EECS 219C: Computer-Aided Verification

Symbolic Model Checking, Optimizations

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Today's Lecture

- Symbolic model checking with BDDs
  - Fairness
  - Counterexample/witness generation for general CTL
- Optimizations in Model checking
  - Abstraction (mostly next week)
  - Symmetry Reduction
  - Compositional Reasoning
- Simulation/Bisimulation

Fairness

- A computation path is defined as fair if a fairness constraint p is true infinitely often along that path
  - Fairness constraint is a state predicate
  - Generalized to set of fairness constraints $\{p_1, p_2, \ldots, p_k\}$ by requiring each element of the subset to be true infinitely often
- Example: Every process in an asynchronous composition must be scheduled infinitely often

Why does Fairness matter?

- We need to model policies that enforce fairness in the model
  - Otherwise, we will get spurious counterexamples
  - Example: A scheduler might use round-robin scheduling amongst processes
    - Instead of verifying the system for a particular fixed fair scheduling strategy, we can verify it for all fair schedulers

Fairness in Symbolic Model Checking of CTL

- Suppose Fairness means that each element of $\{p_1, p_2, \ldots, p_k\}$ must be true infinitely often
- Fair formulation of EG f is: The largest set of states Z such that
  - All of the states in Z satisfy f
  - For all fairness constraints p_i, and all states $s \in Z$, there is a path of length 1 or greater from s to a state in Z satisfying p_i such that all states along that path satisfy f

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  - For all fairness constraints p_i, and all states $s \in Z$,
    - there is a path of length 1 or greater from s to a state in Z satisfying p_i such that all states along that path satisfy f
    - i.e., there is a next state of s satisfying f U $(Z \wedge p_i)$
  - What's the fixpoint formulation of EG f with fairness? For EGf: $\nu Z. [f \wedge \text{EX } Z]$

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  - For all fairness constraints p_i, and all states $s \in Z$,
    - there is a path of length 1 or greater from s to a state in Z satisfying p_i such that all states along that path satisfy f
    - i.e., there is a next state of s satisfying $f \; U \; (Z \wedge p_i)$
  - $\nu Z. \; f \wedge \left(\bigwedge_i EX \; E[\, f \; U \; (Z \wedge p_i)] \right)$

Counterexample Generation under Fairness

- Algorithm needs to be adjusted accordingly
  - Need to find a cycle that visits each fairness constraint p_i at least once
  - See Clarke et al. textbook for details

BDD-related Optimizations – Key Ideas

- Choose a good BDD variable ordering to start with
- Keep the support of computed BDDs as small as possible

What do we need to represent?

- Set of transitions: R(v, v')
- Sets of states: $S_0(v)$, intermediate results of fixpoint computations

Representing R(v, v')

- How should the v and v' variables be ordered in the BDD relative to each other?
- Keep v_i close to v_i' (interleave)

Relational Product

- Recall that reachability analysis involved computing

$S_{i+1}(v) = S_i(v) \vee (\exists v \{ S_i(v) \wedge R(v,v') \}) [v/v']$

- Relational Product operation is
  $\exists v \{ S_i(v) \wedge R(v,v') \}$
- This is done as one primitive BDD operation
  – Rather than an AND followed by EXISTS (why?)

Disjunctive Partitioning

- Suppose we have an asynchronous system composed of k processes
- Then, $R(v, v')$ can be decomposed as

  $\bigvee_i \; R_i(v, v')$

  – Plug into expression for relational product

  – Does $\exists$ distribute over $\vee$? What use is that?

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Conjunctive Partitioning

- Suppose we have an synchronous system composed of k processes
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  – Can we do the same optimization as on the previous slide? If not, is a similar optimization possible?

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Conjunctive Partitioning

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  – Can we do the same optimization as on the previous slide? If not, is a similar optimization possible?
    - We can choose an order in which to quantify out variables and push the quantifiers as far in as possible
    - What order do we pick?

Key Optimizations in (Symbolic) Model Checking

- Abstraction
  – Compute a smaller state graph by "merging states" s.t. if the property holds on the smaller system model, it holds on the larger one
- Symmetry Reduction
  – Group states into equivalence classes by exploiting symmetries in the model
- Compositional Reasoning
  – Compose proofs of correctness of modules to prove the overall system correct

Abstraction

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Abstraction

- Reduce the size of the system model by throwing out information / grouping states
  - If this information is irrelevant to the property of interest (i.e., the property is true on the original model iff it is true on the abstract model) then it is a precise abstraction
  - If the property is true on the original model if it is true on the abstract model, it is a safe abstraction

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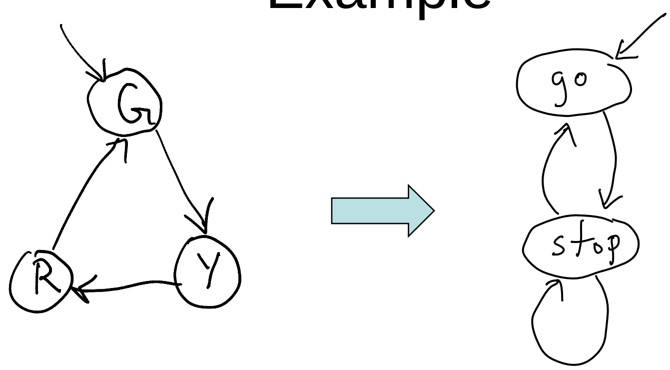
Example



- Abstractions exhibit more behaviors
- Consider the foll 2 properties on the original model and abstraction:

G(go → X stop) G F go

A Simple Form of Abstraction

- Suppose the temporal logic property mentions only a subset of variable V' of the entire set V
- Can I use this information to construct a precise abstraction of the original model?

A Simple Form of Abstraction

- Suppose the temporal logic property mentions only a subset of variable V' of the entire set V
- Can I use this information to construct a precise abstraction of the original model?
  - YES. One such method is the "cone of influence" reduction.
    - Transitively propagate syntactic dependences on variables and "delete" all variables not in the transitive closure

Cone-of-Influence Reduction

- A staple part of all model checkers
- However: often most of the variables remain in the cone-of-influence
  - Need further abstraction
  - Will be covered in class next week