EECS 219C: Computer-Aided Verification

# Symmetry Reduction, Simulation/Bisimulation

Sanjit A. Seshia
EECS, UC Berkeley

## Today's Lecture

- Optimizations in Model checking
  - Symmetry Reduction
- Simulation/Bisimulation

# Simulation and Bisimulation



## Simulation --- Intuition

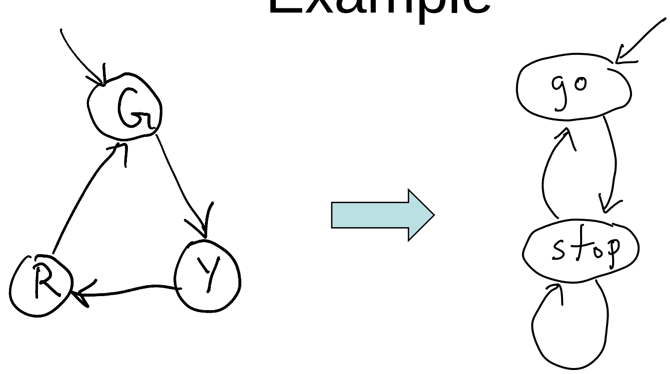
- Two finite state machines (Kripke structures) M and M’
- M’ simulates M if
  - M’ can start in a similarly labeled state as M
  - For every step that M takes from s to t, M’ can mimic it by stepping to a state with similar label as t

## Simulation

- $M = (S, S_0, R, L)$ and $M' = (S', S_0', R', L')$
- A relation $H \subseteq S \times S'$ is a simulation relation between M and M' means that:
  For all (s, s'), if H(s, s') then:
  - $L'(s') = L(s) \cap AP'$
  - For every state t s.t. R(s, t) there is a state t' such that R'(s', t') and H(t, t')
- M' simulates M if
  - there exists a simulation relation H between them, and
  - For each $s_0 \in S_0$, there exists $s_0' \in S_0'$ s.t. $H(s_0, s_0')$



## Example




Atomic propositions: go and stop

Which machine simulates which?

## Bisimulation

- M and M' are bisimulation equivalent (bisimilar) if
  - M can match each step of M' and vice-versa
  - Note: this is NOT the same as "M simulates M' and M' simulates M"
- A relation $H \subseteq S \times S'$ is a bisimulation relation between M and M' means that:
  For all (s, s'), if H(s, s') then:
  - $L'(s') = L(s) \cap AP'$
  - For every state t s.t. R(s, t) there is a state t' such that R'(s', t') and H(t, t')
  - For every state t' s.t. R'(s', t') there is a state t such that R(s, t) and H(t, t')



## (Bi)Simulation and (A)CTL*

- If M' simulates M, then any ACTL* property satisfied by M' is satisfied by M
- If M' and M are bisimilar, any CTL* property satisfied by one is also satisfied by the other

## Symmetry Reduction



## Symmetry

- Many systems have inherent symmetry
  - Overall system might be composed of k identical modules
  - E.g., a multi-processor system with k processors
  - E.g., a multi-threaded program with k threads executing the same code with same inputs
  - Anything with replicated structure
- Question: How can we *detect* and *exploit* the symmetry in the underlying state space for model checking?

# Symmetry in Behavior

- Given a system with two identical modules
  - Run: $s_0, s_1, s_2, \ldots$
  - Trace: $L(s_0), L(s_1), L(s_2), \ldots$
  - Each $s_i = (s_{i1}, s_{i2}, \text{rest})$ comprises *values to variables* of both modules 1 and 2
  - If we can interchange these without changing the set of traces of the overall system, then there is symmetry in the system behavior



# Exploiting Symmetry

- If a state space is symmetric, we can group states into equivalence classes
  - Just as in abstraction
- Resulting state graph/space is called "quotient" graph/space
  - Model check this quotient graph

## Quotient (first attempt)

$M = (S, S_0, R, L)$

Let $\cong$ be an equivalence relation on S

Assume: $s \cong t$ iff $L(s) = L(t)$

$\quad\quad\quad\quad$ & $s \in S_0$ iff $t \in S_0$

Quotient: $M' = ( S', S_0', R', L' )$

- $S' = S/\cong$ , $S_0' = S_0/\cong$ (states are equivalence classes with respect to $\cong$)
- $R'([s], [t])$ whenever $R(s,t)$
- $L'([s]) = L(s)$



## Is that definition enough?

Suppose we want to check an invariant:

Does M satisfy $\varphi$ ?

