model Checking
    - Abstraction + BMC
    - Interpolation-based model checking



# Abstraction

## Abstraction

- Extracting information from a system description that is relevant to proving a property
- Goal: Reduce size of system model

- Terminology:
  - Original model = Concrete system/model

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## Abstraction (2)

- Reduce the size of the system model by throwing out information / grouping states
  - If this information is irrelevant to the property of interest (i.e., the property is true on the original model iff it is true on the abstract model) then it is a precise abstraction
  - If the property is true on the original model if it is true on the abstract model, it is a safe abstraction

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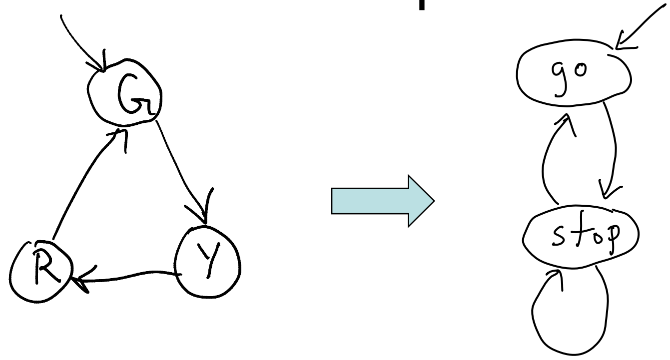
# Example




- Abstractions exhibit more behaviors
- Consider the foll 2 properties on the original model and abstraction:

G(go → X stop)                G F go



# A Simple Form of Abstraction

- Suppose the temporal logic property mentions only a subset of variable V’ of the entire set V
- Can I use this information to construct a precise abstraction of the original model?

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- Suppose the temporal logic property mentions only a subset of variable V' of the entire set V
- Can I use this information to construct a precise abstraction of the original model?
  - YES. One such method is the "cone of influence" reduction.
    - Transitively propagate syntactic dependences on variables and "delete" all variables not in the transitive closure



# Formal Definition

- Abstraction is defined by an abstraction function
- Abstraction function $\alpha : S \rightarrow \hat{S}$
  - S – set of concrete states
  - Ŝ – set of abstract states
- An abstraction induces an equivalence relation over the concrete states
  - Two concrete states are equivalent if they are mapped to the same abstract state

## Formal Definition

- Suppose concrete system is $(S, S_0, R, L)$, and abstract system $(\hat{S}, \hat{S}_0, \hat{R}, \hat{L})$
- Abstraction function $\alpha : S \rightarrow \hat{S}$
  - $S$ – set of concrete states
  - $\hat{S}$ – set of abstract states
- $\hat{S}_0 = \{ t \mid \exists s . S_0(s) \wedge \alpha(s) = t \}$
- $\hat{R}$ = ?
  - How do we algorithmically construct $\hat{S}_0$ and $\hat{R}$ ?
  - How are labels assigned to abstract states?

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## Example of Abstraction

- Our examples in this lecture will be abstractions that extract a subset of state variables
  - State variables partitioned into: visible and invisible
  - An abstract state is an evaluation of visible variables
  - What is $\alpha$ ?
  - Two concrete states that agree on values of visible variables are grouped together

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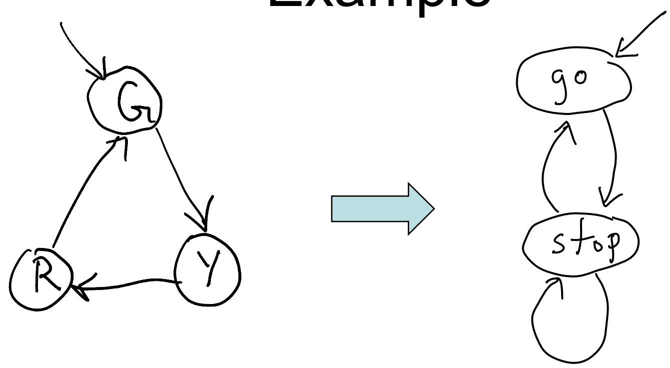
# Example




- Abstractions exhibit more behaviors

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# Abstraction and Properties

- If an LTL property is true on the abstract model, is it necessarily true on the concrete model?
- If an LTL property is false on the abstract model, is it necessarily false on the concrete model?

