EECS 219C: Computer-Aided Verification

Properties as Automata and Explicit-State Model Checking

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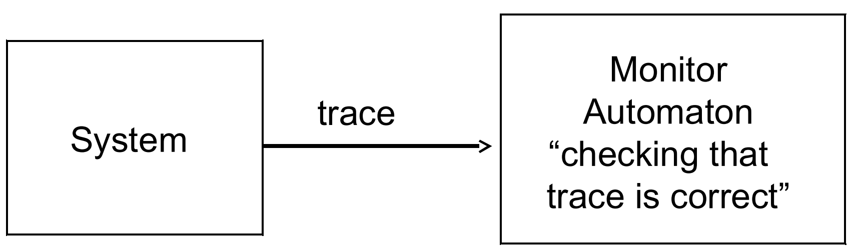
Mental Picture



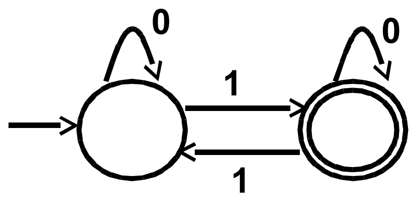
Recap: Automata over Finite Traces

- (Regular) Finite automaton with accepting states
  - All finite traces (words) that take the automaton into the accepting state are "in its language"
- But behaviors (and traces) are infinite length
  - So we need a new notion of acceptance

Automata over Infinite Traces

- What does "Accept" mean?
  - Certain states of the automaton are called "accepting states"
  - The trace must visit an (any) accepting state infinitely often
- Such automata are called Büchi automata
  - Also Omega-automata (written ω-automata)

Example from Class



Language of the automaton = all finite-length binary strings with an odd number of 1s

Reg. expr.: 0*1 (0 + 10*1)*

If you interpret it as a Büchi automaton over infinite words: all infinite-length binary strings with an odd parity of 1s or infinitely many 1s

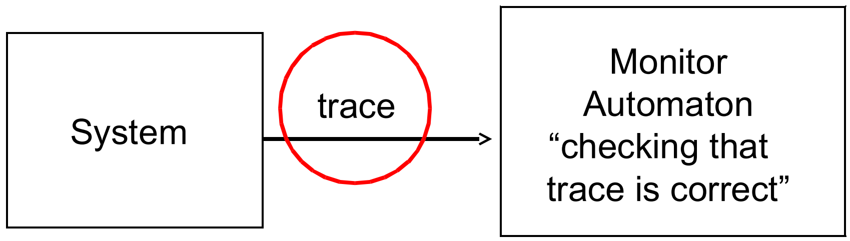
w-regular expr: 0*1 (0 + 10*1)^{w}

Infinitely many repetitions

From Temporal Logic to Monitors

- A monitor for a temporal logic formula
  - is a finite automaton
  - Accepts exactly those behaviors that satisfy the temporal logic formula
    - "Accepts" means that an accepting state is visited infinitely often
- Properties are often specified as automata

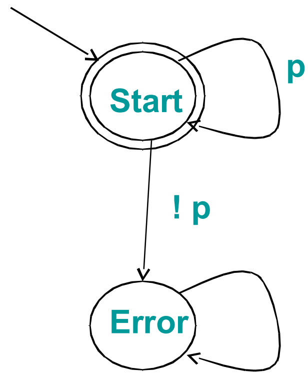
Mental Picture



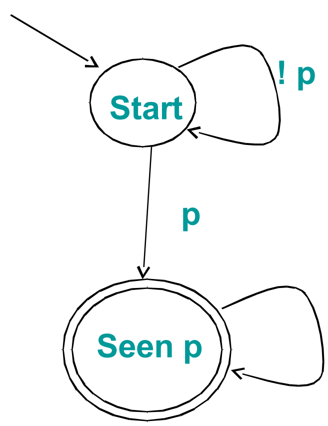
Summary

- A (Büchi) automaton corresponding to a temporal logic formula ϕ *accepts* exactly those traces that satisfy ϕ