Instead if we check:

Does quotient M' satisfy $\varphi$ ?

If M' is constructed using the definition of $\cong$ on the previous slide, will the above check generate spurious counterexamples?

## Stable Equivalences

Equivalence $\cong$ is called stable if:

R (x, y) $\Rightarrow$

for every s in [x]

there exists some t in [y] such that R (s,t)

Claim: Suppose $\cong$ is stable, then:

M satisfies $\varphi$ iff M' satisfies $\varphi$

(Proof idea: show M and M' are bisimilar)



## Detecting Symmetry

- Given symmetry expressed as an equivalence relation between states, we know how to exploit it
- How do we detect/compute this equivalence relation?
  - Need to characterize it more formally

## Symmetry as Permutation

- Symmetry in the state space can be viewed as “equivalence under permutation”
- Permute the set of states so that the set of traces remains the same
  - A subset of states that remains the same under permutation forms the needed equivalence class
- A representation of all possible such permutations represents symmetry in the system



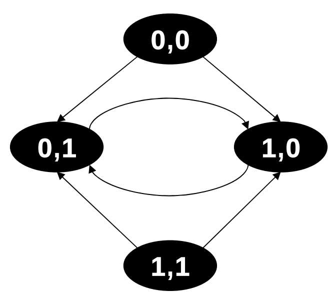
## Automorphisms

A permutation function

$f : S \rightarrow S$

is an automorphism if:

$R(s, t) \Leftrightarrow R(f(s), f(t))$

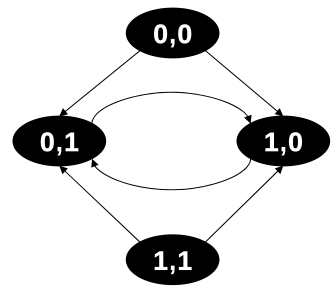



What is an example automorphism for this state space?

## Automorphisms

f: f(0,0) = 1,1 f(1,1) = 0,0
f(0,1) = 0,1 f(1,0) = 1,0

g: g(0,0) = 0,0 g(1,1) = 1,1
g(0,1) = 1,0 g(1,0) = 0,1




$A = \{ f, \ g, \ f \circ g, \ id\}$

The set of all automorphisms forms a group!



## Equivalence using Automorphisms

Let $s \cong t$

if there is some automorphism f such that
$f(s) = t$ (and $L(s) = L(t) \wedge s \in S_0$ iff $t \in S_0$)

The equivalence classes of an automorphism (sets mapped to themselves) are called orbits

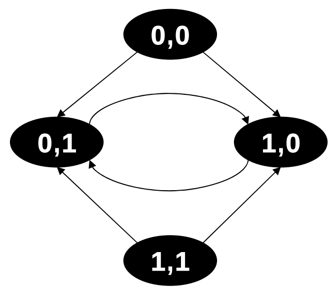
Claim 1: $\cong$ is an equivalence

Claim 2: $\cong$ is stable (why? - HW)

# Orbits

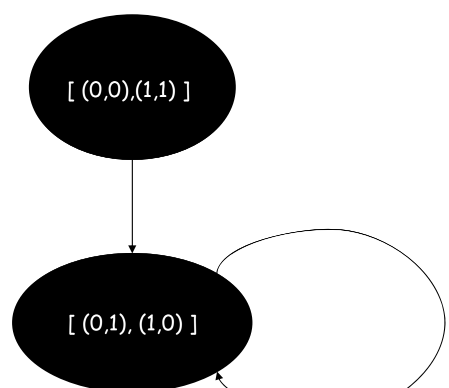
[ (0,0), (1,1) ]

[ (0,1), (1,0) ]






# Symmetry reduction




Map each state to its representative in the orbit

## How Symmetry Reduction works in practice

- A permutation (automorphism) group is *manually* constructed
  - Syntactically specify which modules are identical
- Orbit relation (equivalence relation) automatically generated from this
  - Using fixpoint computation (MC, Sec. 14.3)
- An (lexicographically smallest) element of each equivalence class is picked as its representative
- $S_0$' and R' generated from orbit relation
- Model checking explores only representative states



## Symmetry reduction

- Implemented in many model checkers
  - E.g., SMV, Mur$\varphi$ (finite-state systems), Brutus (security protocols)