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# Cone-of-influence

- Suppose the property $\phi$ mentions a subset of variables V' of the total set V
  - Track variables that V' syntactically depend on, add them to V', and iterate until no new variable dependencies generated
  - Resulting V' is the cone-of-influence and its elements are the visible variables
- Problem: Final V' might be as big as V because it only tracks *syntactic* dependencies
  - But resulting abstraction is precise → if $\phi$ is false in abstract model it is false in concrete model

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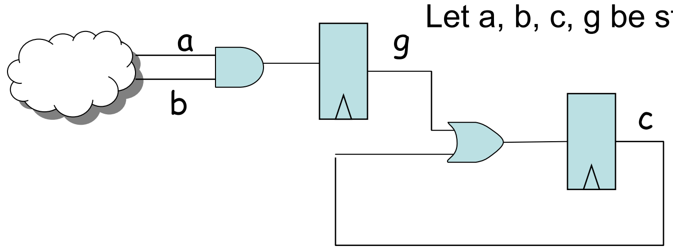
# Example: Cone-of-influence can be conservative

Let a, b, c, g be state variables




What are the expressions for next state variables c' and g' ?

Suppose we want to prove G(c → Xc) . What's the cone of influence?

If we make g invisible, can we still prove the property?
• what about a and b?

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## Another approach to Abstraction

- Start with an *arbitrary* subset of variables as visible
  - An option: the ones mentioned in the property
- Construct abstract model, model check it
  - If property passes, we're done
  - If we get a counterexample, check whether it is a counterexample for the concrete model
    - If yes, we're done
    - If not (spurious counterex.) we must make more variables visible and repeat (REFINEMENT)



## Counter-Example Guided Abstraction-Refinement (CEGAR)

[R. Kurshan, E. Clarke et al.]

- Start with a choice of $\alpha$
- Construct abstract model, model check it
  - If property passes, we're done
  - If we get a counterexample, check whether it's is a counterexample for the concrete model (How do we do this?)
    - If yes, we're done
    - If not (spurious counterex.), we must refine $\alpha$ and repeat

## Intuition about Refinement

- Remember that $\alpha$ partitions the concrete states into equivalence classes
  - $C_1, C_2, \ldots, C_k$
- A refinement $\alpha'$ can further break up the $C_i$'s
  - States that are equivalent under $\alpha'$ should also be equivalent under $\alpha$



## Formal Definition of Refinement

- $\alpha'$ refines $\alpha$ if
  - $\forall s, t \, . \, \alpha'(s) = \alpha'(t) \Rightarrow \alpha(s) = \alpha(t)$
  - $\exists s, t \, . \, \alpha'(s) \neq \alpha'(t) \wedge \alpha(s) = \alpha(t)$
- Given above definition, why will the CEGAR iteration terminate?

## Visible/Invisible Abstraction

- The set of variables is partitioned into visible $V$ and invisible $I$
- Questions:
  - How do we construct the abstract model?
    - Given an arbitrary set of visible variables
  - How do we refine the abstraction?
    - i.e., how do we pick new variables to make visible?
    - We want to pick those that will remove the current spurious counterexample

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## Constructing Abstract Model

- Simply make all invisible variables take arbitrary values
  - Non-deterministically assigned 0 or 1 on each step
- How does this make model checking more efficient?

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## Constructing Abstract Model

- Simply make all invisible variables take arbitrary values
  - Non-deterministically assigned 0 or 1 on each step
- How does this make model checking more efficient?
  - Avoids some existential quantification, simplifies transition relation



## Refining the Abstraction

- The CEGAR approach is most often used today in conjunction with a technique called Bounded Model Checking
- We will study abstraction-refinement in that context

# Bounded Model Checking (BMC)

[Biere, Clarke, Cimatti, Zhu, '99]

- Given
  - A FSM M described by $S_0$, R
  - A property G p and a integer $k \geq 1$
- Determine
  - Does M generate a counterexample to G p of length *k transitions or fewer*?

This problem can be translated to a SAT problem. How?