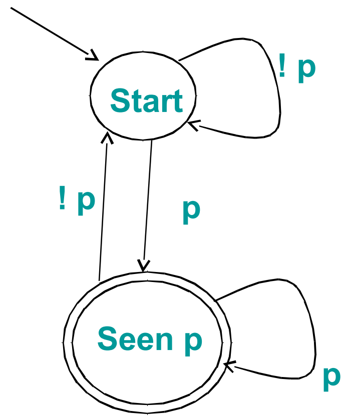
Automaton for G p, p a Boolean formula



Automaton for F p



Automaton for GFp



From LTL to Automata (1)

- Any LTL formula can be translated to a corresponding automaton
- There are many translation algorithms
  - We discus the classical tableaux-based one
  - Reference: Rob Gerth, Doron Peled, Moshe Y. Vardi, Pierre Wolper: *Simple on-the-fly automatic verification of linear temporal logic.* PSTV 1995: 3-18

From LTL to Automata (2)

- The tableaux-based algorithm has three steps:
  - Translating the LTL formula into negation normal form (NNF)
  - Translating NNF to a generalized Büchi automaton
  - Degeneralizing the gen. Büchi automaton

LTL Negation normal form

- Idea: all negations can be pushed inwards

$$
\begin{aligned}
\neg(\psi \vee \phi) &\equiv (\neg\psi) \wedge (\neg\phi) \\
\neg(\psi \wedge \phi) &\equiv (\neg\psi) \vee (\neg\phi) \\
\neg \mathsf{G}\psi &\equiv \mathsf{F}\neg\psi \\
\neg \mathsf{F}\psi &\equiv \mathsf{G}\neg\psi \\
\neg \mathsf{X}\psi &\equiv \mathsf{X}\neg\psi \\
\neg(\psi \,\mathsf{U}\, \phi) &\equiv (\neg\psi)\,\mathsf{R}\,(\neg\phi) \\
\neg(\psi \,\mathsf{R}\, \phi) &\equiv (\neg\psi)\,\mathsf{U}\,(\neg\phi)
\end{aligned}
$$

Consider the LTL formula $\neg((\mathsf{G}\,p)\,\mathsf{U}\,\neg q) \vee \neg(\mathsf{F}\neg(p\,\mathsf{U}\,q))$ for the atomic proposition set $\{p, q\}$. We can rewrite this formula into negation normal form as follows:

$$
\begin{aligned}
& \neg((\mathsf{G}\,p)\,\mathsf{U}\,\neg q) \vee \neg(\mathsf{F}\neg(p\,\mathsf{U}\,q)) \\
\equiv\; & ((\neg\mathsf{G}\,p)\,\mathsf{R}\,\neg\neg q) \vee \mathsf{G}(\neg\neg(p\,\mathsf{U}\,q)) \\
\equiv\; & ((\mathsf{F}\,\neg p)\,\mathsf{R}\,q) \vee \mathsf{G}(p\,\mathsf{U}\,q)
\end{aligned}
$$

Generalized Büchi acceptance

- Idea: We have multiple sets of accepting states, and **all** sets have to be visited infinitely often for the automata to accept a word.

Tableaux construction

- Main Idea:

$$\begin{aligned} \psi \,\mathsf{U}\, \phi &\equiv \phi \vee (\psi \wedge \mathsf{X}\,(\psi \,\mathsf{U}\, \phi)) \\ \psi \,\mathsf{R}\, \phi &\equiv (\psi \wedge \phi) \vee (\phi \wedge \mathsf{X}\,(\psi \,\mathsf{R}\, \phi)) \end{aligned}$$

- Complexity?

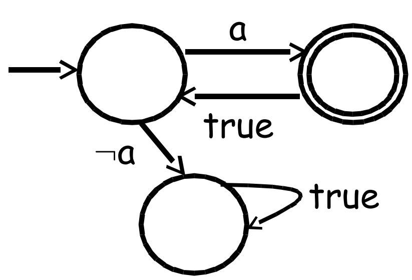
From Automata to LTL

- Any LTL formula can be translated to a corresponding automaton
- How about the other way around?
  - Can an arbitrary Büchi automaton be translated into an LTL formula?

Automaton without LTL counterpart

Automata are more expressive than LTL

What traces does the automaton below accept?



Claim: This cannot be expressed in LTL.

(How about $a \wedge G\,(a \Rightarrow X\,X\,a)$?)