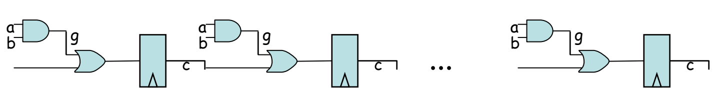


# Unfolding in BMC

- Unfold the model k times:

$$U_k = R_0 \wedge R_1 \wedge \ldots \wedge R_{k-1}$$

$S_0$



$E_k = \neg p$

- Use SAT solver to check satisfiability of

$$S_0 \wedge U_k \wedge E_k$$

- A satisfying assignment is a counterexample of k steps

# Old view on BMC

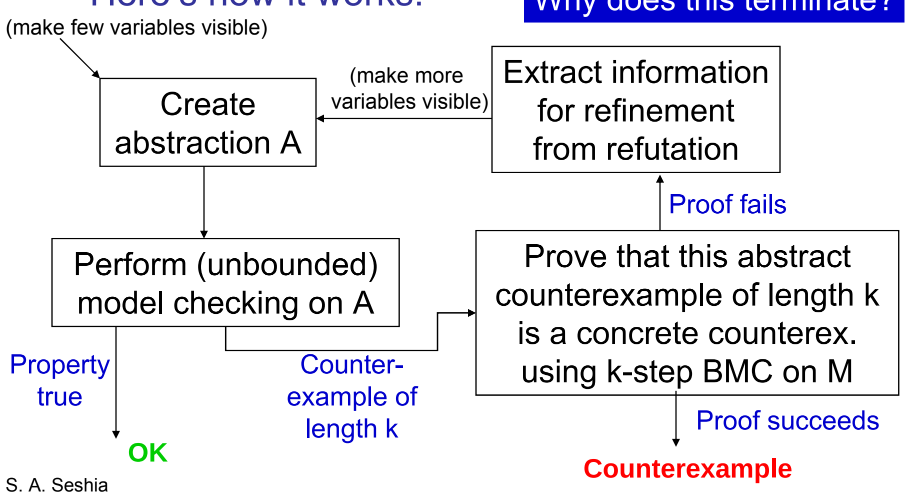
- Originally introduced as a debugging tool
  – By finding counterexamples
- Proving properties:
  – Only possible if a bound on the diameter of the state graph is known
    - The diameter is the maximum over shortest path lengths between any two states.
  – Worst case is exponential in system description.



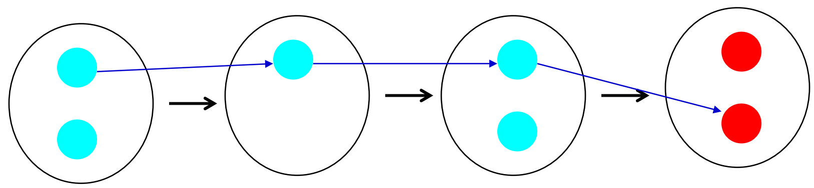
# BMC + CEGAR

- BMC + Abstraction can prove properties too!
- Here's how it works:

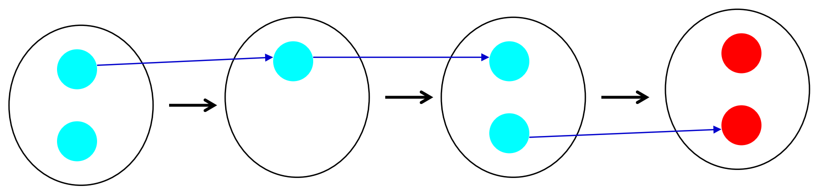
Why does this terminate?



# Abstract/Concrete Error Trace



Abstract trace OK



Abstract trace spurious

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# Steps

1. Create abstraction A ✓
2. Model check A ✓
3. Prove that abstract counterexample is a concrete counterexample using BMC
4. Use refutation of abstract counterexample to do refinement

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## Checking Abstract Counterex.

- Recall: BMC for length k
  - Use SAT solver to check satisfiability of

    $S_0 \wedge U_k \wedge E_k$
- How do we use this to prove the abstract counterexample of length k also holds for concrete model?



## Checking Abstract Counterex.

- Recall: we use BMC for the length k of the abstract counterexample
  - Use SAT solver to check satisfiability of

    $S_0 \wedge U_k \wedge E_k$

    under the partial assignment corresponding to values of the visible variables
  - If SAT solver reports "SAT" we have a concrete counterexample
    - What is a satisfying assignment?
  - If not, we have a refutation ← proof of unsatisfiability

## Refinement

- Given proof of unsatisfiability of

  $S_0 \wedge U_k \wedge E_k$

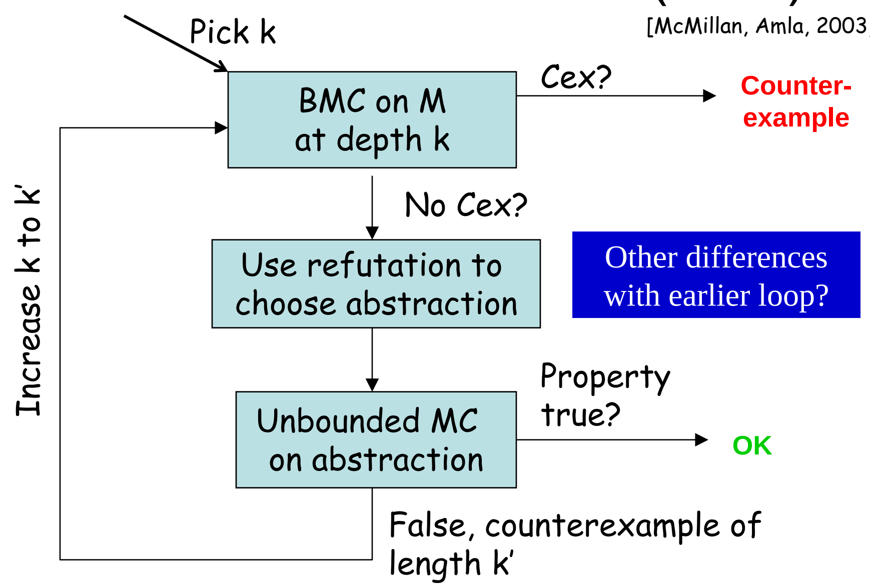
  under the partial assignment corresponding to values of the visible variables
- Look at unsatisfiable core of proof
  - Invisible variables that appear in the core indicate why the abstract counterexample is spurious
  - Make those variables visible



## Modifying the Abstraction-Refinement Loop

- Insight: Why pick an abstraction to start with?
  - Initial abstraction may not be the best start point
  - Why not do BMC initially and use its results to generate abstractions?

# Proof-based Abstraction (PBA)






# Termination of PBA

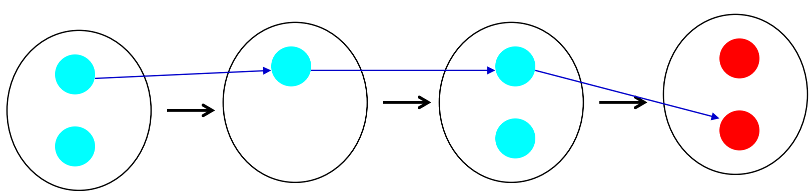
- Depth k increases at each iteration
- Eventually k > diameter d
- If k > d, no counterexample is possible

## CEGAR vs. PBA

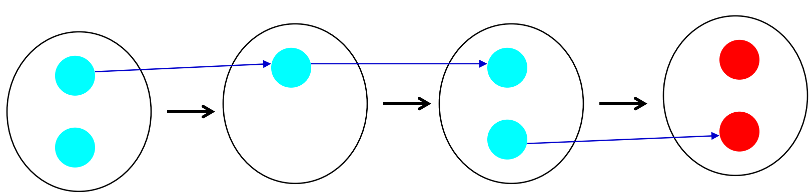
- Refutation via k-step BMC
  - PBA refutes all concrete counterexamples of up to length k
  - CEGAR refutes only the abstract counterexample of length k
- So PBA does more work in the refutation, but usually results in fewer iterations of the loop



## Abstract/Concrete Error Trace



Abstract trace OK



Abstract trace spurious

## Abstraction and Reachability

- An abstraction expands the set of states reachable from the initial state
  - OVER-APPROXIMATION
- Instead of starting by abstracting states, one can *directly abstract the transition relation*
  - Each time you compute the set of next states, you get an over-approximation of the actual set of next states
  - Gives a way of computing an over-approximation of the set of reachable states



## Abstraction using Interpolation

- Abstraction is extracting sufficient/relevant information from a system *to prove a given property*.
- This notion is in some sense closely related to a notion of "interpolant" and a lemma called "Craig's interpolation lemma"

## Interpolation Lemma (Craig, 57)

- If $A \wedge B$ = false, there exists an *interpolant* A' for (A,B) such that:
  (i) $A \Rightarrow A'$
  (ii) $A' \wedge B$ = false
  (iii) A' refers only to common variables of A,B

- Example:
  - $A = p \wedge q$, $B = \neg q \wedge r$, $A' = q$

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## Interpolants from Proofs

(Pudlak,Krajicek,97)

- Interpolant A' for $A \wedge B$:

  $A \Rightarrow A'$

  $A' \wedge B$ = false

  A' refers only to common variables of A,B

- Interpolants can be obtained from proofs
  - given a resolution-based refutation (proof of unsatisfiability) of $A \wedge B$,

    **A' can be derived in time linear in the proof**

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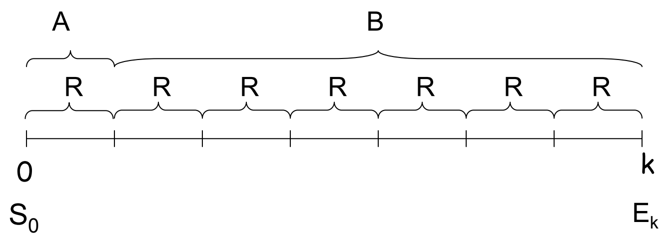
# Interpolation based Model Checking (McMillan, 2003)

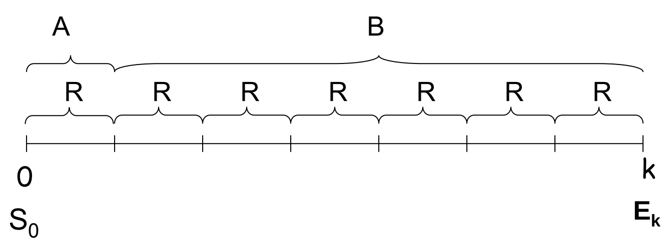
- Main Idea: Pose the problem of over-approximating the set of next states as finding an interpolant




$S_0(v_0) \wedge R(v_0, v_1) \wedge R(v_1, v_2) \wedge \ldots \wedge R(v_{k-1}, v_k) \wedge E_k(v_k)$



# Interpolation based Model Checking




$S_0(v_0) \wedge R(v_0, v_1) \wedge R(v_1, v_2) \wedge \ldots \wedge R(v_{k-1}, v_k) \wedge E_k(v_k)$

$A = S_0(v_0) \wedge R(v_0, v_1)$

$B = R(v_1, v_2) \wedge \ldots \wedge R(v_{k-1}, v_k) \wedge E_k(v_k)$

A' is a function of $v_1$ s.t.
1. A ➔ A'
2. A' $\wedge$ B is unsat

What set of states does A' represent?

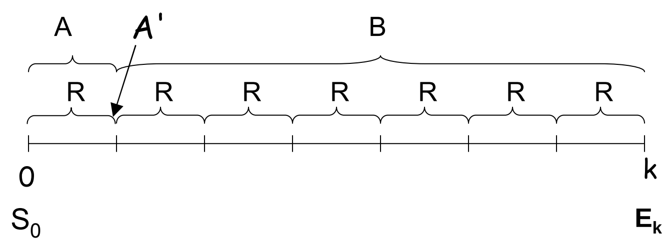
# Interpolation based MC

For a fixed k:

1. Set Z initially to $S_0$
2. Do BMC starting from Z for k steps
   - If SAT: have we found a counterexample?
   - If UNSAT, continue
3. Use interpolation to compute over-approximation of next states of Z and add them back into Z
   – Can newly added states lead to error states in k-1 steps? In k steps?
4. If Z does not increase
   – We've reached a fixed point Z=P. Is the property true?
5. Otherwise, back to step 2



# Intuition




- A' tells us everything the prover deduced about the image of $S_0$ in proving it can't reach an error in k steps.
- Hence, A' is in some sense an abstraction of the image relative to the property *and* the bound k

**The fixed point P is an inductive invariant**

# Inductive Invariant P

- P is true in the initial state
  - $S_0 \Rightarrow P$
- R is maintained by the transition relation
  - $P(s) \wedge R(s,s') \Rightarrow P(s')$
- In other words: every reachable state satisfies P
- The system is deemed to be correct if $P \wedge E$ is UNSAT.



# Refinement

- The procedure may be inconclusive for a fixed k
  - May add a state that reaches error in k steps (getting SAT in step 2 with Z != $S_0$)
- Refinement is just increasing k
  - How big can k get?